A 3-phase 4-pole 50 Hz 2,200 V 1,870 kW star-connected synchronous motor has a synchronous impedance of $(0{\cdot}06 + j0{\cdot}6\,\Omega)$/phase. The motor is to be run in parallel with an inductive load of 1,000 kVA having a power factor of 0·707 lagging, and is to be so excited that the power factor of the combined loads is 0·9 lagging.

If the motor output is 1,870 kW and its efficiency is 0·9 p.u., determine (*a*) the kVAr, kW and kVA input to the motor, (*b*) the input current and power factor to the motor, (*c*) the load angle of the motor in mechanical degrees, and (*d*) the field current of the motor.

*Ans.* 64·8 kVAr; 2,080 kW; 2,170 kVA; 570 A; 0·955 lagging; 7·78 mechanical degrees.

**12.4** A 2,200 V 3-phase star-connected synchronous motor has a resistance of $0{\cdot}6\,\Omega$/phase and a synchronous reactance of $6\,\Omega$/phase. Find graphically or otherwise the generated e.m.f. and the angular retardation of the rotor when the input is 200 kW at (*a*) a power factor of unity, (*b*) a power factor of 0·8 leading. (*C. & G.*)

*Ans.* 2,200 V, $15°_e$; 2,640 V, $13{\cdot}5°_e$.

**12.5** A 400 V 3-phase 50 Hz star-connected synchronous motor has a synchronous impedance per phase of $(1 + j5)\,\Omega$. It takes a line current of 10 A at unity power factor when operating with a certain field current. If the load torque is increased until the line current is 40 A, the field current remaining unchanged, find the new power factor and the gross output power. (*H.N.C.*)

*Ans.* 0·957 lagging, 25 kW.

**12.6** A 150 kW 3-phase induction motor has a full-load efficiency and power factor of 0·91 and 0·89 respectively. A 3-phase star-connected synchronous motor, connected to the same mains, is to be over-excited in order to improve the resultant power factor to unity. The synchronous motor also drives a constant load, its power input being 100 kW. The line voltage is 415 V and the synchronous reactance per phase of the synchronous motor is $0{\cdot}5\,\Omega$, the resistance being negligible. Determine the induced e.m.f. per phase of the synchronous motor.

*Ans.* 306 V. (*H.N.C.*)

**12.7** A synchronous generator operates on constant-voltage constant-frequency busbars. Explain the effect of variation of (*a*) excitation and (*b*) steam supply on power output, power factor, armature current and load angle of the machine.

An 11 kV 3-phase star-connected synchronous generator delivers 4,000 kVA at unity power factor when running on constant-voltage constant-frequency busbars. If the excitation is raised by 20 per cent determine the kVA and power factor at which the machine now works. The steam supply is constant and the synchronous reactance is $30\,\Omega$/phase. Neglect power losses and assume the magnetic circuit to be unsaturated.

*Ans.* 4,280 kVA; 0·935 lagging. (*L.U.*)

**12.8** Describe briefly the procedure for synchronizing and connecting a 3-phase alternator to constant-voltage constant-frequency busbars. How is the output of the machine adjusted?

A single-phase alternator operates on 10 kV 50 Hz busbars. The winding resistance is $1\,\Omega$ and the synchronous impedance $10\,\Omega$. If the excitation is adjusted to give an open-circuit e.m.f. of 12 kV, what is the maximum power output of the machine? Find the armature current and power factor for this condition. (*H.N.C.*)

*Ans.* 10·9 MW; 1,480 A; 0·737 leading.



**12.9** Show that the maximum power that a synchronous generator can supply when connected to constant-voltage constant-frequency busbars increases with the excitation.

An 11 kV 3-phase star-connected turbo-alternator delivers 240 A at unity power factor when running on constant voltage and frequency busbars. If the excitation is increased so that the delivered current rises to 300 A, find the power factor at which the machine now works and the percentage increase in the induced e.m.f. assuming a constant steam supply and unchanged efficiency. The armature resistance is $0{\cdot}5\,\Omega$ per phase and the synchronous reactance $10\,\Omega$ per phase. (*H.N.C.*)

*Ans.* 0·802 lagging; 24 per cent.

**12.10** An 11 kV 300 MVA 3-phase alternator has a steady short-circuit current equal to half its rated value. Determine graphically or otherwise the maximum load the machine can deliver when connected to 11 kV constant-voltage constant-frequency busbars with its field excited to give an open-circuit voltage of 12·7 kV/phase. Find also the armature current and power factor corresponding to this load. Ignore armature resistance. (*H.N.C.*)

*Ans.* 300 MW; 17·4 kA; 0·895 leading.

**12.11** An alternator having a synchronous impedance of $R + jX$ ohms/phase is supplying constant voltage and frequency busbars. Describe, with the aid of complexor diagrams, the changes in current and power factor when the excitation is varied over a wide range, the steam supply remaining unchanged. The complexor diagrams should show the locus of the induced e.m.f.

A star-connected alternator supplies 300 A at unity power factor to 6,600 V constant voltage and frequency busbars. If the induced e.m.f. is now reduced by 20 per cent, the steam supply remaining unchanged, determine the new values of the current and power factor. Assume the synchronous reactance is $5\,\Omega$/phase, the resistance is negligible and the efficiency constant. (*H.N.C.*)

*Ans.* 350 A; 0·85 leading.

**12.12** Deduce an expression for the synchronizing power of an alternator.

Calculate the synchronizing power in kilowatts per degree of mechanical displacement at full load for a 1,000 kVA 6,600 V 0·8 power-factor 50 Hz 8-pole star-connected alternator having a negligible resistance and a synchronous reactance of 60 per cent. (*L.U.*)

*Ans.* 158 kW per mechanical degree.

**12.13** A 40 MVA 50 Hz 3,000 rev/min turbine-driven alternator has an equivalent moment of inertia of 1,310 kg-m², and the machine has a steady short-circuit current of four times its normal full-load current.

Deducing any formula used, estimate the frequency at which hunting may take place when the alternator is connected to an "infinite" grid system. (*H.N.C.*)

*Ans.* 3·14 Hz.

**12.14** An 11 kV 3-phase star-connected turbo-alternator is connected to constant-voltage constant-frequency busbars. The armature resistance is negligible and the synchronous reactance is $10\,\Omega$. The alternator is excited to deliver 300 A

Two transformers of equal voltage ratios but with the following ratings and impedances,

Transformer A—1,000 kVA, 1 per cent resistance, 5 per cent reactance,
Transformer B—1,500 kVA, 1·5 per cent resistance, 4 per cent reactance,

are connected in parallel to feed a load of 1,000 kW at 0·8 p.f. lagging. Determine the kVA in each transformer and its power factor. (*H.N.C.*)
(*Note.* Impedances may be expressed in per-unit form by dividing the percentage impedances by 100.)

*Ans.* A: 448 kVA, 0·73 lagging. B: 804 kVA, 0·834 lagging.

**9.20** Two single-phase transformers work in parallel on a load of 750 A at 0·8 p.f. lagging. Determine the secondary voltage and the output and power factor of each transformer.

Test data are:

Open-circuit: 11,000 V/3,300 V for each transformer
Short-circuit with h.v. winding short-circuited:

Transformer A: secondary input 200 V, 400 A, 15 kW
Transformer B: secondary input 100 V, 400 A, 20 kW (*L.U.*)

*Ans.* 3,190 V; A: 807 kVA, 0·65 lagging. B: 1,615 kVA, 0·86 lagging.

# Chapter 10

# GENERAL PRINCIPLES OF ROTATING MACHINES

Rotating machines vary greatly in size, ranging from a few watts to 600 MW and above—a ratio of power outputs of over $10^7$. They also vary greatly in type depending on the number and interconnexion of their windings and the nature of electrical supply to which they are to be connected. Despite these differences of size and type their general principles of operation are the same, and it is the purpose of this chapter to examine these common principles. Three succeeding chapters give a more detailed treatment of particular types of machine.

## 10.1 Modes of Operation

There are three distinguishable ways or modes of operation of rotating machines and these are illustrated in the block diagrams of Fig. 10.1. The three modes, motoring, generating and braking, are specified below.

### MOTORING MODE

Electrical energy is supplied to the main or *armature winding* of the machine and a mechanical energy output is available at a rotating shaft. This mode of operation is illustrated in Fig. 10.1(*a*), which takes the form of a 2-port representation of a machine, one port being electrical and the other mechanical.

An externally applied voltage $v$ drives a current $i$ through the armature winding against an internally induced e.m.f. $e$. The process of induction of e.m.f. is discussed in Section 10.4. The winding is thus enabled to absorb electrical energy at the rate $ei$. At least some of this energy is available for conversion (some may be stored in associated magnetic fields). The armature winding gives rise to an instantaneous torque $T_A'$* which drives the rotating member of the machine (the *rotor*) at an angular velocity $\omega_r$, and mechanical energy is delivered at the rate of $\omega_r T_A'$. The process of torque production is discussed in Section 10.5. An externally applied load torque

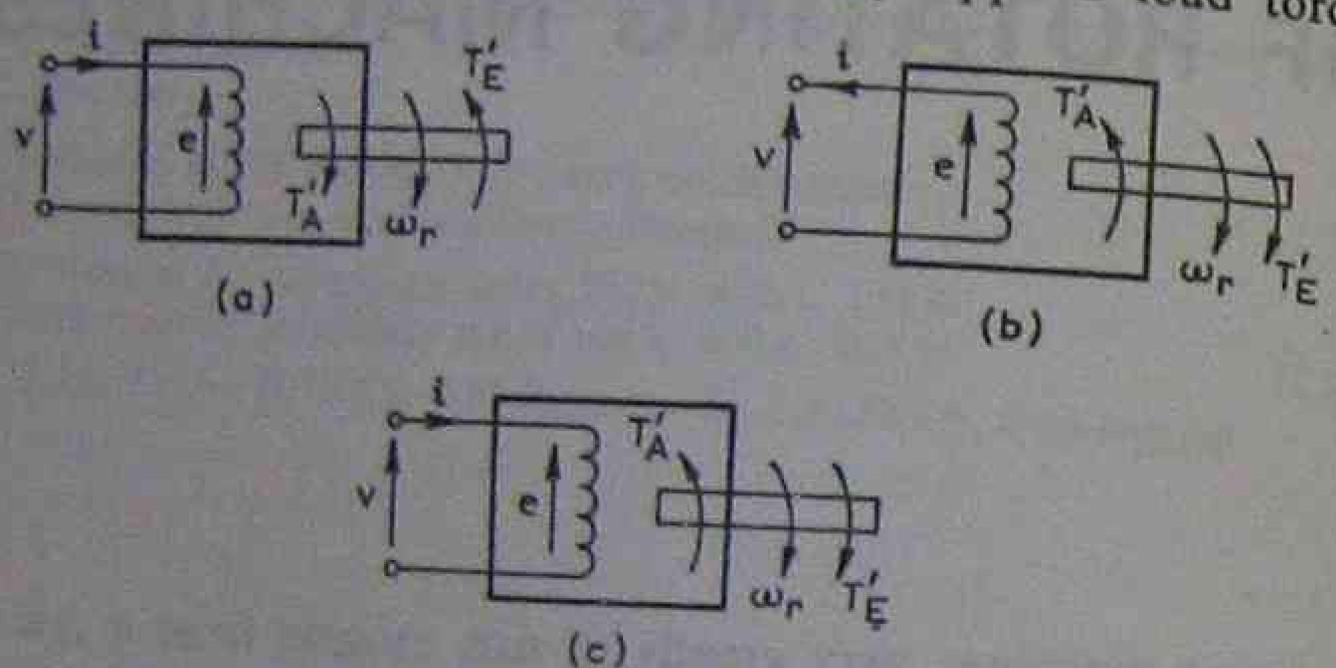



*Fig. 10.1* MODES OF OPERATION OF ROTATING MACHINES
(a) Motoring mode
(b) Generating mode
(c) Braking mode

$T_E'$ acting in a direction opposite to that of rotation enables the load to absorb mechanical energy:

$$T_A' - T_E' = J\frac{d\omega_r}{dt} \tag{10.1}$$

$J$ being the moment of inertia of the rotor and its mechanical load.

If the windage and friction torque is included in the external applied load torque, then, following eqn. (10.1), if $T_A'$ and $T_E'$ are equal and opposite, $d\omega_r/dt = 0$ and the machine will rotate at a steady speed.

When steady-state operation prevails, provided a sufficient period of time is considered,

$$(\omega_r T_A')_{mean} = (ei)_{mean} \tag{10.2}$$

Since the armature winding must develop torque and have an e.m.f. induced in it, a magnetic field is required. In very small

* To avoid confusion with $t$ for time, instantaneous torque will be represented by $T'$.



machines this may be provided by permanent magnets, but in most machines it is provided electromagnetically.

Some machines have a separate *field winding* to produce the required magnetic field. For such machines the block diagram of Fig. 10.1(*a*) would require a second electrical port. For the sake of simplicity this has been omitted. The energy fed to the field winding is either dissipated as loss in the field winding or is stored in the associated magnetic field and does not enter into the conversion process.

GENERATING MODE

Mechanical energy is supplied to the shaft of the machine by a prime mover and an electrical energy output is available at the armature-winding terminals. This mode of operation is illustrated in Fig. 10.1(*b*). The shaft of the machine is driven at an angular velocity $\omega_r$ in the direction of the applied external instantaneous torque $T_E'$ and in opposition to the torque $T_A'$ due to the armature winding, enabling the machine to absorb mechanical energy. The armature winding has an e.m.f. $e$ induced in it which drives a current through an external load of terminal voltage $v$. Eqns. (10.1) and (10.2) apply equally to generator action.

BRAKING MODE

In this mode of operation the machine has both a mechanical energy input and an electrical energy input. The total energy input is dissipated as loss in the machine. This mode is of limited practical application but occurs sometimes in the operation of induction and other machines.

## 10.2 Rotating Machine Structures

Rotating electrical machines have two members, a stationary member called the *stator* and a rotating member called the *rotor*. The stator and rotor together constitute the magnetic circuit or core of the machine and both are made of magnetic material so that magnetic flux is obtained for moderate values of m.m.f. The rotor is basically a cylinder and the stator a hollow cylinder. The rotor and stator are separated by a small air-gap as shown in Fig. 10.2. Compared with the rotor diameter the radial air-gap length is small. The stator and rotor magnetic cores are usually, but not invariably, built up from laminations (typically 0·35 mm thick) in order to reduce eddy-current loss.

If the rotor is to rotate, a mutual torque has to be sustained between the rotor and stator. A winding capable of carrying current and of sustaining torque is required on at least one member and usually, but not always, on both. One method of arranging windings in a rotating machine is to place coils in uniformly distributed slots on both the stator and the rotor. This method is illustrated in Fig.

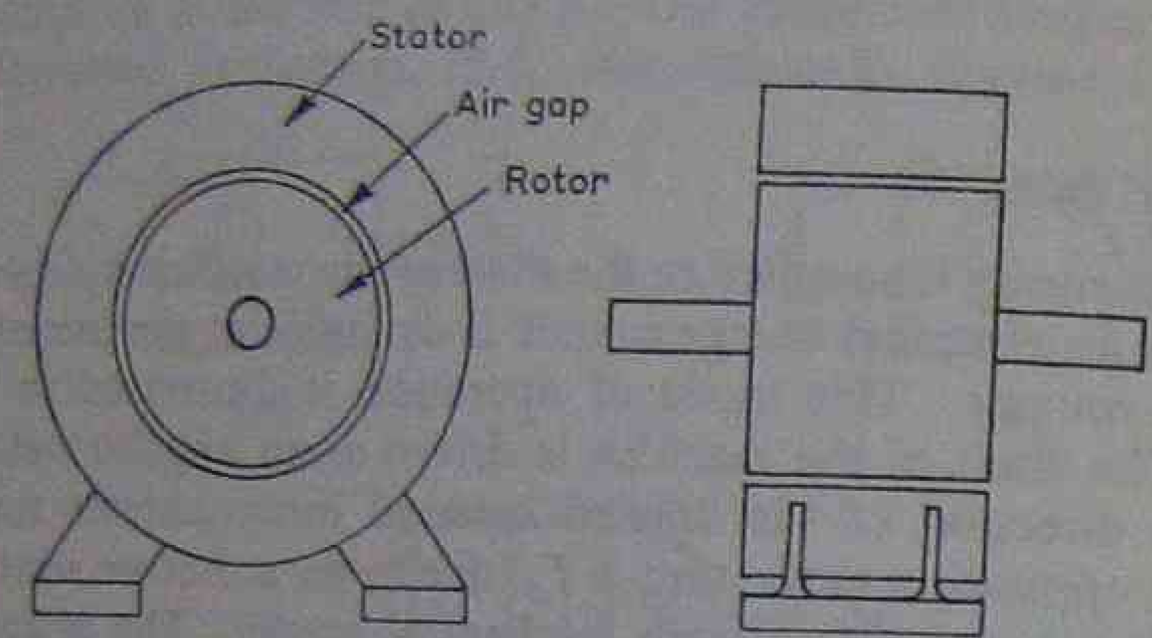



*Fig. 10.2* BASIC ARRANGEMENT OF A ROTATING MACHINE

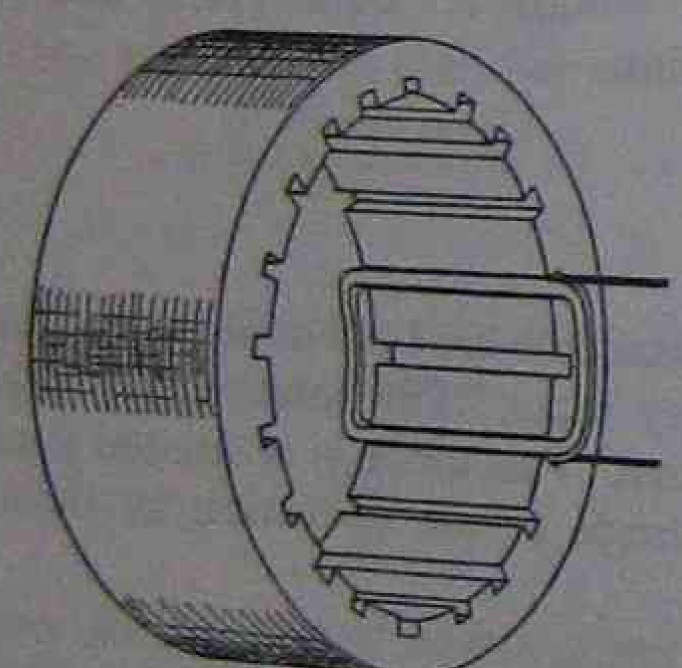


*Fig. 10.3* STATOR AND ONE OF ITS COILS

10.3, where, for clarity, only the stator is shown. An arrangement of this sort is commonly used in induction machines. The distance between the coil sides is usually about one pole pitch.

To make a complete winding, similar coils are placed in other pairs of slots and all the coils are then connected together in groups. The groups of coils may then be connected in series or in parallel, and in 3-phase machines in star or mesh.

Some windings may be double-layer windings. In such windings each slot contains two coil sides, one at the top and the other at the



bottom of a slot. Each coil has one coil side at the top of the slot and the other at the bottom.

An alternative arrangement to having uniform slotting on both sides of the air-gap is to have salient poles around which are wound concentrated coils to provide the field winding. The salient poles may be on either the stator or the rotor, and such arrangements are illustrated in Figs. 10.4 and 10.5(*a*).

The salient-pole stator arrangement is commonly used for direct-current machines and occasionally for small sizes of synchronous

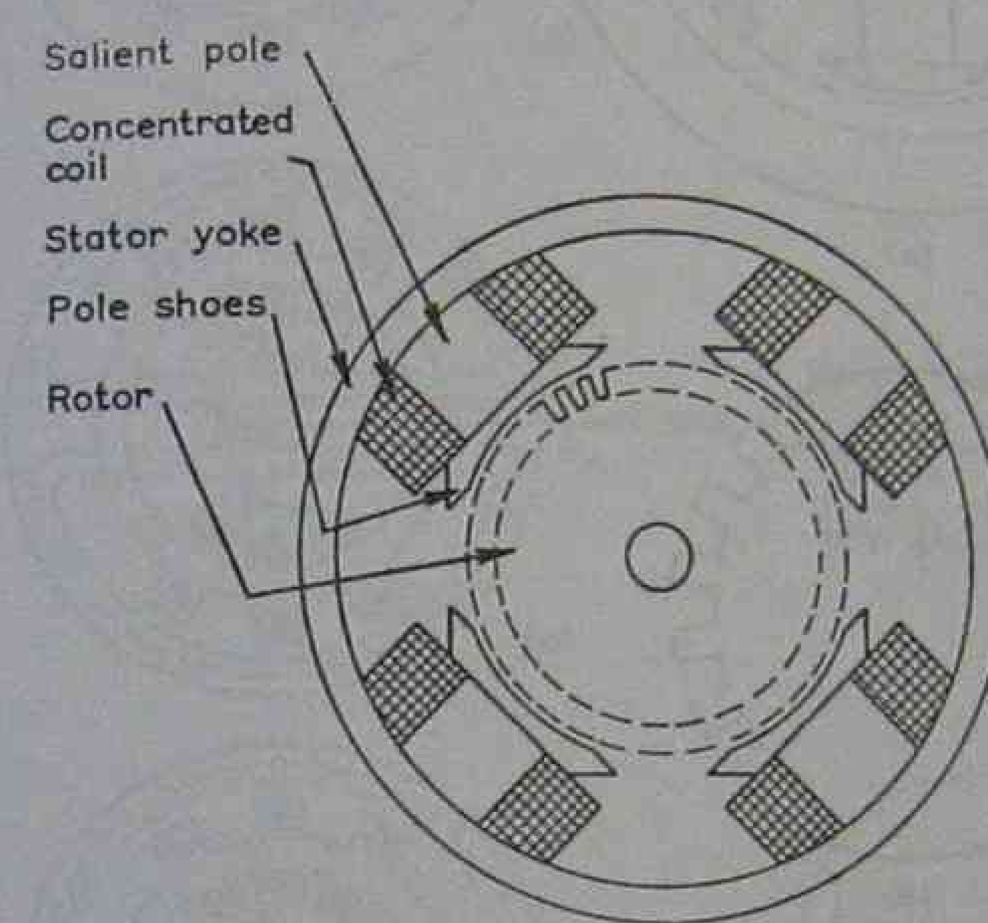



*Fig. 10.4* SALIENT-POLE STATOR

machine. As far as d.c. machines are concerned the stator is most often referred to as the field and the rotor as the armature. The main winding in such a machine is on the rotor and is called the armature winding.

The salient pole rotor arrangement is most often used for synchronous machines. In such machines the main winding is on the stator but it is often called the armature winding.

In general, rotating machines can have any even number of poles. The concentrated coil windings surrounding the poles are excited so as to make successive poles of alternate north and south polarity.

The salient-pole rotor structure is unsuitable for large high-speed turbo-alternators used in the supply industry because of the high stress in the rotor due to centrifugal force. In such machines a cylindrical rotor is used as shown in Fig. 10.5(*b*). Uniform slotting occupies two thirds of the rotor surface, the remaining third being unslotted. Such rotors are usually solid steel forgings.

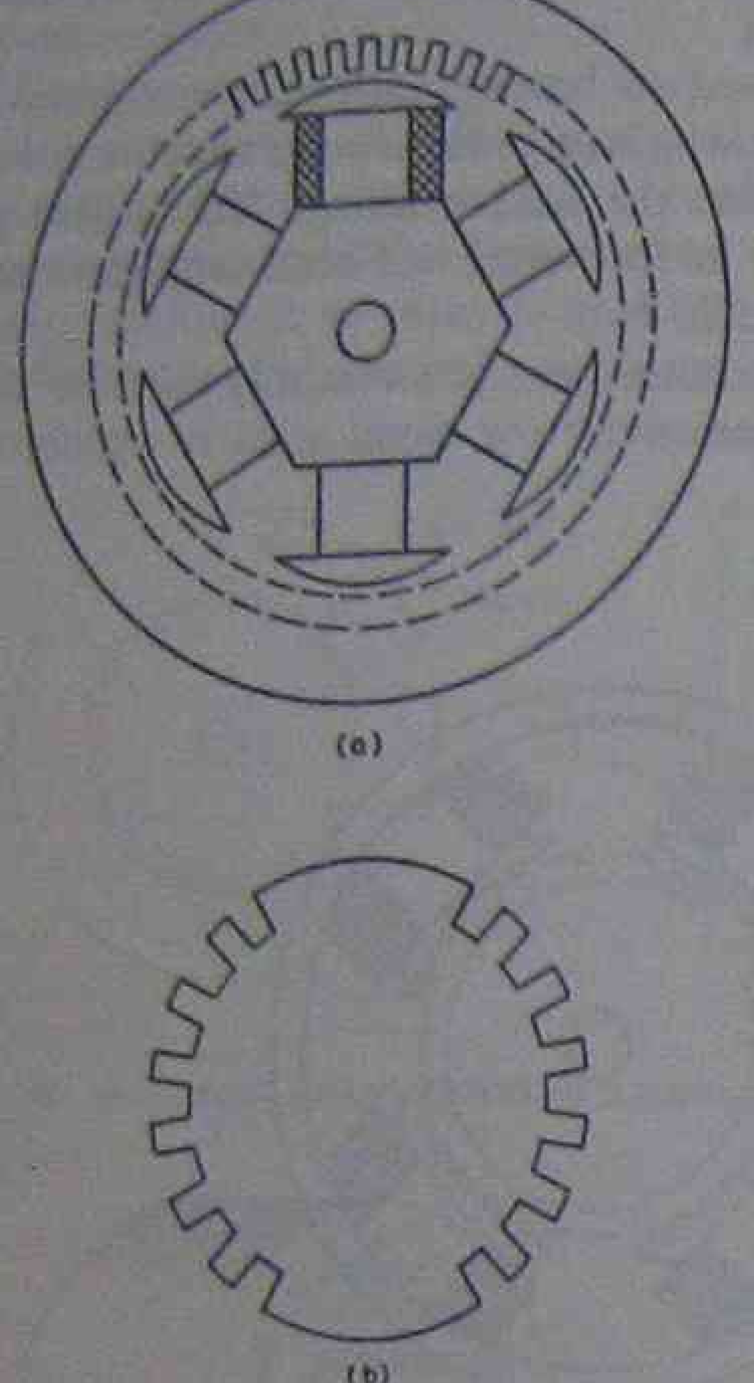



*Fig. 10.5* ROTORS
(*a*) Salient pole (*b*) Cylindrical

## 10.3 Self- and Mutual Inductance of Stator and Rotor Windings

The simplest rotating machine structure is a 2-pole machine with a uniform air-gap as shown in Fig. 10.2 which does not exhibit "saliency" (i.e. does not have salient poles) on either side of the air-gap. In this and all succeeding sections of this chapter only 2-pole machines will be considered.

Fig. 10.6(*a*) shows such a machine. The stator winding axis is chosen to correspond with a horizontal angular reference axis called the *direct axis* (*d*-axis) at which $\theta = 0$.

A convention for positive current in a coil must be established and this is done in the following way. Consider a winding such as the stator winding of Fig. 10.6(*a*) whose axis corresponds with the *d*-axis. Positive current is taken to produce an m.m.f. acting in the positive direction of the *d*-axis. Thus the stator winding in Fig. 10.6(*a*) is excited by positive current. The angular position, $\theta$, of a



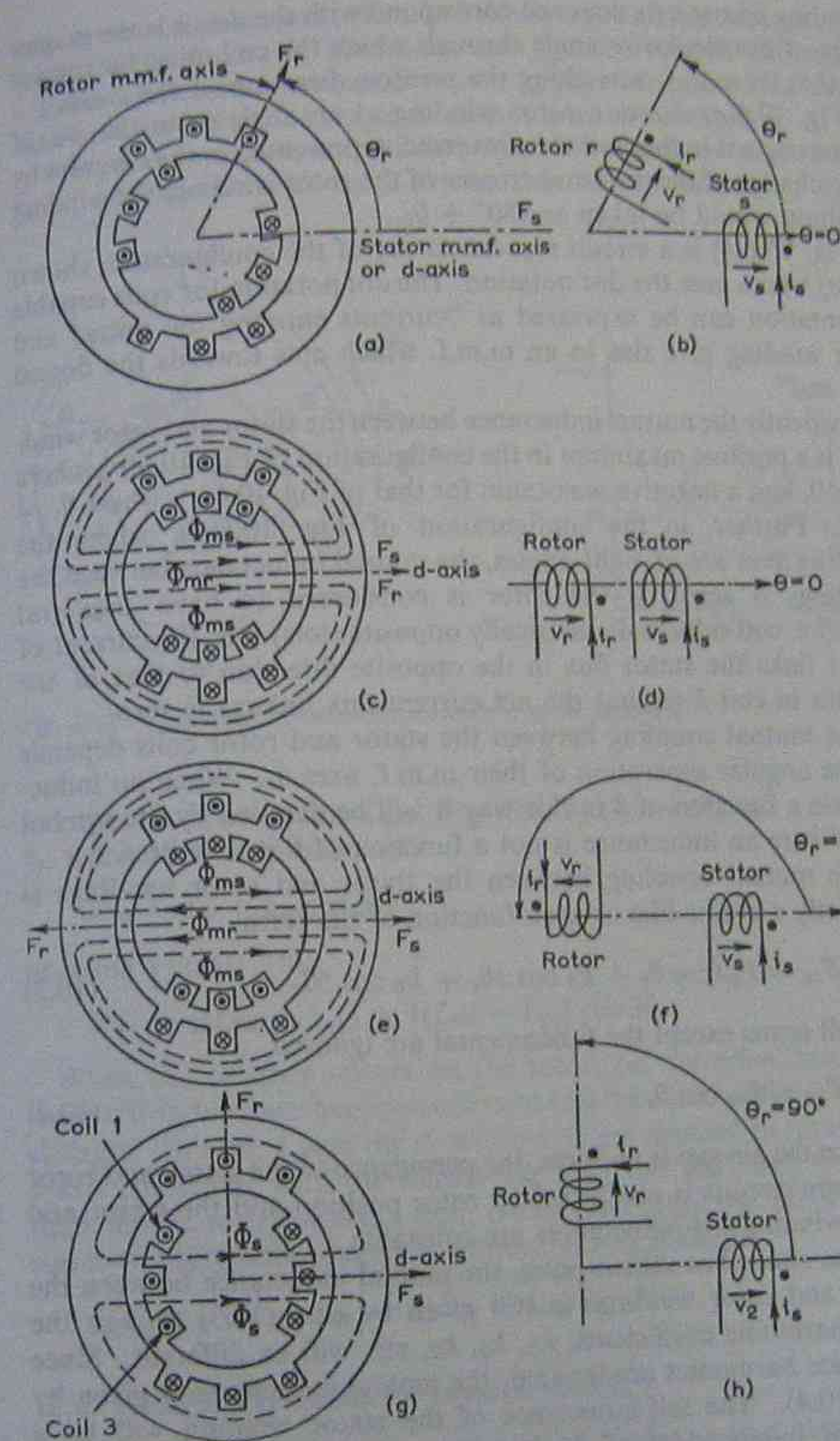



*Fig. 10.6* MUTUAL COUPLING OF STATOR AND ROTOR COILS

winding whose axis does not correspond with the $d$-axis is the magnitude of the clockwise angle through which the coil must be rotated so that its m.m.f. acts along the positive direction of the $d$-axis.

Fig. 10.6($a$) shows a rotor winding at an angle $\theta_r$ to the $d$-axis. If the current in this coil were reversed, represented in the diagram by interchange of the dots and crosses of the rotor winding, the winding position would be taken as $180° + \theta_r$.

Fig. 10.6($b$) is a circuit representation of the configuration shown at ($a$) which uses the dot notation. The dot notation for coils capable of rotation can be expressed as "currents entering the dotted end of a winding give rise to an m.m.f. which acts towards the dotted coil end".

Evidently the mutual inductance between the stator and rotor windings is a positive maximum in the configuration of Fig. 10.6($c$), where $\theta_r = 0$, and a negative maximum for that of Fig. 10.6($e$), where $\theta_r = 180°$. Further, in the configuration of Fig. 10.6($g$), where the winding axes are at right angles, the mutual inductance between the windings is zero. If the rotor is considered to have diametral coils (i.e. coil sides in diametrically opposite slots) then the current of coil 1 links the stator flux in the opposite direction to that of the current in coil 3 so that the net current-flux linkage is zero.

The mutual coupling between the stator and rotor coils depends on the angular separation of their m.m.f. axes $\theta_r$. When an inductance is a function of $\theta$ in this way it will be denoted by the symbol $\mathscr{L}$. Where an inductance is not a function of $\theta$ it is written $L$.

The mutual coupling between the stator and rotor windings is evidently a cosine-like or even function of the form

$$\mathscr{L}_{sr} = L_{sr}(\cos\theta_r + k_3\cos 3\theta_r + k_5\cos 5\theta_r \ldots) \tag{10.3}$$

If all terms except the fundamental are ignored,

$$\mathscr{L}_{sr} = L_{sr}\cos\theta_r \tag{10.4}$$

Since the air-gap is uniform, the permeance of the stator and rotor magnetic circuits is unaffected by rotor position and the stator and rotor winding self-inductances are constants.

If the stator has salient poles, the mutual inductance between the stator and rotor windings is still given by eqn. (10.3) though the space-harmonic coefficients, $k_3$, $k_5$, $k_7$, etc. will be different. Since the space harmonics are ignored, the mutual inductance is given by eqn. (10.4). The self-inductance of the stator winding will be a constant independent of $\theta_r$, but the self-inductance of the rotor coil will depend on the rotor position (see Fig. 10.7). When the rotor m.m.f. axis is lined up with the $d$-axis, its self-inductance will be a



maximum, $L_{dd}$, say, but when it is lined up with an axis at right angles to the $d$-axis, the *quadrature axis* ($q$-axis), the self-inductance will have fallen to a minimum value $L_{qq}$, say, because of the much lower permeance of the magnetic circuit centred on this axis.

After the rotor has turned through 180° the rotor m.m.f. axis again corresponds with the $d$-axis so that its self-inductance again

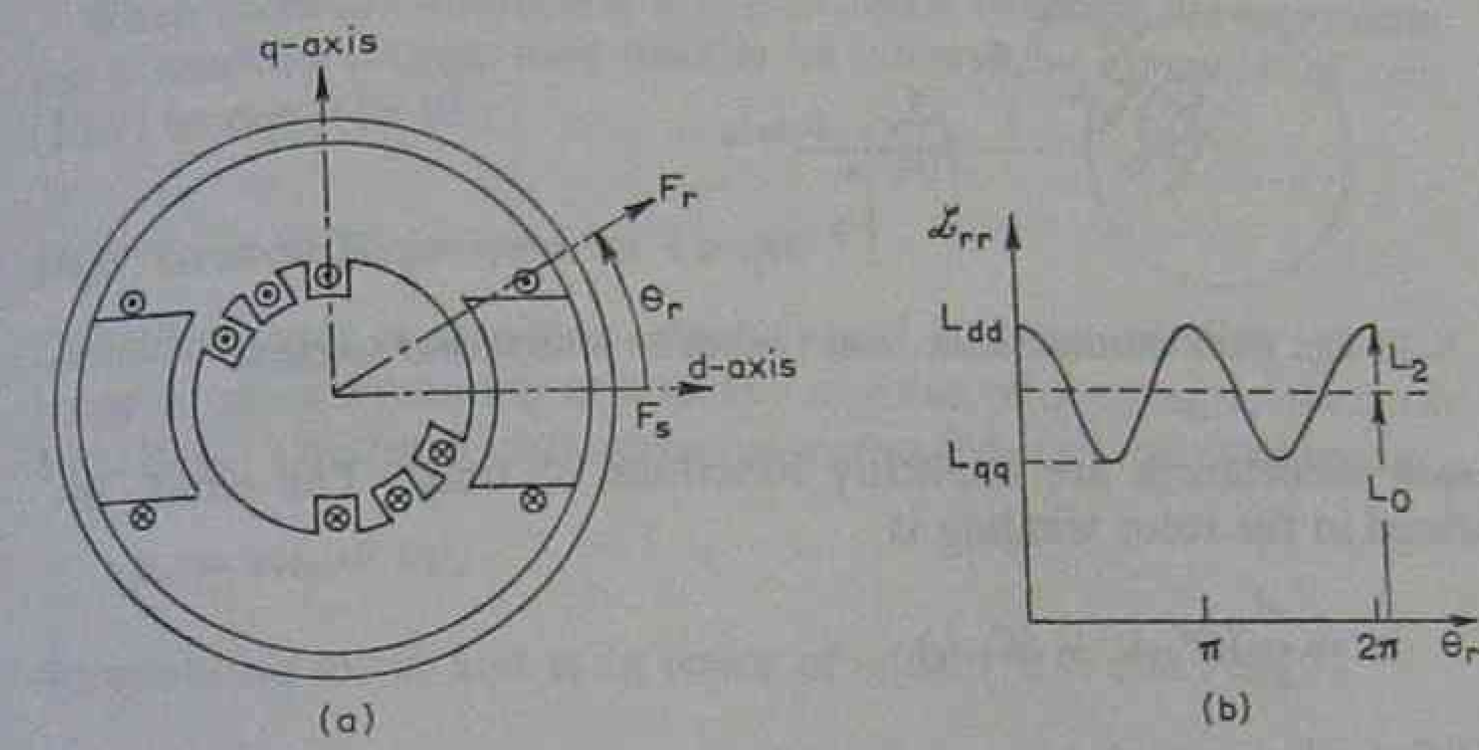



*Fig. 10.7* VARIATION OF SELF-INDUCTANCE DUE TO SALIENCY

has the maximum value $L_{dd}$. Fig. 10.7($b$) shows $\mathscr{L}_{rr}$ to a base of $\theta_r$, from which approximately

$$\mathscr{L}_{rr} = L_0 + L_2\cos 2\theta_r \tag{10.5}$$

or, from Fig. 10.7($b$),

$$\mathscr{L}_{rr} = \tfrac{1}{2}(L_{dd} + L_{qq}) + \tfrac{1}{2}(L_{dd} - L_{qq})\cos 2\theta_r \tag{10.6}$$

When the saliency occurs on the rotor, on the other hand, the rotor self-inductance becomes constant and the stator self-inductance changes. In such a case the $d$- and $q$-axis are deemed to rotate, the $d$-axis coinciding with the salient-pole axis and the $q$-axis being at right angles to the $d$-axis. Eqn. (10.6) then gives the variation of stator-winding self-inductance without modification as

$$\mathscr{L}_{ss} = \tfrac{1}{2}(L_{dd} + L_{qq}) + \tfrac{1}{2}(L_{dd} - L_{qq})\cos 2\theta_r \tag{10.7}$$

## 10.4 General Expression for Induced E.M.F.

Consider a stator winding $s$, and a rotor winding $r$ that rotates at a steady angular velocity $\omega_r = d\theta_r/dt$ with respect to the stator winding, as shown in Fig. 10.8. Let the windings carry instantaneous currents $i_s$, $i_r$ which, in general, will be functions of time. The mutual inductance of the windings is a function of $\theta_r$ as also will be one of the self-inductances if saliency exists on either side of the air-gap. Since the position of the rotor winding $\theta_r$ is a function of time,

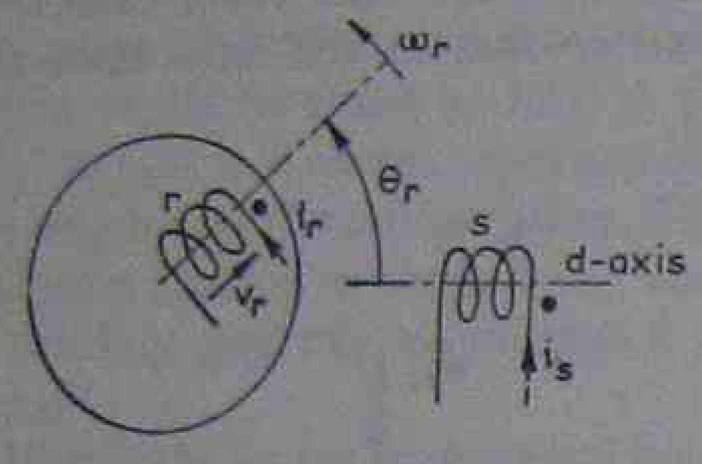



*Fig. 10.8* INDUCED E.M.F. AND TORQUE IN A ROTATING MACHINE

such inductances are implicitly functions of time. The e.m.f. induced in the rotor winding is

$$e_r = \frac{d}{dt}(\mathscr{L}_{rr}i_r + \mathscr{L}_{sr}i_s)$$

Differentiating each term as a product,

$$e_r = \frac{\partial \mathscr{L}_{rr}}{\partial \theta_r}\frac{d\theta_r}{dt}i_r + \mathscr{L}_{rr}\frac{di_r}{dt} + \frac{\partial \mathscr{L}_{sr}}{\partial \theta_r}\frac{d\theta_r}{dt}i_s + \mathscr{L}_{sr}\frac{di_s}{dt}$$

$$e_r = \underbrace{\mathscr{L}_{rr}'\omega_r i_r + \mathscr{L}_{sr}'\omega_r i_s}_{\text{Rotational voltages}} + \underbrace{\mathscr{L}_{rr}\frac{di_r}{dt} + \mathscr{L}_{sr}\frac{di_s}{dt}}_{\text{Transformer voltages}} \qquad (10.8)$$

where $\mathscr{L}_{rr}' = \partial \mathscr{L}_{rr}/\partial\theta$ etc. and $\omega_r = d\theta_r/dt$.

It will be seen that the expression for $e_r$ contains terms of two distinct types: (*a*) voltages proportional to the rotor angular velocity and called *rotational voltages*, and (*b*) voltages proportional to the rate of change of the winding currents. These latter are often called *transformer voltages*.

Including the voltage drop in the rotor winding resistance the voltage applied to that winding is

$$v_r = r_r i_r + \mathscr{L}_{rr}'\omega_r i_r + \mathscr{L}_{sr}'\omega_r i_s + \mathscr{L}_{rr}\frac{di_r}{dt} + \mathscr{L}_{sr}\frac{di_s}{dt} \qquad (10.9)$$

Similarly, the voltage applied to the stator winding is

$$v_s = r_s i_s + \mathscr{L}_{ss}'\omega_r i_s + \mathscr{L}_{sr}'\omega_r i_r + \mathscr{L}_{ss}\frac{di_s}{dt} + \mathscr{L}_{sr}\frac{di_r}{dt} \qquad (10.10)$$



In general, for $n$ coupled windings the voltage applied to the $j$th winding is

$$v_j = r_j i_j + \sum_{k=1}^{k=n}\frac{d}{dt}(\mathscr{L}_{jk}i_k) \qquad (10.11)$$

When there are additional stator or rotor windings the expression for a winding voltage may usually be inferred by extension of eqn. (10.9) or eqn. (10.10).

### 10.5 General Expression for Torque

Consider again a machine consisting of a stator winding $s$ and a rotor winding $r$ rotating at a steady angular velocity $\omega_r$, as shown in Fig. 10.8. The total instantaneous power fed into the machine is

$$p_e = v_r i_r + v_s i_s$$

Substituting for $v_s$ and $v_r$ in terms of eqns. (10.9) and (10.10),

$$p_e = r_r i_r^2 + \mathscr{L}_{rr}'\omega_r i_r^2 + \mathscr{L}_{sr}'\omega_r i_r i_s + \mathscr{L}_{rr}i_r\frac{di_r}{dt} + \mathscr{L}_{sr}i_r\frac{di_s}{dt}$$
$$+ r_s i_s^2 + \mathscr{L}_{ss}'\omega_r i_s^2 + \mathscr{L}_{sr}'\omega_r i_r i_s + \mathscr{L}_{ss}i_s\frac{di_s}{dt} + \mathscr{L}_{sr}i_s\frac{di_r}{dt} \qquad (10.12)$$

In this equation the terms $r_r i_r^2$ and $r_s i_s^2$ represent power loss in the winding resistances.

The energy stored in the magnetic fields associated with the two coils is

$$w_f = \tfrac{1}{2}\mathscr{L}_{rr}i_r^2 + \tfrac{1}{2}\mathscr{L}_{ss}i_s^2 + \mathscr{L}_{sr}i_s i_r$$

The rate at which energy is stored in the magnetic field is

$$\frac{dw_f}{dt} = \tfrac{1}{2}i_r^2\mathscr{L}_{rr}'\omega_r + \mathscr{L}_{rr}i_r\frac{di_r}{dt} + \tfrac{1}{2}i_s^2\mathscr{L}_{ss}'\omega_r + \mathscr{L}_{ss}i_s\frac{di_s}{dt}$$
$$+ i_s i_r\mathscr{L}_{sr}'\omega_r + \mathscr{L}_{sr}i_s\frac{di_r}{dt} + \mathscr{L}_{sr}i_r\frac{di_s}{dt} \qquad (10.13)$$

where again $\mathscr{L}_{rr}' = \partial\mathscr{L}_{rr}/\partial\theta_r$, etc., and $\omega_r = d\theta_r/dt$.

There is an instantaneous mechanical power output corresponding to that portion of the instantaneous electrical power input which is neither dissipated in the winding resistances nor used to store energy in the magnetic field. If the instantaneous torque on the rotor is $T'$, then

$$p_m = \omega_r T' = p_e - r_r i_r^2 - r_s i_s^2 - \frac{dw_f}{dt}$$
$$= \tfrac{1}{2}\mathscr{L}_{rr}'\omega_r i_r^2 + \mathscr{L}_{sr}'\omega_r i_r i_s + \tfrac{1}{2}\mathscr{L}_{ss}'\omega_r i_s^2$$

or

$$T' = \tfrac{1}{2}\mathscr{L}_{rr}' i_r^2 + \mathscr{L}_{sr}' i_r i_s + \tfrac{1}{2}\mathscr{L}_{ss}' i_s^2 \qquad (10.14)$$

When there are additional stator or rotor windings the total torque acting on the rotor or stator may be inferred by extension of eqn. (10.14).

**EXAMPLE 10.1** A torque motor has a uniform air-gap. The stator and rotor each carry windings and the axis of the rotor coil may rotate relative to that of the stator coil. The mutual inductance between the coils is such that

$$\mathscr{L}_{sr} = L_{sr}\cos\theta_r$$

(*a*) Show that, when the axes of the coils are lined up on the *d*-axis and each coil carries conventionally positive current, the coils are in a position of stable equilibrium.
(*b*) If with the coils so aligned the current in either coil is reversed show that the position is one of unstable equilibrium.
(*c*) In such an arrangement the rotor and stator coils are in series, the rotor coil axis is at $\theta_r = 135°$ and the maximum mutual inductance is 1 H. Calculate the coil currents if the mutual torque on the rotor is to be 100 N-m in the $-\theta$ direction and the coils are excited with direct current.

The instantaneous torque is given by eqn. (10.14) as

$$T' = \tfrac{1}{2}\mathscr{L}_{rr}' i_r^2 + \mathscr{L}_{sr}' i_r i_s + \tfrac{1}{2}\mathscr{L}_{ss}' i_s^2 \qquad (10.14)$$

Since the air-gap is uniform the rotor and stator winding self-inductances are constants and their angular rates of change are zero, i.e. $\mathscr{L}_{rr}' = 0$ and $\mathscr{L}_{ss}' = 0$.

$$\mathscr{L}_{sr}' = \frac{d}{d\theta_r}(\mathscr{L}_{sr}) = \frac{d}{d\theta_r}(L_{sr}\cos\theta_r) = -L_{sr}\sin\theta_r$$

Substituting these conditions into the expression for instantaneous torque,

$$T' = -L_{sr} i_r i_s \sin\theta_r \qquad (10.15)$$

With the rotor coil axis aligned with the stator coil axis $\theta_r = 0$ and the torque is zero. If the rotor coil is given a small deflection $+\delta\theta_r$ from this position, with positive currents flowing in both windings the torque takes on a negative value and acts in the $-\theta_r$ direction to restore the rotor to its initial position. Similarly, if the rotor is given a small deflection $-\delta\theta_r$ the torque takes on a positive value and acts in the $+\theta_r$ direction to restore the rotor to its initial position.

If one of the coil currents is reversed, however, the opposite result occurs and a small deflection in either direction leads to a torque acting so as to increase the deflection. If the rotor is free to move in this case it will take up an equilibrium position at $\theta_r = 180°$, where the torque is again zero.

It will be noted that in both cases the tendency is for the coils to align themselves in the positive maximum mutual inductance configuration.



From eqn. (10.15) for $i_s = i_r = I$, the steady torque when the axis of the rotor winding is at any angle $\theta_r$ is

$$T = -L_{sr}I^2\sin\theta_r$$

As the torque on the rotor is to act in the $-\theta$ direction, $T = -100$ N-m. This gives

$$I = \sqrt{\frac{100}{1\times\sin 135°}} = \underline{\underline{11{\cdot}9\,\text{A}}}$$

**EXAMPLE 10.2** An electrodynamic wattmeter has a fixed current coil and a rotatable voltage coil. The magnetic circuit of the device does not exhibit saliency. The following are details of a particular wattmeter:

| | |
|---|---|
| Full-scale deflection | 110° |
| Control-spring constant | $10^{-7}$ N-m/deg |
| Maximum current-coil current (r.m.s.) | 10 A |
| Maximum voltage-coil voltage (r.m.s.) | 60 V |
| Voltage-coil resistance | 600 Ω |

The mutual inductance between the coils varies cosinusoidally with the angle of separation of the coil axes. The zero on the instrument corresponds to a voltage-coil position of $\theta_r = 145°$.

(*a*) Determine the direct current flowing in the current coil when a direct voltage of 60 V is applied to the voltage coil and the angular deflection is 100° from the instrument scale zero.
(*b*) For a sinusoidally varying current-coil current of 6 A (r.m.s.) and a voltage-coil voltage of 60 V (r.m.s.) of the same frequency as the current, determine the phase angle by which the current lags the voltage when the voltage-coil deflection is 60° from the instrument scale zero.

The reactance of the voltage coil is negligible compared with its resistance. A diagram of the arrangement is given in Fig. 10.9.

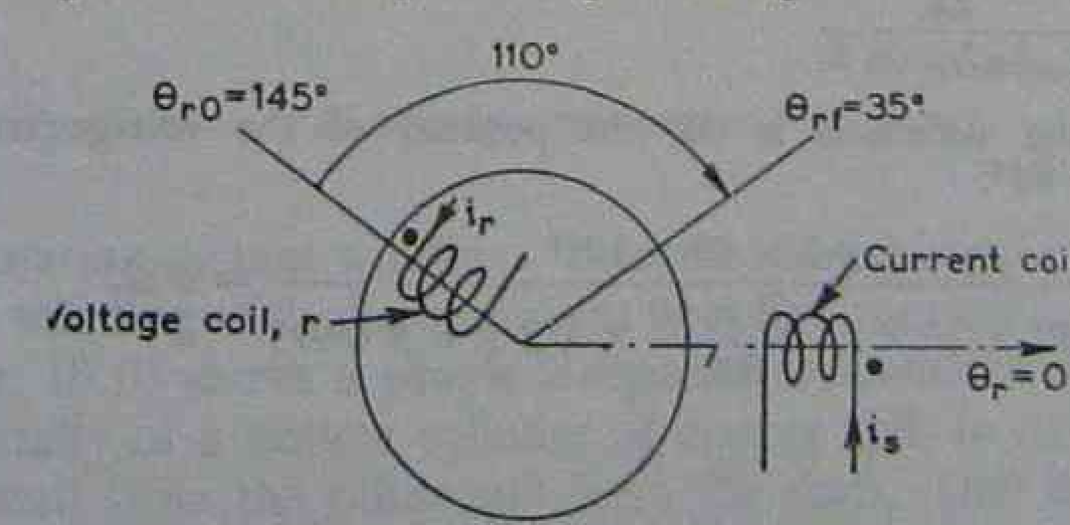



Fig. 10.9

Since there is no saliency and $\mathscr{L}_{sr} = L_{sr}\cos\theta_r$, the expression for the instantaneous torque is the same as that found in Example 10.2, namely

$$T' = -L_{sr} i_r i_s \sin\theta_r \qquad (10.15)$$

If the conditions for full-scale deflection are substituted the value of $L_{sr}$, the maximum possible mutual inductance between the coils, is found. Since the deflection is in the $-\theta_r$ direction the torque is negative.

$$T = -110 \times 10^{-7}\,\text{N-m},\ i_r = \frac{60}{600} = 0{\cdot}1\,\text{A},\ i_s = 10\,\text{A},\ \theta_{rf} = 145^\circ - 110^\circ = 35^\circ$$

whence

$$L_{sr} = \frac{110 \times 10^{-7}}{0{\cdot}1 \times 10 \times \sin 35^\circ} = 192 \times 10^{-7}\,\text{H}$$

(*a*) When the angular deflection is 100° the position of the voltage coil is $\theta_r = 145 - 100 = 45^\circ$. This gives, in eqn. (10.15),

$$-100 \times 10^{-7} = -192 \times 10^{-7} \times 0{\cdot}1 \times I_s \sin 45^\circ$$

so that

$$I_s = \frac{100}{192 \times 0{\cdot}1 \times 0{\cdot}707} = \underline{\underline{7{\cdot}37\,\text{A}}}$$

(*b*) If the voltage-coil current is taken as reference, the current-coil current is

$$i_s = I_{sm} \cos(\omega t - \phi)$$

The instantaneous torque is, from eqn. (10.15),

$$\begin{aligned} T' &= -L_{sr} I_{rm} \cos \omega t I_{sm} \cos(\omega t - \phi) \sin \theta_r \\ &= -\tfrac{1}{2} L_{sr} I_{rm} I_{sm} \sin \theta_r [\cos \phi + \cos(2\omega t - \phi)] \end{aligned}$$

This expression shows that the instantaneous torque consists of two components: (*a*) a steady component, and (*b*) an alternating component which oscillates at twice the frequency of the currents in the two coils. The inertia of the rotating system will prevent its responding to the alternating component. The average torque is therefore

$$T = -\tfrac{1}{2} L_{sr} I_{rm} I_{sm} \sin \theta_r \cos \phi$$

whence

$$\cos \phi = -\frac{2T}{L_{sr} I_{rm} I_{sm} \sin \theta_r}$$

When the angular deflection is 60° the position of the voltage coil is $\theta_r = 145 - 60 = 85^\circ$:

$$\cos \phi = -\frac{-2 \times 60 \times 10^{-7}}{192 \times 10^{-7} \times \dfrac{\sqrt{2} \times 60}{600} \times \sqrt{2} \times 6 \times 0{\cdot}995} = 0{\cdot}525$$

so that

$$\phi = \underline{\underline{58{\cdot}3^\circ}}$$

## 10.6 The Alignment Principle

Example 10.1 has shown that the torque acting on the rotor of a simple rotating machine structure consisting of a stator and a rotor coil is such as to tend to align the coils in their maximum positive mutual inductance position. The mutual torque on the system is then zero. If a continuously rotating machine is to be produced, therefore, some method must be found of maintaining a constant angular displacement of the axes of the rotor and stator winding m.m.f.s under steady conditions despite the rotation of the rotor and its winding. Several different methods exist for achieving this; the particular method chosen determines the type of machine. The rest of the chapter is devoted to considering how this constant angular displacement of the axes of the winding m.m.f.s is brought about in some common types of machine.



## 10.7 The Commutator

In some types of machine the stator winding is excited with direct current and so the axis of the stator m.m.f., $F_s$, is fixed. If a constant angle is to be maintained between the axes of the stator and rotor winding m.m.f.s the rotor winding m.m.f. must be stationary despite

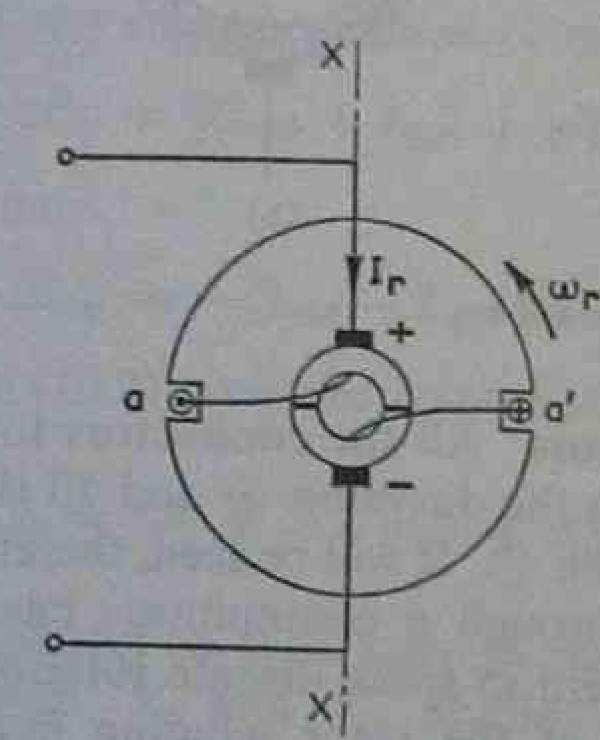


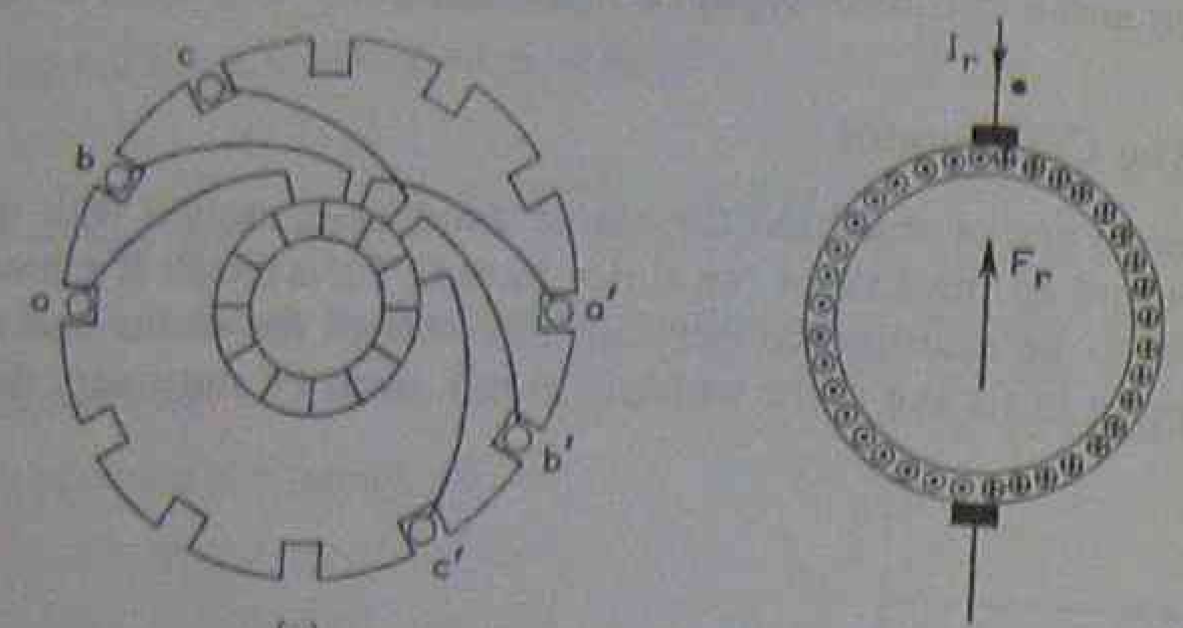

*Fig. 10.10* A TWO-SEGMENT COMMUTATOR

the rotation of that winding. This may be achieved by exciting the rotor winding with direct current supplied through a commutator.

Fig. 10.10 shows a simple 2-segment commutator which consists essentially of a hollow cylinder of copper split in half, each being insulated from the other and from the shaft. One end of a rotor coil is joined to each commutator segment. Two brushes, fixed in space, make alternate contact with each segment of the commutator as it rotates. Although the current in the coil reverses twice in each revolution, it will be seen that whichever of the coil sides, *a* or *a'*, lies to the left of the brush axis XX will carry current in the direction indicated by ⊙, whereas whichever coil side lies to the right of XX carries current in the direction indicated by ⊕.

A rotor winding consisting of many coils wound into uniformly distributed slots may also be supplied through a commutator. Each of the two ends of each coil is connected to two different commutator segments. The rotor coils are connected in series, the ends of successive coils being joined at the commutator as shown in Fig. 10.11(*a*). Such windings are double-layer windings.

Fig. 10.11(*b*) is a conventional representation of such a winding where the commutator is not shown and the brushes are thought




(a) (b)

*Fig. 10.11* A MULTI-SEGMENT COMMUTATOR

of as bearing directly on the conductors. All the conductors to the left of the brush axis carry current in the direction ⊙ and all those to its right carry current in the direction ⊕. It will be seen, therefore, that supplying the rotor winding through a commutator has the effect of fixing a certain current pattern in space despite rotation of the winding. As a result the axis of the rotor winding m.m.f., $F_r$, is fixed in space and coincides with the brush axis. The positive brush at which the current enters the winding corresponds to the dotted end of the winding.

### 10.8 Separately Excited D.C. Machine

The d.c. machine has almost invariably a salient pole structure on the stator and a non-salient pole rotor. The stator has a concentrated coil winding; the rotor winding is distributed in slots. Fig. 10.4 shows the structure commonly adopted for the d.c. machine. The stator winding is excited with direct current, and the rotor winding is supplied with direct current through a commutator, thus maintaining a constant angular displacement between the axes of the stator and rotor winding m.m.f.s as is required for torque maintenance.

Since the rotor is not salient pole the stator winding self-inductance



does not vary with the angular position of the rotor as explained in Section 10.3, i.e.

$$\mathscr{L}_{ss} = L_{ss} \tag{10.16}$$

so that

$$\mathscr{L}_{ss}' = \frac{\partial \mathscr{L}_{ss}}{\partial \theta} = 0 \tag{10.17}$$

The mutual inductance between the stator and rotor windings is

$$\mathscr{L}_{sr} = L_{sr} \cos \theta_r \tag{10.4}$$

so that

$$\mathscr{L}_{sr}' = -L_{sr} \sin \theta_r \tag{10.18}$$

Since there is saliency on the stator the rotor self-inductance varies with the angular position of the rotor and is given by eqn. (10.6) as

$$\mathscr{L}_{rr} = \tfrac{1}{2}(L_{dd} + L_{qq}) + \tfrac{1}{2}(L_{dd} - L_{qq}) \cos 2\theta_r \tag{10.6}$$

so that

$$\mathscr{L}_{rr}' = -(L_{dd} - L_{qq}) \sin 2\theta_r \tag{10.19}$$

Eqn. (10.9) gives the voltage to the rotor winding as

$$v_r = r_r i_r + \mathscr{L}_{rr}'\omega_r i_r + \mathscr{L}_{sr}'\omega_r i_s + \mathscr{L}_{rr}\frac{di_r}{dt} + \mathscr{L}_{sr}\frac{di_s}{dt} \tag{10.9}$$

Consider a separately excited d.c. machine with steady, direct voltages, $V_s$ and $V_r$, applied to the stator and rotor windings. Let the steady, direct currents in the stator and rotor windings be $I_s$ and $I_r$. The time rates of change of these steady currents are zero so that the voltage applied to the rotor winding is

$$V_r = r_r I_r + \mathscr{L}_{rr}'\omega_r I_r + \mathscr{L}_{sr}'\omega_r I_s \tag{10.20}$$

or

$$V_r = r_r I_r - (L_{dd} - L_{qq}) \sin 2\theta_r \omega_r I_r - L_{sr} \sin \theta_r \omega_r I_s \tag{10.21}$$

In this equation the terms involving $\omega_r$ are rotational voltages, and the larger these terms are, for a given angular velocity and for given winding currents, the more effective the machine will be as an energy convertor. For the commutation of the rotor winding current to take place without sparking between brushes and commutator, the brush axis must be approximately at right angles to the stator winding m.m.f. This condition is represented by substituting the value $\theta_r = -\pi/2$ in eqn. (10.21) and has the effect of making the

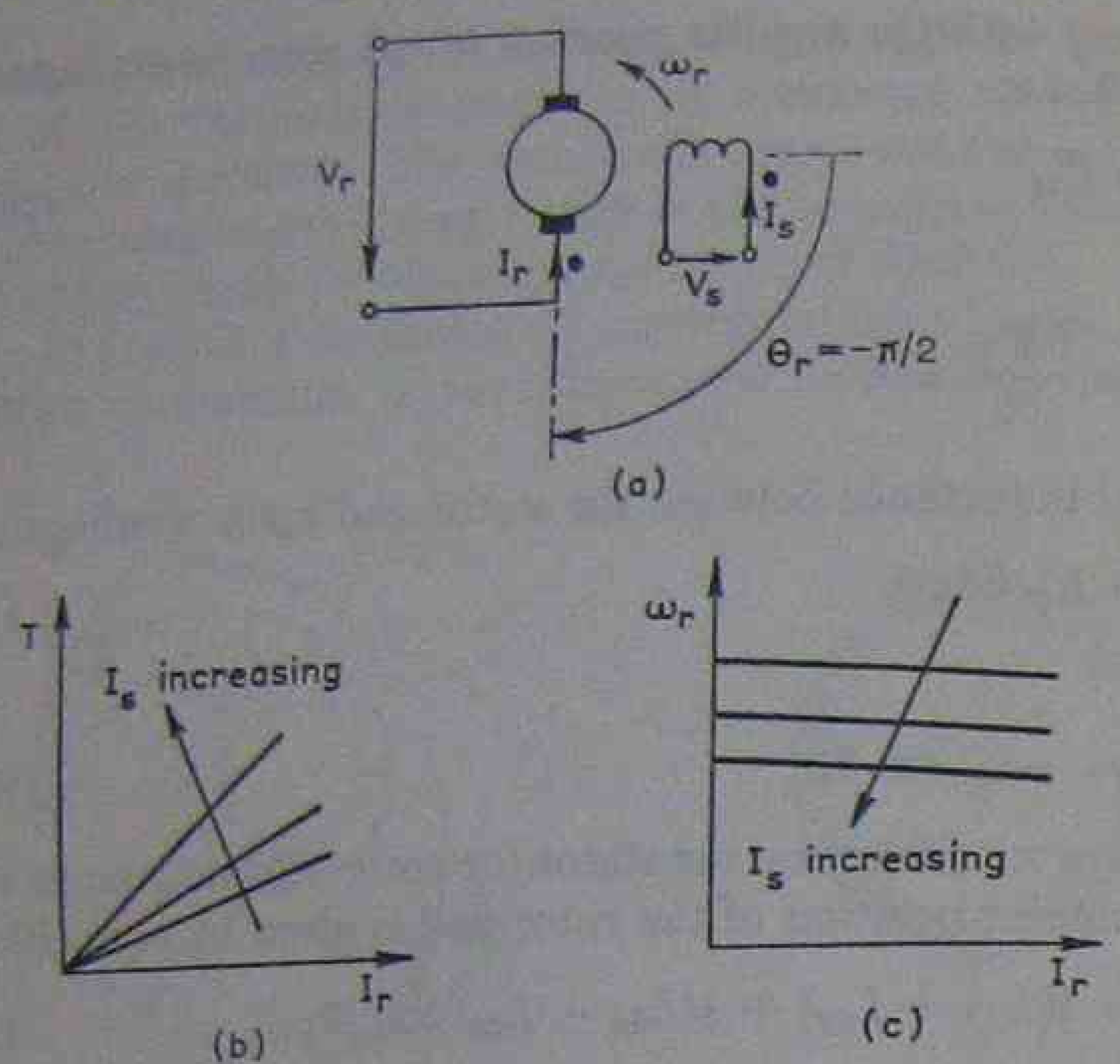



*Fig. 10.12* SEPARATELY EXCITED D.C. MOTOR

rotational voltage involving the term $(L_{dd} - L_{qq})$ zero but the rotational voltage involving the the term $L_{sr}$ a maximum. Substituting $\theta_r = -\pi/2$ in eqn. (10.21) has the advantage of removing from the equations minus signs which could be a source of confusion. Assigning the value $\theta_r = -\pi/2$ means that the dotted end of the rotor winding (i.e. the positive commutator brush) is placed at

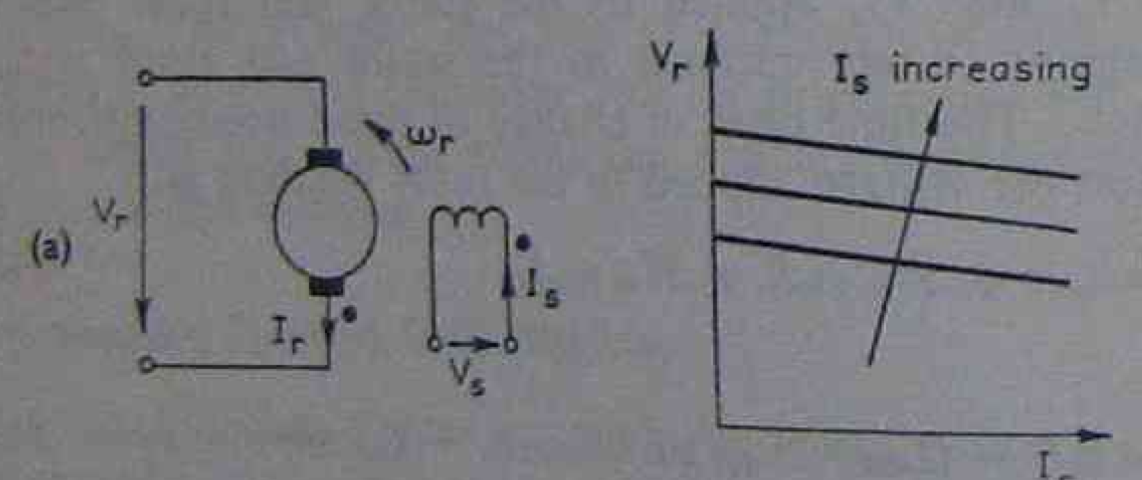



*Fig. 10.13* SEPARATELY EXCITED D.C. GENERATOR

$\theta_r = -\pi/2$, as shown in Figs. 10.12, 10.13 and 10.14. Carrying out this substitution gives

$$V_r = r_r I_r + L_{sr}\omega_r I_s \tag{10.22}$$

The machine configuration is shown in Fig. 10.12(*a*).



The voltage $L_{sr}\omega_r I_s$ is a steady, direct rotational voltage due to the rotation of the rotor winding in a magnetic field set up by the stator winding. The fact that it is a direct voltage is due to the effect of the commutator.

When attention is directed to the voltage applied to the stator winding it will be realized that no rotational voltage will appear in the winding. This is so because, in spite of rotor rotation, the rotor winding m.m.f. axis is fixed in space by the action of the commutator, and so the rotor flux does not change its linkage with the stator winding even when the rotor rotates. The voltage applied to the stator winding is therefore given by eqn. (10.10):

$$V_s = r_s I_s \tag{10.23}$$

The instantaneous torque developed is given by eqn. (10.14) as

$$T' = \tfrac{1}{2}\mathscr{L}_{rr}' i_r^2 + \mathscr{L}_{sr}' i_r i_s + \tfrac{1}{2}\mathscr{L}_{ss}' i_s^2 \tag{10.14}$$

As previously noted,

$$\mathscr{L}_{ss}' = 0 \tag{10.17}$$

$$\mathscr{L}_{sr}' = -L_{sr}\sin\theta_r \tag{10.18}$$

$$\mathscr{L}_{rr}' = -(L_{dd} - L_{qq})\sin 2\theta_r \tag{10.19}$$

The positive rotor brush or terminal is located at $\theta_r = -\pi/2$, and the rotor and stator windings carry steady currents $I_r$ and $I_s$. The steady torque $T$ is then given by substitution in eqn. (10.14) as

$$\begin{aligned} T &= -\tfrac{1}{2}(L_{dd} - L_{qq})\sin(-\pi)I_r^2 - L_{sr}\sin(-\pi/2)I_r I_s \\ &= L_{sr} I_r I_s \end{aligned} \tag{10.24}$$

The torque/rotor-winding-current characteristic is shown in Fig. 10.12(*b*).

The result shown in eqn. (10.24) may be confirmed by multiplying eqn. (10.22) by $I_r$, which gives

$$V_r I_r = r_r I_r^2 + L_{sr}\omega_r I_r I_s$$

The term $V_r I_r$ represents the input power to the rotor winding, and $L_{sr}\omega_r I_r I_s$ the portion which is available for conversion to mechanical power. Therefore

$$\omega_r T = L_{sr}\omega_r I_r I_s$$

or the torque on the rotor is

$$T = L_{sr} I_r I_s \tag{10.24}$$

From eqn. (10.22),

$$\omega_r = \frac{V_r - r_r I_r}{L_{sr} I_s} \tag{10.25}$$

Eqns. (10.22), (10.23) and (10.24) have been set up for conventionally positive current entering the windings corresponding to electrical power input and therefore motoring mode operation. Eqn. (10.24) shows the torque developed as positive, i.e. acting in the $+\theta_r$ direction. For motor operation it is to be expected that rotation will take place in the same direction as that in which the torque acts, and this is confirmed by the positive sign of $\omega_r$ given by eqn. (10.25).

For a constant applied rotor-winding voltage and a constant stator current, eqn. (10.25) shows that the speed of the separately excited d.c. machine operating in the motoring mode will remain almost constant as the rotor winding current varies with load, since the internal voltage drop $r_r I_r$ will be small compared with $V_r$ in any efficient machine. The speed/rotor-winding-current characteristic is shown in Fig. 10.12(*c*).

Eqns. (10.22), (10.23) and (10.24) apply equally to generator action. In this $I_r$ will be taken to emerge from the dotted end of the rotor winding and will be negative. As a result the torque due to the rotor winding will be negative and will act in the $-\theta_r$ direction. Therefore the rotor must be assumed to be driven in the $+\theta_r$ direction by the prime mover, to be consistent with this assumed current direction. Changing the sign of $I_r$ in eqn. (10.22),

$$V_r = -r_r I_r + L_{sr}\omega_r I_s \tag{10.26}$$

The $V_r/I_r$ characteristic of the separately excited d.c. generator is shown in Fig. 10.13(*b*).

**EXAMPLE 10.3** A separately excited d.c. machine is rotated at 500 rev/min by a prime mover. When the field (stator winding) current is 1 A the armature (rotor winding) generated voltage is 125 V with the armature open-circuited. The armature resistance is 0·1 Ω and the field resistance is 250 Ω. Determine:

(*a*) The rotational voltage coefficient, $L_{sr}\omega_r$.
(*b*) The maximum mutual inductance between the stator and rotor windings.
(*c*) The armature terminal voltage if the machine acts as a generator delivering a current of 200 A at a speed of 1,000 rev/min and the field current is 2 A.
(*d*) The input current and speed if the machine acts as a motor and develops a gross torque of 1,000 N-m. The armature and field windings are each excited from a 500 V supply.

Neglect iron loss and the effect of magnetic saturation.



Using the assumptions previously made the steady-state operating equations are

$$V_r = r_r I_r + L_{sr}\omega_r I_s \tag{10.22}$$

$$V_s = r_s I_s \tag{10.23}$$

$$T = L_{sr} I_r I_s \tag{10.24}$$

(*a*) Adhering to the previous sign conventions and considering the armature winding open-circuit test, $V_r = 125\,\text{V}$, $I_r = 0$, $I_s = 1\,\text{A}$. Substituting in eqn. (10.22),

$$125 = (0{\cdot}1 \times 0) + (L_{sr}\omega_{r1} \times 1)$$

Therefore

$$L_{sr}\omega_{r1} = \underline{\underline{125\,\text{V/A}}}$$

(*b*) $$L_{sr} = \frac{125}{\omega_{r1}} = \frac{125}{2\pi 500/60} = \underline{\underline{2{\cdot}39\,\text{H}}}$$

(*c*) When the speed of the machine is doubled, this will double the value of the voltage coefficient. Substituting the given data for generator action in (10.22),

$$V_r = \{0{\cdot}1 \times (-200)\} + \left\{125 \times \frac{1{,}000}{500} \times 2\right\} = \underline{\underline{480\,\text{V}}}$$

Note that $I_r = -200\,\text{A}$ for generator action.

(*d*) From eqn. (10.25),

$$I_s = \frac{V_s}{r_s} = \frac{500}{250} = 2\,\text{A}$$

In eqn. (10.24),

$$1{,}000 = 2{\cdot}39 \times I_r \times 2 \quad \text{so that} \quad I_r = \frac{1{,}000}{2{\cdot}39 \times 2} = \underline{\underline{209\,\text{A}}}$$

If $\omega_{r2}$ is the new angular velocity, then from (10.22) the new voltage coefficient, $L_{sr}\omega_{r2}$, is

$$L_{sr}\omega_{r2} = \frac{V_r - r_r I_r}{I_s} = \frac{500 - (0{\cdot}1 \times 209)}{2} = 240\,\text{V/A}$$

The new speed is

$$n_2 = n_1\left(\frac{L_{sr}\omega_{r2}}{L_{sr}\omega_{r1}}\right) = 500 \times \frac{240}{125} = \underline{\underline{960\,\text{rev/min}}}$$

## 10.9 Shunt and Series D.C. Machines

The stator winding of a d.c. machine is usually excited from the same supply as the rotor winding. The stator winding may be connected in parallel with the rotor winding across the supply to form a d.c. *shunt machine* or in series with the rotor winding to form a d.c. *series machine*.

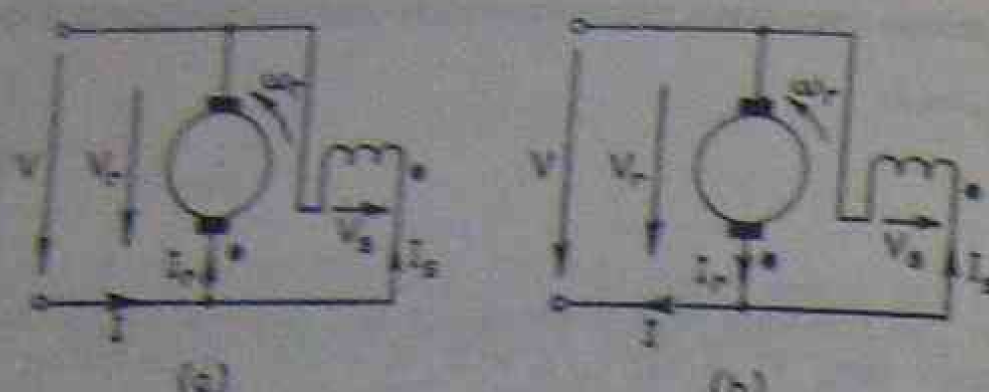



*Fig. 10.14* D.C. SHUNT MACHINE

Fig. 10.14(*a*) shows the connexion diagram for a d.c. shunt machine operating in the motoring mode, and Fig. 10.14(*b*) shows it operating in the generating mode. The operating equations for the shunt machine may be obtained from those of the separately excited machine. In eqns. (10.22) and (10.23), putting $V_r = V$ and $V_s = V$ gives

$$V = r_r I_r + L_{sr}\omega_r I_s \qquad (10.27)$$

$$V = r_s I_s \qquad (10.28)$$

The torque equation remains unchanged as

$$T = L_{sr} I_r I_s \qquad (10.24)$$

Referring to Fig. 10.14(*a*) for motor-mode operation,

$$I = I_r + I_s \qquad (10.29)$$

The equations for generating action are obtained by putting $I_r = -I_r$ in eqns. (10.27) and (10.24). In addition, for generator action,

$$I_r = I + I_s \qquad (10.30)$$

The characteristics of the shunt machine are similar to those of the separately excited machine shown in Figs. 10.12 and 10.13. The establishment of a stable output voltage for shunt generator operation requires some saturation of the magnetic circuit.

Fig. 10.15(*a*) shows the connexion diagram for a d.c. series machine operating in the motoring mode. The operating equations for the series machine may also be obtained from those of the separately excited machine. Substituting $I_r = I$ and $I_s = I$ in eqns. (10.22), (10.23) and (10.24) gives

$$V_r = r_r I + L_{sr}\omega_r I \qquad (10.31)$$

$$V_s = r_s I \qquad (10.32)$$

$$T = L_{sr} I^2 \qquad (10.33)$$



The torque/current characteristic of the d.c. series motor is shown in Fig. 10.15(*b*). From Fig. 10.15(*a*),

$$V = V_r + V_s = r_r I + L_{sr}\omega_r I + r_s I$$

$$= (r_r + r_s)I + L_{sr}\omega_r I \qquad (10.34)$$

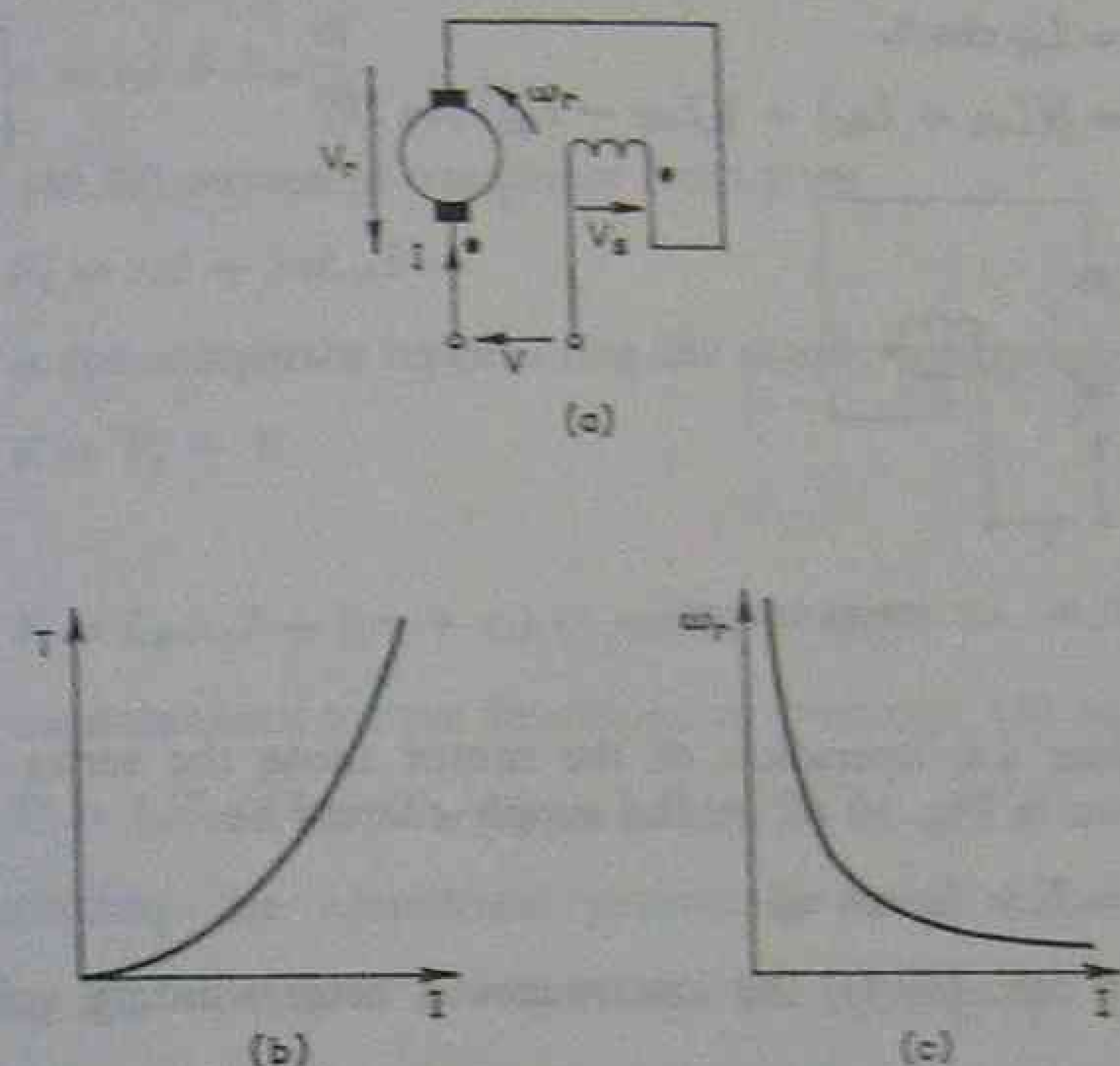



*Fig. 10.15* D.C. SERIES MOTOR

From this equation,

$$\omega_r = \frac{V - (r_r + r_s)I}{L_{sr} I} \qquad (10.35)$$

Since $(r_r + r_s)I$ is very much smaller than $V$, the speed of the d.c. series motor is approximately inversely proportional to the input current. Therefore, on light loads dangerously high speeds could be reached. In practical applications of the motor, protective devices are used to guard against this contingency. The speed/current characteristic is shown in Fig. 10.15(*c*).

The output voltage of a d.c. series generator is approximately proportional to the output current. The establishment of this output voltage also is dependent upon there being some saturation of the magnetic circuit.

## 10.10 Universal Motor

The universal motor is a series connected motor suitable for operation on either a.c. or d.c. supplies.

As previously, the inductance coefficients are

$$\mathcal{L}_{ss} = L_{ss}$$

$$\mathcal{L}_{sr} = L_{sr}\cos\theta_r \qquad (10.16)$$

$$\mathcal{L}_{rr} = \tfrac{1}{2}(L_{dd} + L_{qq}) + \tfrac{1}{2}(L_{dd} - L_{qq})\cos 2\theta_r \qquad (10.4)$$

(10.6)

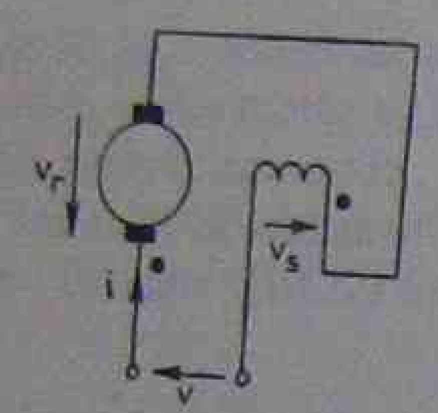



*Fig. 10.16* A.C. SERIES MOTOR

To consider a.c. operation of the motor using the series connexion shown in Fig. 10.16, let the supply current be

$$i = i_r = i_s = I_m\cos\omega t$$

Following eqn. (10.9), the instantaneous rotor-winding voltage is

$$v_r = r_r i + \mathcal{L}_{rr}'\omega_r i + \mathcal{L}_{sr}'\omega_r i + \mathcal{L}_{rr}\frac{di}{dt} + \mathcal{L}_{sr}\frac{di}{dt} \qquad (10.9)$$

The dotted end of the rotor winding is at $\theta_r = -\pi/2$; when substituted in the above equations this gives $\mathcal{L}_{rr}' = 0$, $\mathcal{L}_{sr}' = L_{sr}$, $\mathcal{L}_{rr} = \tfrac{1}{2}(L_{dd} + L_{qq}) = L_{rr}$, say, and $\mathcal{L}_{sr} = 0$. Eqn. (10.9) then becomes

$$v_r = r_r i + L_{sr}\omega_r i + L_{rr}\frac{di}{dt} \qquad (10.36)$$

This equation may be written in complexor form. Let $V_r$ be the complexor corresponding to $v_r$ and $I$ the complexor corresponding to $i$. Then

$$V_r = r_r I + L_{sr}\omega_r I + j\omega L_{rr} I \qquad (10.37)$$

Due to the action of the commutator in fixing the axis of the rotor-winding m.m.f., no rotational voltages appear in the stator winding whether operation is from a d.c. or an a.c. supply. The



instantaneous stator-winding voltage is therefore, from eqn. (10.10),

$$v_s = r_s i + \mathcal{L}_{ss}\frac{di}{dt} + \mathcal{L}_{sr}\frac{di}{dt} \qquad (10.38)$$

$\mathcal{L}_{ss} = L_{ss}$, and with the rotor winding at $\theta_r = -\pi/2$, $\mathcal{L}_{sr} = 0$. This gives

$$v_s = r_s i + L_{ss}\frac{di}{dt} \qquad (10.39)$$

Eqn. (10.38) written in complexor form gives

$$V_s = r_s I + j\omega L_{ss} I \qquad (10.40)$$

If $V$ is the complexor representing the supply voltage, then

$$V = V_r + V_s$$

i.e.

$$V = L_{sr}\omega_r I + [(r_r + r_s) + j\omega(L_{rr} + L_{ss})]I \qquad (10.41)$$

The instantaneous torque developed is, from eqn. (10.14),

$$T' = \tfrac{1}{2}\mathcal{L}_{rr}' i_r^2 + \mathcal{L}_{sr}' i_r i_s + \tfrac{1}{2}\mathcal{L}_{ss}' i_s^2 \qquad (10.14)$$

Substituting the conditions previously found ($\mathcal{L}_{rr}' = 0$, $\mathcal{L}_{sr}' = L_{sr}$, $\mathcal{L}_{ss}' = 0$, $i_r = i_s = i$) in eqn. (10.14) gives

$$T' = L_{sr} i^2 = L_{sr} I_m^2 \cos^2\omega t = L_{sr}\frac{I_m^2}{2}(\cos 2\omega t + 1) \qquad (10.42)$$

This equation therefore shows that for a.c. operation the torque developed by the machine consists of two components, a steady torque and one that pulsates at twice the supply frequency. The average torque is

$$T = L_{sr} I^2 \qquad (10.43)$$

where $I$ is the r.m.s. value of $i$. The torque/current characteristic is therefore the same as for d.c. operation.

Universal motors usually have a compensating winding on the stator with its m.m.f. axis coinciding with the rotor brush axis. The compensating winding is connected in series opposition with the rotor winding and serves to reduce the voltage drop in the internal reactance as well as assisting commutation. It has been neglected in the above analysis. A laminated stator construction is essential for a.c. operation.

**EXAMPLE 10.4** A 0·1 kW series motor has the following constants:

| | | |
|---|---|---|
| Armature resistance | $r_r$ | $= 12\,\Omega$ |
| Series field resistance | $r_s$ | $= 36\,\Omega$ |
| Effective armature inductance | $L_{rr}$ | $= 0{\cdot}3\,H$* |
| Series field inductance | $L_{ss}$ | $= 0{\cdot}34\,H$ |
| Maximum mutual inductance between rotor and stator windings | $L_{sr}$ | $= 0{\cdot}71\,H$ |

Determine the input current and speed when the load torque applied to the motor is 0·18 N-m (*a*) when connected to a 200 V d.c. supply, and (*b*) when connected to a 200 V 50 Hz a.c. supply.

Neglect windage and friction and all core losses.

(*a*) Considering first d.c. operation, the applied voltage is

$$V = (r_r + r_s)I + L_{sr}\omega_r I \tag{10.34}$$

and the torque developed is

$$T = L_{sr}I^2 \tag{10.33}$$

From eqn. (10.33),

$$I = \sqrt{\frac{T}{L_{sr}}} = \sqrt{\frac{0{\cdot}18}{0{\cdot}71}} = \underline{\underline{0{\cdot}504\,A}}$$

From eqn. (10.34),

$$\omega_r = \frac{V - (r_r + r_s)I}{L_{sr}I} = \frac{200 - (48 \times 0{\cdot}504)}{0{\cdot}71 \times 0{\cdot}504} = 492\,\text{rad/s}$$

$$n_r = \frac{\omega_r}{2\pi} \times 60 = \frac{492}{2\pi} \times 60 = \underline{\underline{4{,}700\,\text{rev/min}}}$$

(*b*) Considering a.c. operation, the r.m.s. current is, from eqn. (10.43),

$$I = \sqrt{\frac{T}{L_{sr}}} = \sqrt{\frac{0{\cdot}18}{0{\cdot}71}} = \underline{\underline{0{\cdot}504\,A}}$$

The applied voltage is

$$V = L_{sr}\omega_r I + [(r_r + r_s) + j\omega(L_{rr} + L_{ss})]I$$

Taking $I$ as the reference complexor,

$$200\underline{/\theta} = 0{\cdot}71 \times 0{\cdot}504\omega_r\underline{/0^\circ} + [48 + j2\pi \times 50(0{\cdot}3 + 0{\cdot}34)]0{\cdot}504\underline{/0^\circ}$$

$$200\cos\theta + j200\sin\theta = 0{\cdot}358\omega_r + 24{\cdot}2 + j101$$

Equating quadrate parts in this equation,

$$200\sin\theta = 101$$

whence $\sin\theta = 0{\cdot}505$, $\cos\theta = 0{\cdot}864$.

Equating reference parts,

$$200\cos\theta = 0{\cdot}358\omega_r + 24$$

$$\omega_r = \frac{200 \times 0{\cdot}864 - 24{\cdot}2}{0{\cdot}358} = 415\,\text{rad/s}$$

$$n_r = \frac{415}{2\pi} \times 60 = \underline{\underline{3{,}970\,\text{rev/min}}}$$

* The actual armature inductance $\approx L^2_{sr}/L_{ss} \approx 1{\cdot}48$ H. The effective value is reduced to 0·3 H due to the effect of a compensating winding connected in series opposition with the armature.



## 10.11 Rotating Field due to a Three-phase Winding

Fig. 10.17 shows a stator winding with three diametral coils $aa'$, $bb'$ and $cc'$, each having $N_s$ turns. The dots and crosses indicate the direction of conventionally positive current in each coil as explained

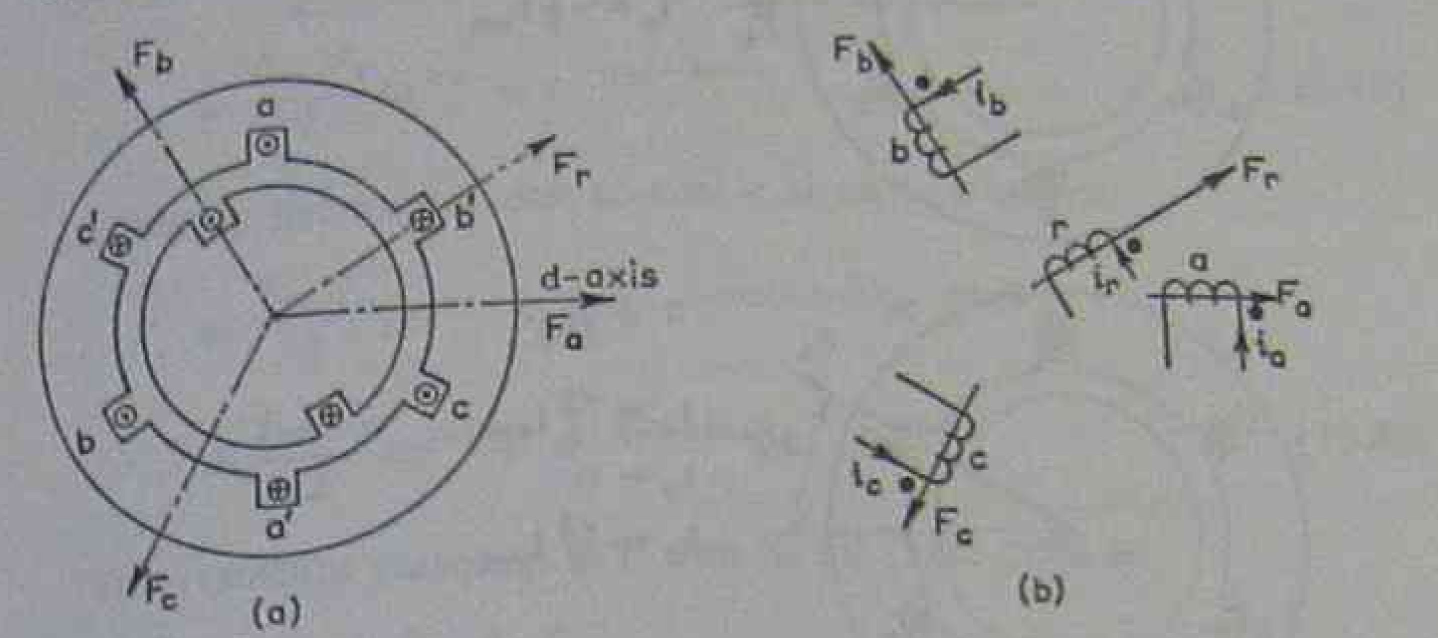



*Fig. 10.17* M.M.F. DUE TO A 3-PHASE WINDING

in Section 10.3. The axes of the coil m.m.f.s are therefore mutually displaced by $2\pi/3$ radians, as shown in Fig. 10.17.

Suppose the three coils are supplied with balanced 3-phase currents, $i_a$, $i_b$ and $i_c$, such that

$$i_a = I_{sm}\cos\omega t = \frac{I_{sm}}{2}(e^{j\omega t} + e^{-j\omega t}) \tag{10.44}$$

$$i_b = I_{sm}\cos(\omega t - 2\pi/3) = \frac{I_{sm}}{2}(e^{j(\omega t - 2\pi/3)} + e^{-j(\omega t - 2\pi/3)}) \tag{10.45}$$

$$i_c = I_{sm}\cos(\omega t + 2\pi/3) = \frac{I_{sm}}{2}(e^{j(\omega t + 2\pi/3)} + e^{-j(\omega t + 2\pi/3)}) \tag{10.46}$$

The m.m.f. of coil $a$ is directed in the reference direction when $i_a$ is positive. The instantaneous value of this m.m.f. is therefore

$$F_a' = \frac{I_{sm}N_s}{2}(e^{j\omega t} + e^{-j\omega t})e^{j0} \tag{10.47*}$$

This expression has been multiplied by $e^{j0}$ (= 1) to indicate that it acts in the space reference direction.

* To avoid confusion with $f$ for frequency, instantaneous m.m.f. will be represented by $F'$.

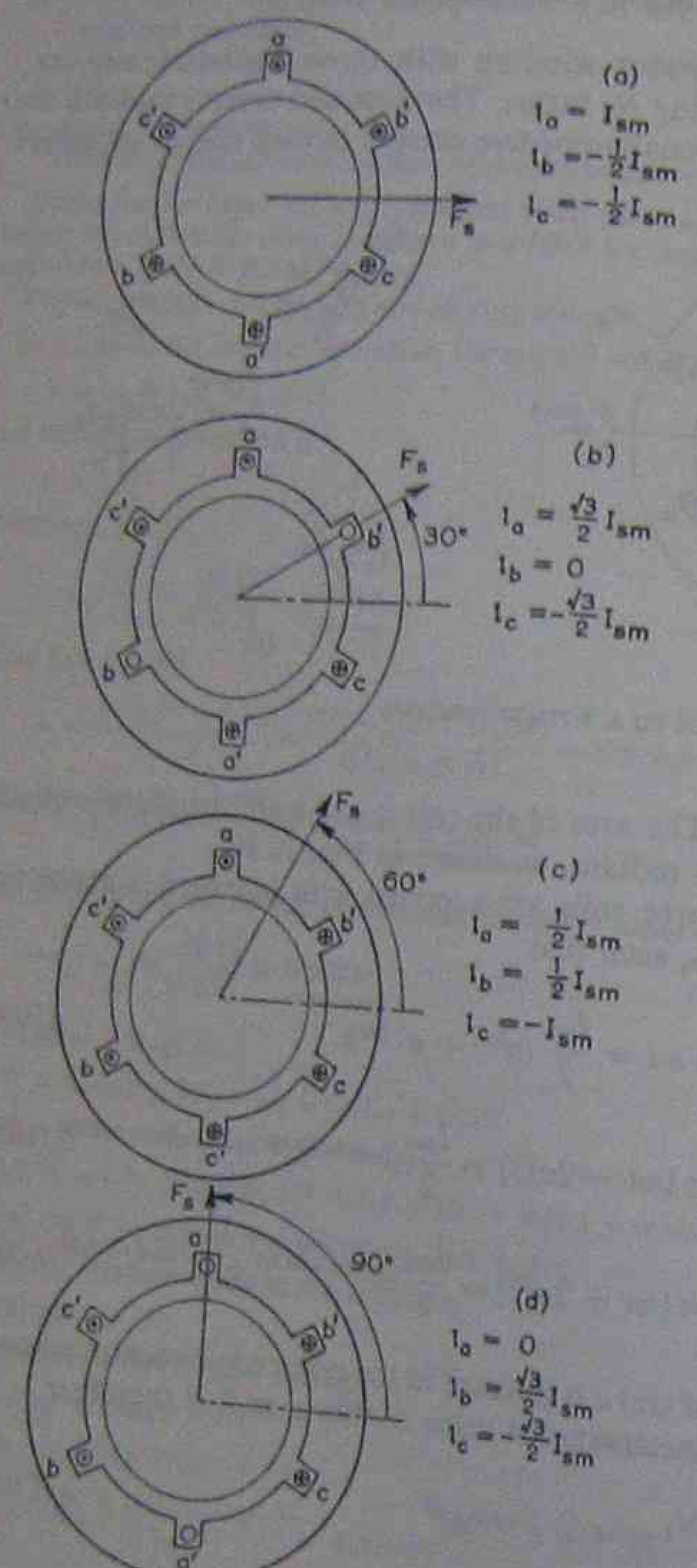



*Fig. 10.18* M.M.F. DUE TO A 3-PHASE WINDING AT DIFFERENT INSTANTS



The m.m.f. of coil $b$ is directed along an axis $+2\pi/3$ radians from the reference direction when $i_b$ is positive. The instantaneous value of this m.m.f. is therefore

$$F_b' = \frac{I_{sm}N_s}{2}(e^{j(\omega t - 2\pi/3)} + e^{-j(\omega t - 2\pi/3)})e^{j2\pi/3}$$

$$= \frac{I_{sm}N_s}{2}(e^{j\omega t} + e^{-j(\omega t - 4\pi/3)}) \tag{10.48}$$

Similarly the m.m.f. due to coil $c$ at any instant is

$$F_c' = \frac{I_{sm}N_s}{2}(e^{j(\omega t + 2\pi/3)} + e^{-j(\omega t + 2\pi/3)})e^{-j2\pi/3}$$

$$= \frac{I_{sm}N_s}{2}(e^{j\omega t} + e^{-j(\omega t + 4\pi/3)}) \tag{10.49}$$

The resultant stator m.m.f. due to all three coils is

$$F_s' = F_a' + F_b' + F_c'$$

$$= \frac{I_{sm}N_s}{2}[e^{j\omega t} + e^{-j\omega t} + e^{j\omega t} + e^{-j(\omega t - 4\pi/3)} + e^{j\omega t} + e^{-j(\omega t + 4\pi/3)}]$$

Since $e^{-j\omega t} + e^{-j(\omega t - 4\pi/3)} + e^{-j(\omega t + 4\pi/3)} = 0$,

$$F_s' = \tfrac{3}{2} I_{sm}N_s e^{j\omega t} \tag{10.50}$$

This equation shows that, when three coils are so positioned that their m.m.f. axes are mutually displaced by $2\pi/3$ radians and are then supplied with balanced 3-phase currents, an m.m.f. of constant magnitude results and the m.m.f. axis rotates at an angular velocity of $\omega$ radians per second.

For the coil configuration and phase sequence chosen the direction of rotation is in the $+\theta$ direction. It will be found that, if the phase sequence is reversed, the direction of rotation of the resultant m.m.f. axis is also reversed.

Fig. 10.18 shows the m.m.f. due to a 3-phase winding supplied with balanced 3-phase currents for a number of different instants. At (*a*) the current in phase $a$ is positive maximum value and the currents in the two other phases are half the negative maximum value. The negative currents are indicated by showing the current in the cross direction in coil sides $b$ and $c$, and in the dot direction in coil sides $b'$ and $c'$. $F_s$ is shown acting along the stator m.m.f. axis.

Figs. 10.18(*b*), (*c*) and (*d*) show successive instants in the 3-phase cycle corresponding to 30° rotations of the complexor diagram.

It will be seen that the axis of the stator m.m.f. is also displaced by successive steps of 30° in the $+\theta$ direction, so that $F_s$ completes one revolution in each cycle and thus must rotate with an angular velocity of $\omega$ radians per second. This is in agreement with eqn. (10.50).

Eqn. (10.50) also shows that the m.m.f. due to a 3-phase winding when excited by balanced 3-phase currents could be represented as the m.m.f. of a single winding of $N_s$ turns and excited with a direct current of value $\frac{3}{2}I_{sm}$, where the winding is considered to rotate at an angular velocity $\omega$ and $N_s$ represents the number of turns of each stator phase.

In Chapter 11 the resultant m.m.f. due to 3-phase distributed windings is considered, and the effect of space harmonics is discussed. These are ignored in the present treatment.

### 10.12 Three-phase Synchronous Machine

In the previous section it has been shown that, when three coils have their m.m.f. axes mutually displaced by $2\pi/3$ and are then supplied with balanced 3-phase currents, an m.m.f. of constant magnitude results, the m.m.f. axis rotating at $\omega$ radians per second. If a constant angular displacement is to be maintained between the resultant stator and rotor m.m.f.s, as is required for the continuous production of torque, the rotor m.m.f. must also rotate at $\omega$ in the same direction as the stator m.m.f.

This rotation of the rotor m.m.f. may be brought about in a number of different ways. In the synchronous machine the rotor winding is excited with direct current supplied through slip rings. The axis of the rotor m.m.f. then rotates at the same speed as the rotor itself, so that the condition for continuous torque production is that the rotor should rotate at $\omega$ in the same direction as the resultant stator m.m.f. axis.

The rotors of synchronous machines are often of the salient-pole type shown in Fig. 10.5(*a*), but for simplicity only the non-salient pole type of rotor as shown in Fig. 10.5(*b*) will be considered. Instead of the stator phase windings consisting of the single coils considered in Section 10.11, the phase windings consist of several coils distributed in slots and occupying the whole stator periphery as shown in Fig. 10.19. The effect of this distribution of the winding is to introduce constants called *distribution factors* into equations relating to the operation of the machine. These constants are ignored here; in most practical cases they have numerical values close to unity.

The coupling between the d.c. excited rotor winding $r$ (see Fig. 10.17) and the stator reference phase $a$ is a cosine-like or even

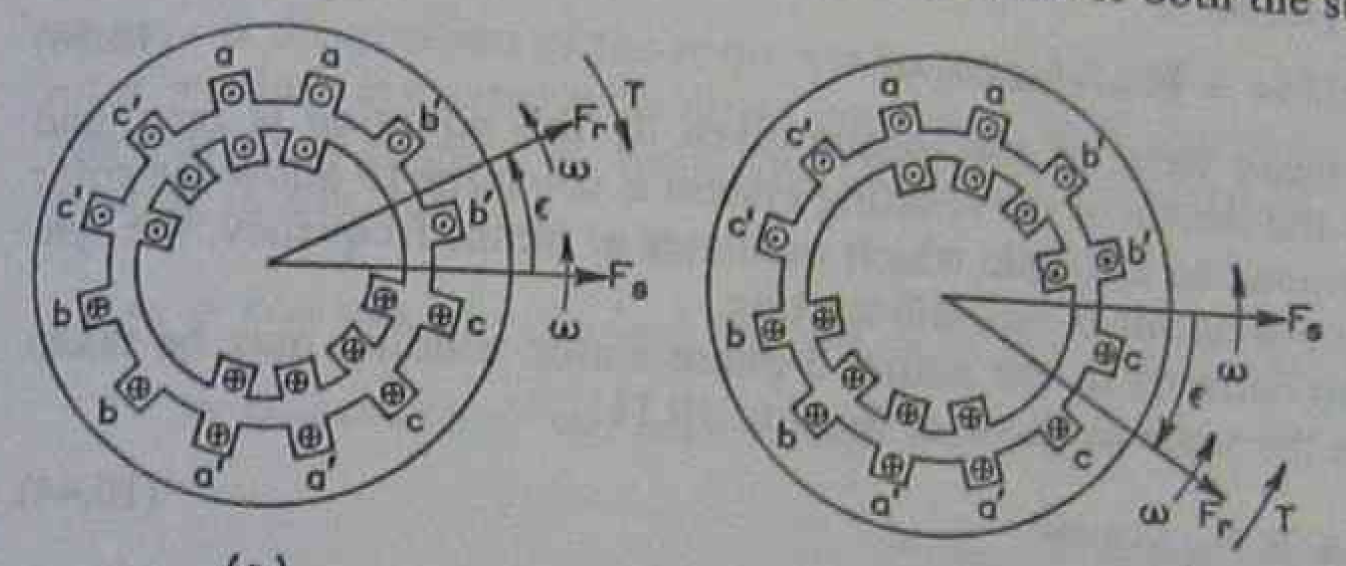


function which, ignoring space harmonics and taking $\theta_r$ as the instantaneous angle of the axis of the rotor winding with respect to phase $a$, is

$$\mathscr{L}_{ar} = L_{sr}\cos\theta_r \tag{10.51}$$

Since it is assumed that there are no salient poles, all the self-inductances are constant and their angular rates of change are zero.

As explained in the previous section, the 3-phase stator winding may be considered to be replaced by a representative stator windings of $N_s$ turns, excited by a direct current of $\frac{3}{2}I_{sm}$ and rotating at an angular velocity of $\omega$ radians per second. The axes of both the stator



(a) (b)

*Fig. 10.19* SYNCHRONOUS MACHINE
(*a*) Generating (*b*) Motoring

and rotor windings therefore rotate at $\omega$. If the angular displacement between these axes is $\epsilon$ as shown in Fig. 10.19, then, from eqn. (10.51),

$$\mathscr{L}_{sr} = L_{sr}\cos\epsilon \tag{10.52}$$

When $\epsilon$ is positive the rotor m.m.f. axis is displaced anticlockwise from the stator m.m.f. axis. The angular rate of change of this mutual inductance is

$$\mathscr{L}_{sr}' = -L_{sr}\sin\epsilon \tag{10.53}$$

The instantaneous torque on the rotor is given by eqn. (10.14) as

$$T' = \tfrac{1}{2}\mathscr{L}_{rr}'i_r^2 + \mathscr{L}_{sr}'i_ri_s + \tfrac{1}{2}\mathscr{L}_{ss}'i_s^2 \tag{10.14}$$

The rotor winding current $i_r$ is $I_r$, a steady, direct current, and the representative stator winding carries a current of $\frac{3}{2}I_{sm}$, where $I_{sm}$ is the maximum current per phase in the actual 3-phase winding. Substituting for these currents and for the angular rates of change of inductance in eqn. (10.14),

$$T' = -\tfrac{3}{2}L_{sr}I_rI_{sm}\sin\epsilon \tag{10.54}$$

For a steady angular displacement $\epsilon_s$ between the axes of the stator and rotor m.m.f.s, the mean torque on the rotor is

$$T = -\tfrac{3}{2} L_{sr} I_r I_{sm} \sin \epsilon_s \qquad (10.55)$$

It is to be noted that under steady conditions the 3-phase machine, unlike the single-phase machine, does not produce an oscillating component of torque.

At starting, as a motor, the rotor angular velocity $\omega_r$ is zero, so that the displacement between the rotor and stator m.m.f. axes is $\epsilon = \omega t$, and the instantaneous torque on the rotor given by eqn. (10.54) is

$$T' = -\tfrac{3}{2} L_{sr} I_r I_{sm} \sin \omega t \qquad (10.56)$$

The mean value of the torque given by this equation is zero, and since the inertia of the rotating system is too large to allow the rotor to respond to a torque which oscillates at mains frequency, the synchronous motor is not self-starting.

The currents in the actual 3-phase stator winding may be taken to be the same as those of Section 10.11:

$$i_a = I_{sm} \cos \omega t \qquad (10.44)$$

$$i_b = I_{sm} \cos (\omega t - 2\pi/3) \qquad (10.45)$$

$$i_c = I_{sm} \cos (\omega t + 2\pi/3) \qquad (10.46)$$

For the stator phase currents so chosen, the stator m.m.f. axis at $t = 0$ is along the positive direction of the $d$-axis, and therefore $\epsilon_s$ is the angle of separation of the rotor and stator m.m.f. axes as shown in Fig. 10.19. When $\epsilon_s$ is positive the rotor m.m.f. axis is displaced anticlockwise from the stator m.m.f. axis, and as shown by eqn. (10.55), the torque on the rotor acts in the $-\theta$ direction, i.e. in the direction opposite to rotation. Under such circumstances the machine acts as a generator, the rotor being driven against the direction of the torque developed on it by a prime mover. When $\epsilon_s$ is negative the torque acts in the $+\theta$ direction, i.e. in the same direction as rotation, and the machine acts as a motor.

When the machine is unloaded $\epsilon_s = 0$, corresponding to the alignment of the rotor and stator m.m.f. axes. As load is imposed the value of $\epsilon_s$ increases, the rotor and stator m.m.f. axes are displaced and the appropriate torque is developed.

The operation of the synchronous machine is illustrated in Figs. 10.19($a$) and ($b$). In both diagrams the stator current distribution is drawn for the instant in the 3-phase cycle when $i_a = I_{sm}$ and $i_b = i_c = -\tfrac{1}{2} I_{sm}$, so that the axis of the stator m.m.f. is along the



positive direction of the $d$-axis. Fig. 10.19($a$) illustrates generator action and Fig. 10.19($b$) motor action.

If the stator phases $b$ and $c$ are assumed to be open-circuited, then the voltage applied to stator phase $a$ may be obtained by adapting the subscripts of eqn. (10.10) as

$$v_a = r i_a + \mathcal{L}_{ar}' \omega_r i_r + \mathcal{L}_{aa} \frac{di_a}{dt} + \mathcal{L}_{ar} \frac{di_r}{dt} \qquad (10.57)$$

$$\mathcal{L}_{ar} = L_{sr} \cos \theta_r \qquad (10.51)$$

$$\mathcal{L}_{ar}' = -L_{sr} \sin \theta_r = -L_{sr} \sin (\omega_r t + \epsilon) \qquad (10.58)$$

where $\epsilon$ is the position of the rotor winding axis at $t = 0$. Since the rotor winding is excited with direct current, $i_r = I_r$ and $di_r/dt = 0$. The current in phase $a$ is, from eqn. (10.44), $i_a = I_{sm} \cos \omega t$. Substituting in eqn. (10.57),

$$\begin{aligned} v_a &= r I_{sm} \cos \omega t - \omega_r L_{sr} I_r \sin (\omega_r t + \epsilon) + L_{aa} \frac{d}{dt} (I_{sm} \cos \omega t) \\ &= r I_{sm} \cos \omega t + \omega_r L_{sr} I_r \cos (\omega_r t + \epsilon + \pi/2) \\ &\quad + L_{aa} \frac{d}{dt} (I_{sm} \cos \omega t) \end{aligned} \qquad (10.59)$$

Under normal operating conditions all three phases carry current, and under balanced conditions this has the effect of increasing the effective inductance per phase by approximately 50 per cent because of the mutual inductance between phases. If the effective inductance per phase is $L_{ss}$, eqn. (10.59) becomes

$$\begin{aligned} v_a &= r I_{sm} \cos \omega t + \omega_r L_{sr} I_r \cos (\omega_r t + \epsilon + \pi/2) \\ &\quad + L_{ss} \frac{d}{dt} (I_{sm} \cos \omega t) \end{aligned} \qquad (10.60)$$

In complexor form eqn. (10.60) becomes

$$V_s e^{j\delta} = \frac{\omega_r L_{sr} I_r}{\sqrt{2}} e^{j(\epsilon + \pi/2)} + (r + j\omega L_{ss}) \frac{I_{sm}}{\sqrt{2}} e^{j0} \qquad (10.61)$$

or

$$V_s = E_s + Z_s I \qquad (10.62)$$

**EXAMPLE 10.5** A 2-pole 1,000 V 50 Hz synchronous machine has a 3-phase star-connected stator winding each phase of which has an effective inductance of 0·01 H and negligible resistance. The maximum mutual inductance between the rotor winding and a stator phase is 0·4 H.

($a$) Determine the developed torque, the stator phase current, the rotor winding current, the angle between the stator and rotor m.m.f. axes and the induced rotational voltage per phase when the machine acts as a motor with an output power of 224 kW. The input power factor is unity and the stator line voltage is 1,000 V. Neglect all losses.

(b) Determine the load current and output power when the machine acts as a generator if the rotor current is 12 A, the output power factor is 0·8 lagging and the stator line terminal voltage is 1,000 V. Find also the angle between the rotor and stator m.m.f. axes and the phase angle between the stator phase terminal voltage and the stator phase induced rotational voltage. Neglect the effects of distribution of the windings and of magnetic saturation.

(a) Rotor angular velocity, $\omega_r = 2\pi f = 2\pi \times 50 = 314\,\text{rad/s}$.

Developed torque, $T = \dfrac{P}{\omega_r} = \dfrac{224{,}000}{314} = \underline{\underline{712\,\text{N-m}}}$

Neglecting all losses,

$$\sqrt{3}V_L I_L \cos\phi = \text{Power output}$$

For the star connexion,

$$I_p = I_L = \frac{224{,}000}{\sqrt{3} \times 1{,}000 \times 1} = \underline{\underline{129\,\text{A}}}$$

From eqn. (10.61) the impedance per phase is

$$Z_s = r + j\omega L_{ss} = 0 + (j314 \times 0{\cdot}01) = j3{\cdot}14\,\Omega$$

From eqn. (10.62),

$$V = E + ZI$$

Taking the stator current as the reference complexor, and remembering that the input power factor is unity,

$$\frac{1{,}000}{\sqrt{3}}\underline{/0^\circ} = E\underline{/\epsilon + 90^\circ} + (j3{\cdot}14 \times 129)\underline{/0^\circ}$$

Therefore

$$E\underline{/\epsilon + 90^\circ} = 577 - j405 = 706\underline{/-35^\circ}$$

Thus the magnitude of the induced rotational voltage per phase is $\underline{\underline{706\,\text{V}}}$

The angle between the stator and rotor m.m.f. axes is

$$\epsilon = -35 - 90 = \underline{\underline{-125^\circ}}$$

From eqn. (10.55) the mean torque is

$$T = -\tfrac{3}{2}L_{sr}I_r I_{sm}\sin\epsilon \qquad (10.55)$$

Thus the rotor current is

$$I_r = -\frac{712}{\frac{3}{2} \times 0{\cdot}4 \times \sqrt{2} \times 129\sin(-125^\circ)} = \underline{\underline{7{\cdot}95\,\text{A}}}$$

As a check, from eqn. (10.73),

$$E = \frac{L_{sr}\omega_r I_r}{\sqrt{2}} = \frac{0{\cdot}4 \times 314 \times 7{\cdot}95}{\sqrt{2}} = 706\,\text{V}$$

(b) From eqn. (10.62),

$$I = \frac{V - E}{Z}$$



Taking the stator current as the reference complexor, then for an output power factor of 0·8 lagging, the phase voltage will *lead* the current by $\cos^{-1} 0{\cdot}8 = 36{\cdot}9^\circ$. Therefore

$$I\underline{/0^\circ} = \frac{\dfrac{577\underline{/+36{\cdot}9^\circ} + 0{\cdot}4 \times 314 \times 12\underline{/\epsilon + 90^\circ}}{\sqrt{2}}}{j3{\cdot}14}$$

and

$$I = 184\underline{/-53{\cdot}1^\circ} - 339\underline{/\epsilon}$$
$$= 110 - j147 - (339\cos\epsilon + j339\sin\epsilon)$$

The quadrate part of the complex expression for the current is zero; hence

$$-147 - 339\sin\epsilon = 0$$

$$\sin\epsilon = -\frac{147}{339} = -0{\cdot}433 \quad \epsilon = -25{\cdot}7^\circ \quad \text{and} \quad \cos\epsilon = 0{\cdot}901$$

Thus

$$I = 110 - (339 \times 0{\cdot}901) = \underline{\underline{-196\,\text{A}}}$$

and

$$I = \underline{\underline{196\underline{/180^\circ}\,\text{A}}}$$

The negative value of current corresponds to generating action and the reversal of current with respect to the terminal voltage. The axis of the stator m.m.f. at $t = 0$ is therefore at 180° whereas that of the rotor m.m.f. is at $\epsilon = -25{\cdot}7^\circ$. The axis of the rotor m.m.f. is thus displaced from that of the stator m.m.f. by $180 - 25{\cdot}7 = 154{\cdot}3^\circ$ in the $+\theta$ direction as is to be expected for generator action.

$$\text{Output power} = \frac{3 \times 577 \times 196 \times 0{\cdot}8}{1{,}000} = \underline{\underline{271\,\text{kW}}}$$

The phase angle between the induced rotational voltage per phase and the terminal voltage is

$$90 - 25{\cdot}7 - 36{\cdot}9 = \underline{\underline{27{\cdot}4^\circ}}$$

## 10.13 Three-phase Induction Machine

The 3-phase induction machine has a uniformly slotted stator and rotor. The stator has a 3-phase winding like that of the synchronous machine. Unlike that of the synchronous machine the rotor winding is not excited with direct current and may be supposed to consist of short-circuited coils. The effects of distribution are again neglected, and the three stator phase windings and three rotor phase windings are each treated as if they were single, concentrated coils. The arrangement is shown in Fig. 10.20(*a*).

The stator winding is excited with balanced 3-phase currents, which, as explained in Section 10.11, set up an m.m.f. of constant magnitude the axis of which rotates at $\omega$ radians per second in the $+\theta$ direction for the 2-pole configuration shown in Fig. 10.20(*a*) and for the stator phase currents given by

$$i_a = I_{sm} \cos \omega t \tag{10.44}$$

$$i_b = I_{sm} \cos (\omega t - 2\pi/3) \tag{10.45}$$

$$i_c = I_{sm} \cos (\omega t + 2\pi/3) \tag{10.46}$$

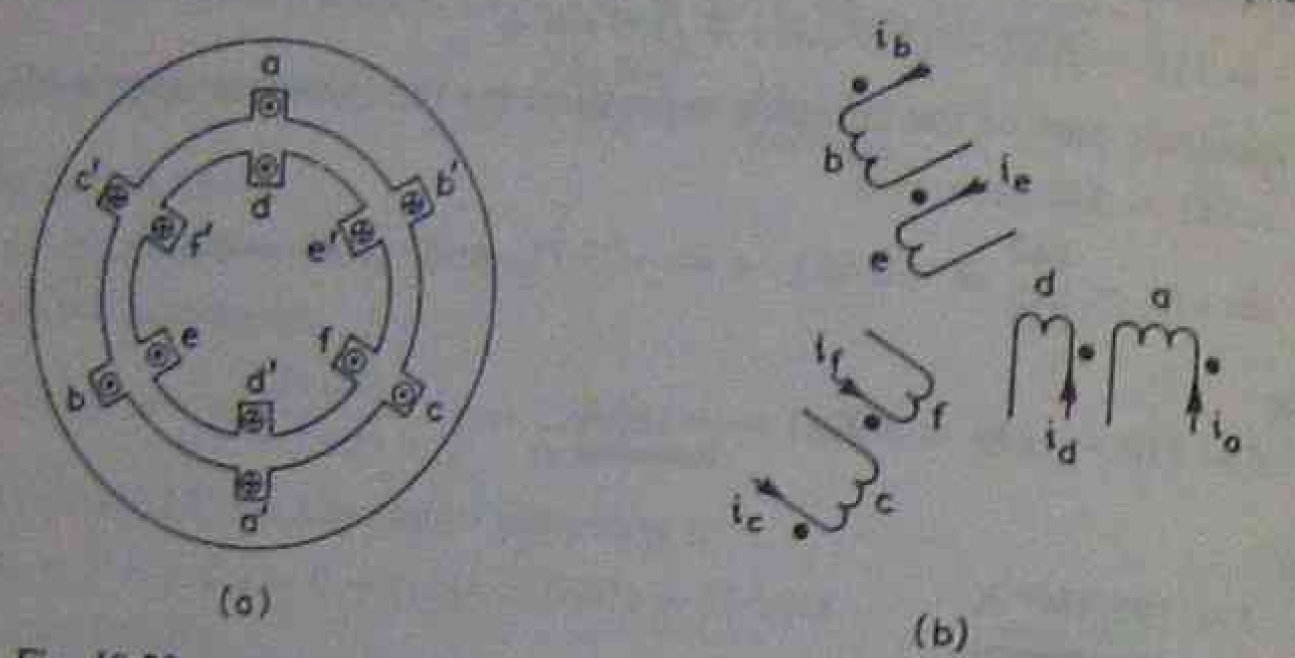



*Fig. 10.20* MUTUAL INDUCTANCES OF THE 3-PHASE INDUCTION MACHINE

If the machine is to produce torque there must be a rotor winding current. Since the resultant stator m.m.f. is constant in magnitude and rotates at constant speed, a rotor winding current will only be obtained if the angular velocity of the rotor, $\omega_r$, differs from that of the stator m.m.f., $\omega$, because only in this way can a rotational voltage be obtained in the rotor winding.

The magnitude and frequency of the induced currents in the rotor windings are clearly proportional to $\omega - \omega_r$, the rotor angular velocity relative to that of the axis of the stator m.m.f. It is usual to analyse the action of induction machines in terms of the *per-unit slip*, *s*, which is defined as

$$s = \frac{\omega - \omega_r}{\omega} \tag{10.63}$$

The voltage applied to any winding is

$$v_j = r_j i_j + \sum_{k=1}^{k=n} \frac{d}{dt} (\mathscr{L}_{jk} i_k) \tag{10.11}$$

Each of the rotor coils is short-circuited so that the above equation may be used to obtain expressions for the rotor currents. For the configuration of Fig. 10.20, however, there are six self-inductances



and nine mutual inductances. The resulting algebraic work though not difficult is extremely tedious. More sophisticated methods of analysis exist which greatly simplify the algebraic manipulation but are outside the scope of the present chapter.

Without carrying out the actual algebraic work, however, it is not difficult to anticipate the form of the expression for the current in any rotor phase.

Rotor current/phase $\propto$ Induced rotor e.m.f. per phase

Rotor e.m.f. per phase $\propto$ Relative angular velocity of stator m.m.f., $s\omega$
$\propto$ Maximum mutual inductance between a rotor and stator phase winding, $L_{sr}$
$\propto$ Maximum stator current per phase, $I_{sm}$.

The angular frequency of the rotor current per phase is also directly proportional to the relative angular velocity of the stator m.m.f., $s\omega$.

The current in phase *d* of the rotor winding is thus of the form

$$i_d = \frac{ks\omega L_{sr} I_{sm}}{Z_r} \cos (s\omega t - \phi_r + \alpha) \tag{10.64}$$

where *k* and $\alpha$ are constants, and $Z_r$ is the rotor impedance per phase:

$$Z_r = r_r + js\omega L_{rr} \tag{10.65}$$

$$\phi_r = \tan^{-1} \frac{s\omega L_{rr}}{r_r} \tag{10.66}$$

$L_{rr}$ is the total effective rotor inductance per phase. This includes a contribution due to mutual coupling with the two other rotor phases.

Analysis shows that $k = \frac{3}{2}$ and that if rotor phase *d* is at $\theta_r = 0$ at $t = 0$, then $\alpha = -\pi/2$. Therefore

$$i_d = \frac{\frac{3}{2} s\omega L_{sr} I_{sm}}{Z_r} \cos (s\omega t - \phi_r - \pi/2) \tag{10.67}$$

The currents in the two other rotor phases are such as to form, with that in rotor phase *d*, a balanced 3-phase system of currents. Therefore, as shown by eqn. (10.50), the 3-phase rotor winding will give rise to an m.m.f. of constant magnitude rotating at an angular velocity $s\omega$ in the $+\theta$ direction. Following eqn. (10.50), the rotor m.m.f. is

$$F_r = \tfrac{3}{2} I_{rm} N_r e^{j(s\omega t - \phi_r - \pi/2)} \tag{10.68}$$

where $I_{rm}$ is the maximum rotor current per phase and $N_r$ is the number of turns per rotor phase:

$$I_{rm} = \frac{\tfrac{3}{2} s\omega L_{sr} I_{sm}}{Z_r} \tag{10.69}$$

Since the rotor winding itself is rotating at $\omega_r = (1 - s)\omega$, the axis of the rotor m.m.f. has an angular velocity in space given by

Absolute angular velocity of rotor m.m.f.
$= (1 - s)\omega + s\omega = \omega$

i.e. the angular velocity of the rotor m.m.f. is constant and independent of rotor speed. The axes of the stator and rotor m.m.f.s both rotate at the same angular velocity with an angular displacement $-\phi - \pi/2$ (obtained by comparing eqns. (10.50) and (10.68)). The machine will therefore produce a steady torque.

Just as the m.m.f. due to a 3-phase winding when excited by balanced 3-phase currents can be represented as the m.m.f. of a single winding of $N_s$ turns excited by a direct current of $3I_{sm}$, where the winding is considered to rotate at $\omega$ radians per second, so also may the m.m.f. due to a 3-phase rotor winding when excited by balanced 3-phase currents be represented as the m.m.f. of a winding of $N_r$ turns carrying a direct current $\tfrac{3}{2} I_{rm}$ and rotating at an angular velocity $\omega$. The mutual inductance between a stator phase $a$ and a rotor phase $r$ is

$$\mathscr{L}_{ar} = L_{sr} \cos \theta_r$$

The mutual inductance between the two equivalent windings of $N_s$ and $N_r$ turns carrying direct currents $\tfrac{3}{2} I_{sm}$ and $\tfrac{3}{2} I_{rm}$ respectively is

$$\mathscr{L}_{sr} = L_{sr} \cos \epsilon \qquad \text{whence} \qquad \mathscr{L}_{sr}' = -L_{sr} \sin \epsilon$$

The angular displacement of the axes of the stator and rotor m.m.f.s is

$$\epsilon = -\phi_r - \pi/2$$

The instantaneous torque developed on the rotor may be obtained from an application of eqn. (10.14):

$$\begin{aligned} T' &= \tfrac{1}{2} \mathscr{L}_{rr}' i_r^2 + \mathscr{L}_{sr}' i_r i_s + \tfrac{1}{2} \mathscr{L}_{ss}' i_s^2 \\ &= -L_{sr} \sin(-\phi_r - \pi/2) \tfrac{3}{2} I_{rm} \tfrac{3}{2} I_{sm} \end{aligned} \tag{10.14}$$





Substituting for $I_{rm}$ from eqn. (10.69),

$$T' = -\left(\frac{3}{2}\right)^3 \frac{s\omega L_{sr}^2 I_{sm}^2}{Z_r} \sin(-\phi_r - \pi/2)$$

Since none of these terms varies with time the mean torque is

$$T = \left(\frac{3}{2}\right)^3 \frac{s\omega L_{sr}^2 I_{sm}^2 \cos \phi_r}{Z_r} \tag{10.70}$$

The per-unit slip, $s$, is positive when $\omega > \omega_r$, and when this condition obtains the torque developed on the rotor is positive and acts in the $+\theta$ direction, the direction of assumed rotor rotation. The machine therefore acts in the motoring mode for positive values of $s$. If the rotor is coupled to a prime mover and driven so that $\omega_r > \omega$ the slip and torque become negative and the machine acts in the generating mode.

Eqn. (10.70) shows that, like the 3-phase synchronous motor, the 3-phase induction motor produces a non-oscillatory torque. Unlike the 3-phase synchronous motor, however, the 3-phase induction motor is self-starting. At starting $\omega_r = 0$, and from eqn. (10.63),

$$s = \frac{\omega - 0}{\omega} = 1$$

Eqn. (10.70) shows that a torque will be developed on the rotor for this value of $s$.

## PROBLEMS

**10.1** An electrodynamic ammeter consists of a fixed coil and a moving coil connected in series. The self-inductance of the fixed coil is 400$\mu$H and that of the moving coil 200$\mu$H. The mutual inductance between the coils is

$$\mathscr{L}_{sr} = 100 \times 10^{-6} \cos \theta_r \quad \text{henry}$$

where $\theta_r$ is the position of the axis of the moving coil relative to that of the fixed one. The zero on the instrument scale corresponds to a position of the axis of the moving coil $\theta_r = 145°$. Full-scale deflection is 110° from the scale zero. The control constant is $5{\cdot}22 \times 10^{-5}$ N-m per degree of deflection. Determine the direct current required for full-scale deflection.

If an alternating current of 5 A r.m.s. and of frequency 50 Hz passes through the ammeter coils, what is the voltage drop across the instrument terminals? The resistance of the windings may be neglected.

*Ans.* 10 A; 0·91 V. (The angular deflection of the moving coil is approximately 48° from the instrument zero.)

**10.2** A rotating relay consists of a stator coil of self-inductance 2·0 H and a rotor coil of self-inductance 1·0 H. The axes of the rotor and stator coils are displaced by 30°, and in this configuration the mutual inductance of the coils is 1·0 H. Neither stator nor rotor has salient poles. A current $i_s = 14{\cdot}14 \cos 314t$

amperes is passed through the stator coil. The rotor coil is short-circuited. Draw a diagram to show the relative directions of the stator and rotor coil currents and calculate the r.m.s. value of the rotor current. Determine also the r.m.s. value of voltage applied to the stator coil. Describe the direction in which the torque acts. The resistance of each winding can be neglected.

*Ans.* 10 A; 3,140 V. The torque acts so as to tend to align the coils.

**10.3** A 2-pole d.c. machine has a field (stator) winding resistance of 200 Ω and an armature (rotor) resistance of 0·1 Ω. When operating as a generator the output voltage is 240 V when the armature winding current is 100 A, the field winding current 2 A, and the speed 500 rev/min.

Determine the armature current and speed when the machine is connected as a shunt motor to a 400 V d.c. supply and the total load torque imposed on the motor is 1,000 N-m. Assume that the machine is linear.

*Ans.* 209 A; 758 rev/min.

**10.4** A 93 W 2-pole series motor has the following constants:

| | |
|---|---|
| Armature resistance . . . . . . . . | $r_r = 12\,\Omega$ |
| Series field resistance . . . . . . . . | $r_s = 36\,\Omega$ |
| Effective armature inductance . . . . . . | $L_{rr} = 0{\cdot}3$ H |
| Series field inductance . . . . . . . . | $L_{ss} = 0{\cdot}34$ H |
| Maximum mutual inductance between rotor and stator windings | $L_{sr} = 0{\cdot}71$ H |

Determine the speed, output power, input current and power factor when the motor is connected to a 200 V 50 Hz supply and the load torque applied is 0·25 N-m. Neglect windage and friction and iron losses.

*Ans.* 3,000 rev/min; 78·5 W; 0·594 A; 0·802 lagging.

**10.5** A 100 V 3-phase 9 kVA 2-pole 50 Hz star-connected alternator has a total effective self-inductance per phase of 1·5 mH. The maximum mutual inductance between the rotor (field winding) and a stator phase winding is 90 mH. Determine the field current required to give an open-circuit line voltage of 100 V when the machine is driven at 3,000 rev/min. Find also, for this speed and field current, the terminal line voltage when the synchronous generator delivers rated full load current at (*a*) a power factor of 0·8 lagging; (*b*) unity power factor; (*c*) a power factor of 0·8 leading. The armature resistance per phase is negligible.

*Ans.* 2·89 A; 68·6 V; 89·1 V; 123 V.

**10.6** The synchronous machine of Problem 10·5 is run as a motor connected to 3-phase 100 V 50 Hz busbars. Determine the field current required if the input power factor is to be unity when the gross torque imposed is 20 N-m. Neglect all losses.

*Ans.* 3·01 A.

Chapter 11

# THREE-PHASE WINDINGS AND FIELDS

In an a.c. machine the armature (or main) winding may be either on the stator (i.e. the stationary part of the machine) or on the rotor, the same form of winding being used in each case. The simplest form of 3-phase winding has concentrated coils each spanning one pole pitch, and with the starts of each spaced 120° (electrical) apart on the stator or rotor. These coils may be connected in star or delta as required.

In most machines the coils are not concentrated but are distributed in slots over the surface of the stator or rotor, and it is this type of winding which will now be considered. The same type of winding is common to both synchronous and asynchronous (induction) machines.

## 11.1 Flux Density Distributions

In all a.c. machines an attempt is made to secure a sinusoidal flux density distribution in the air-gap. This may be achieved approximately by the distribution of the winding in slots round the air-gap or by using salient poles with shaped pole shoes.

In Fig. 11.1(*a*) a section of a multipolar machine is shown. If the flux density in the air-gap is to be sinusoidally distributed, the flux density must be zero on the inter-polar axes such as OA, OC and OE, and maximum on the polar axes OB and OD. Since successive poles are of alternate north and south polarities, the maximum flux densities along OB and OD are oppositely directed. Thus a complete cycle of variation of the flux density takes place in a

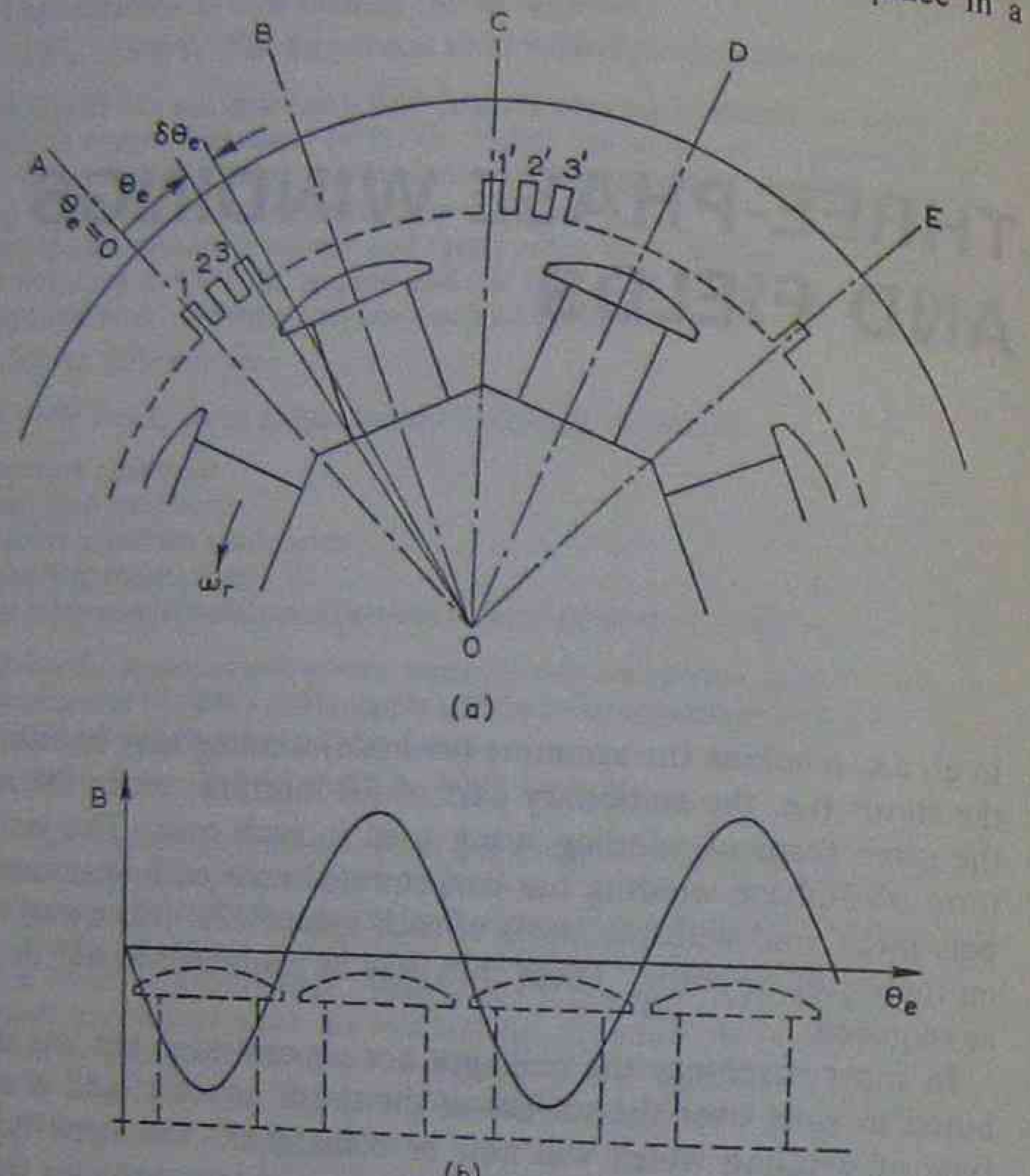



*Fig. 11.1* SINUSOIDAL FLUX DENSITY DISTRIBUTION

double pole pitch from the axis OA to the axis OE. This is shown in Fig. 11.1(*b*).

Taking axis OA as the datum for angular measurements, the flux density at any point in the air-gap is

$$B = B_m \sin \theta_e \qquad (11.1)$$

where $\theta_e$ is the angle from the origin measured in electrical radians or electrical degrees. Since one cycle of variation of the flux density occurs in a double pole pitch,

1 double pole pitch $\equiv 2\pi$ electrical radians or 360 electrical degrees



If the machine has $2p$ poles or $p$ double pole pitches,

$$\theta_e = p\theta_m \qquad (11.2)$$

where $\theta_m$ is the angular measure in mechanical radians or degrees.

## 11.2 Three-phase Single-layer Concentric Windings

The two sides of an armature coil must be placed in slots which are approximately a pole pitch (180 electrical degrees) apart so that the e.m.f.s in the coil sides are cumulative. In addition, in 3-phase machines the starts of each phase winding must be 120 electrical degrees apart.

In single-layer windings one coil side occupies the whole of a slot. As a result, difficulty is experienced in arranging the end connectors, or overhangs. In concentric and split-concentric windings differently shaped coils having different spans are necessary. To preserve e.m.f. balance in each of the phases, each phase must contain the same number of each shape of coil.

Fig. 11.2(*a*) represents a developed stator with 24 stator slots, and it is desired to place a 4-pole 3-phase concentric winding in them:

$$\text{Number of slots per pole} = \frac{24}{4} = 6$$

$$\text{Number of slots per pole and phase} = \frac{24}{4 \times 3} = 2$$

Fig. 11.2(*a*) shows the coil arrangement for the red phase as a thin full line. The start and finish (marked S and F respectively) of the phase winding are brought out, all the coils in the one phase being connected in series. For a phase sequence RYB, the yellow phase (shown dotted) must start 120 electrical degrees after the red phase. One pole pitch contains six slots and is equivalent to 180 electrical degrees. Hence a slot pitch is equivalent, in this case, to 30 electrical degrees.

The red phase starts in slot 1 and therefore the yellow phase must start in slot 5. In the same way the blue phase is 240 electrical degrees out of space phase with the red phase. The blue phase must therefore start in slot 9.

In Fig. 11.2 the finishes of the three phases have been commoned, making a star-connected winding. It would have been equally correct to common the three starts. The winding might also have been mesh-connected, in which case the finish of the red phase would have been connected to the start of the yellow phase, the finish of the yellow to the start of the blue, the finish of the blue to the start of the red, three connectors to the three junctions being brought out to terminals.

It will be observed that each phase has coils of each of the four different sizes used, thus maintaining balance between the phases.

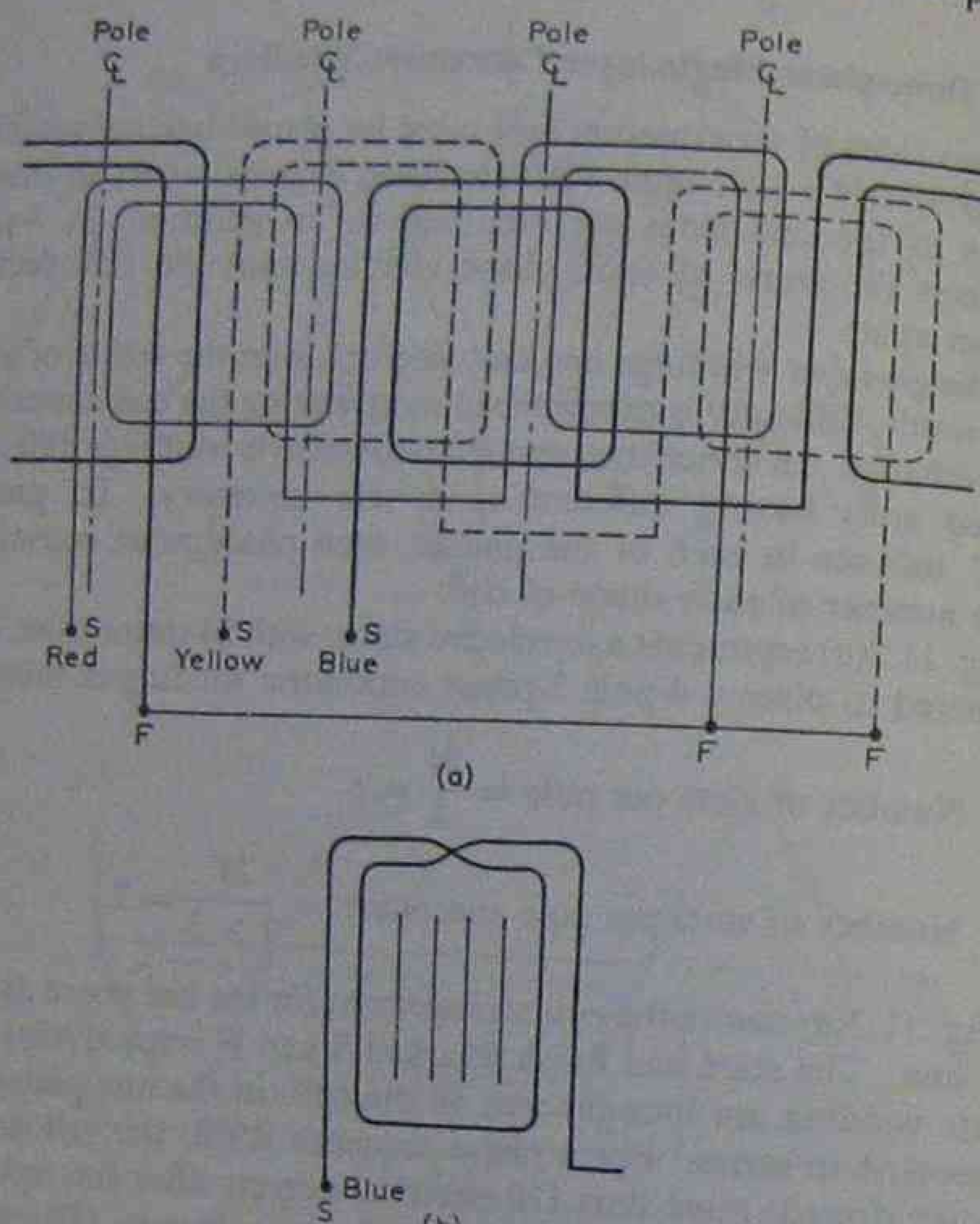



*Fig. 11.2* FOUR-POLE 3-PHASE SINGLE-LAYER CONCENTRIC WINDING

It will also be seen that a coil group of any one phase consists of two coils per double pole pitch, one coil being greater than a pole pitch by one slot pitch and the other being less than a pole pitch by the same amount. If the end connexions of these two coils were crossed over as shown in Fig. 11.2(*b*) two full-pitch coils (i.e. having a span of exactly one-pole pitch) would be formed. Therefore each such coil group is the equivalent, electrically, of two full-pitch coils joined in series. All single-layer windings are effectively composed of full-pitch coils.



## 11.3 Three-phase Single-layer Mush Winding

Fig. 11.3 shows a 4-pole 3-phase single-layer mush winding. The distinctive feature of the mush winding is the utilization of constant-span coils. The overhangs are arranged in a similar manner to those of a conventional double-layer winding.

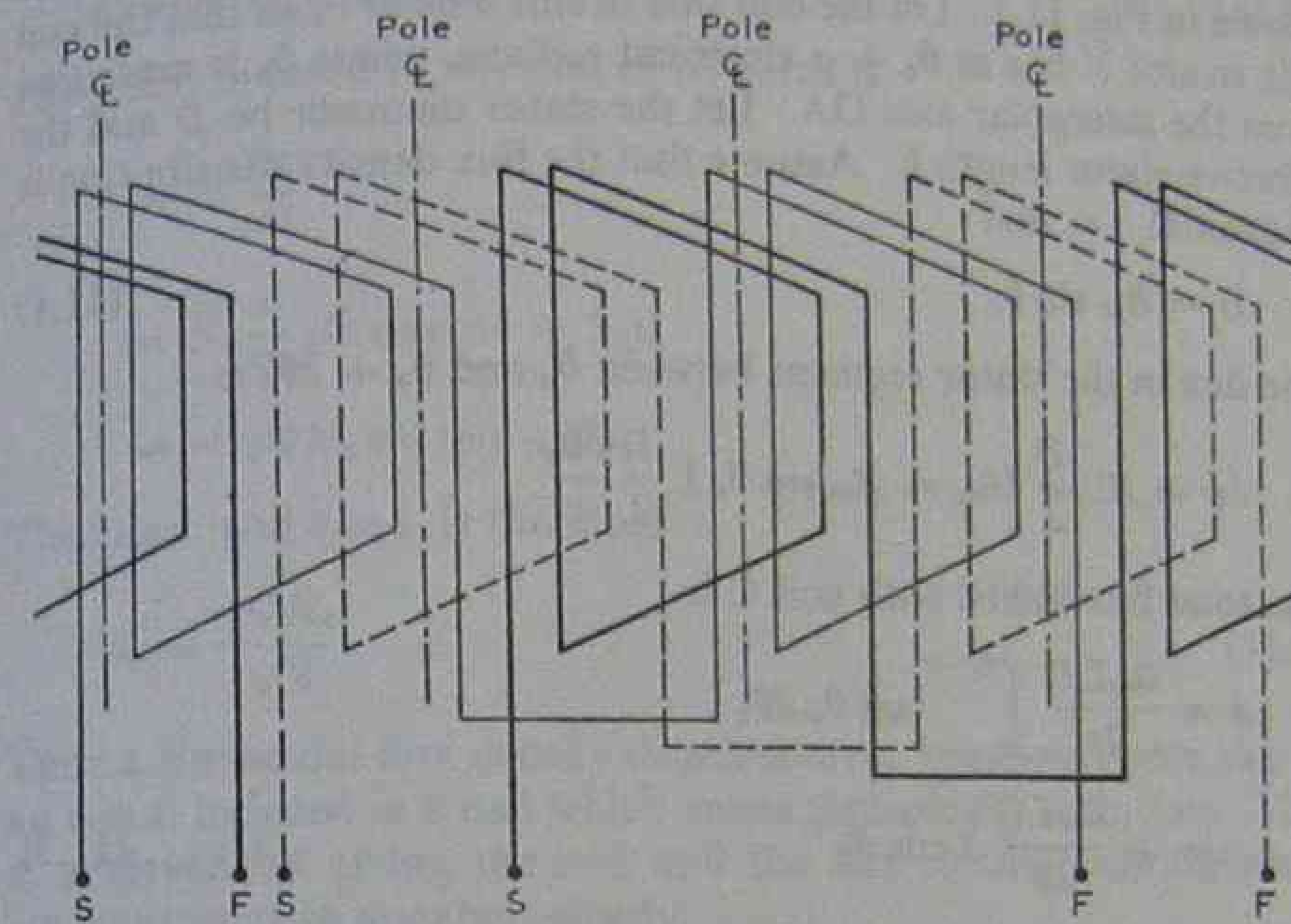



*Fig. 11.3* FOUR-POLE 3-PHASE SINGLE-LAYER MUSH WINDING

## 11.4 Three-phase Double-layer Windings

The double-layer windings used in 3-phase machines are essentially similar to those used in d.c. machines except that no connexions to a commutator are required.

Since each phase must be balanced, all must contain equal numbers of coils and the starts of each phase must be displaced by 120 electrical degrees. If a number of groups of coils are to be connected in parallel, then similar parts in the winding at equal potentials must be available, a condition obtainable only in machines having a number of poles divisible by three when a wave winding is used.

On the other hand, tooth ripple, which arises where there are an integral number of slots per pole, resulting in the same relative positions of equivalent slots under each pole, may be avoided in double-layer windings by the use of winding pitches different from the pole pitch, thus giving a fractional number of slots per pole. A further advantage of the double-layer winding is the possibility of using constant-span coils. Only single-layer windings are considered in the rest of this chapter.

### 11.5 E.M.F. Induced in a Full-pitch Coil

Consider a full-pitch coil C with coil sides lying in slots 3 and 3′ as shown in Fig. 11.1. Let the coil side in slot 3 lie at $\theta_e$ so that the coil side in slot 3′ lies at $\theta_e + \pi$ electrical radians, where $\theta_e$ is measured from the interpolar axis OA. Let the stator diameter be $D$ and the effective stator length $L$. Assume that the flux density distribution is sinusoidal, i.e. that

$$B = B_m \sin \theta_e \tag{11.1}$$

The flux in the stator segment between $\theta_e$ and $\theta_e + \delta\theta_e$ is

$$\delta\phi = BL\frac{D}{2}\,\delta\theta_m = B_m \sin \theta_e\, L\,\frac{D}{2}\,\frac{\delta\theta_e}{p}$$

The total flux linked with coil C is

$$\phi = \frac{B_m LD}{2p}\int_{\theta_e}^{\theta_e+\pi} \sin \theta_e\, d\theta_e$$

$$= +\frac{B_m LD}{2p}\,2\cos\theta_e \tag{11.3}$$

If a coil lies with its sides on the interpolar axes, as, for example, the coil lying in slots 1 and 1′ of Fig. 11.1, then the coil links the total flux per pole, $\Phi$:

$$\Phi = \frac{B_m LD}{2p}\int_0^{\pi} \sin \theta_e\, d\theta_e$$

$$= +\frac{B_m LD}{2p}\,2 \tag{11.4}$$

The flux linked with coil C is therefore, by substitution in eqn. (11.3),

$$\phi = \Phi \cos \theta_e \tag{11.5}$$

Suppose the pole system rotates in the direction shown at a uniform angular velocity

$$\omega_r = 2\pi n_0 \quad \text{radians/second} \tag{11.6}$$

where $n_0$ is the rotor speed in revolutions per second. The position of any coil such as C at any instant, in electrical radians, is

$$\theta_e = \omega t + \theta_0$$



where $\theta_0$ is the position of the coil at $t = 0$, and

$$\omega = p\omega_r = 2\pi n_0 p \quad \text{electrical radians/second} \tag{11.7}$$

Substituting for $\theta_e$ in eqn. (11.5), the flux linking any coil such as C at any time $t$ is

$$\phi = \Phi \cos(\omega t + \theta_0) \tag{11.8}$$

The e.m.f. induced in any coil of $N_c$ turns is

$$e = N_c \frac{d\phi}{dt}$$

$$= N_c \frac{d}{dt}\{\Phi \cos(\omega t + \theta_0)\}$$

$$= -\omega\Phi N_c \sin(\omega t + \theta_0)$$

The r.m.s. coil e.m.f. is therefore

$$E_c = \frac{\omega \Phi N_c}{\sqrt{2}} \tag{11.9}$$

Thus a sinusoidal flux density distribution in space may give rise to an e.m.f. induced in a coil which varies sinusoidally with time. This is achieved by giving the coil and the flux density distribution a constant relative angular velocity.

The frequency of the induced e.m.f. is

$$f = \frac{\omega}{2\pi} = \frac{2\pi n_0 p}{2\pi} = n_0 p \tag{11.10}$$

$n_0$ is called the *synchronous speed*. In this equation it is measured in revolutions per second.

### 11.6 Distribution (or Breadth) Factor and E.M.F. Equation

Suppose that under each pole pair each phase of the winding has $g$ coils connected in series, each coil side being in a separate slot. The e.m.f. per phase and pole pair is the complexor sum of the coil voltages. These will not be in time phase with one another since successive coils are displaced round the armature, and hence will not be linked by the same value of flux at the same instant. $E_1$, $E_2$, $E_3$ . . . $E_g$ (as shown in Fig. 11.4(*a*)) represent the r.m.s. values of the e.m.f.s in successive coils. The phase displacement between successive e.m.f.s is $\psi$, which depends on the electrical angular displacement between successive slots on the armature.

Suppose the machine has a total of $S$ slots and $2p$ poles. Then

$$\text{Number of slots per pole} = \frac{S}{2p}$$

The slot pitch (electrical angle between slot centre lines) is

$$\psi = \frac{180°_e}{S/2p} \quad \text{(since 1 pole pitch} = 180°_e) \tag{11.11}$$

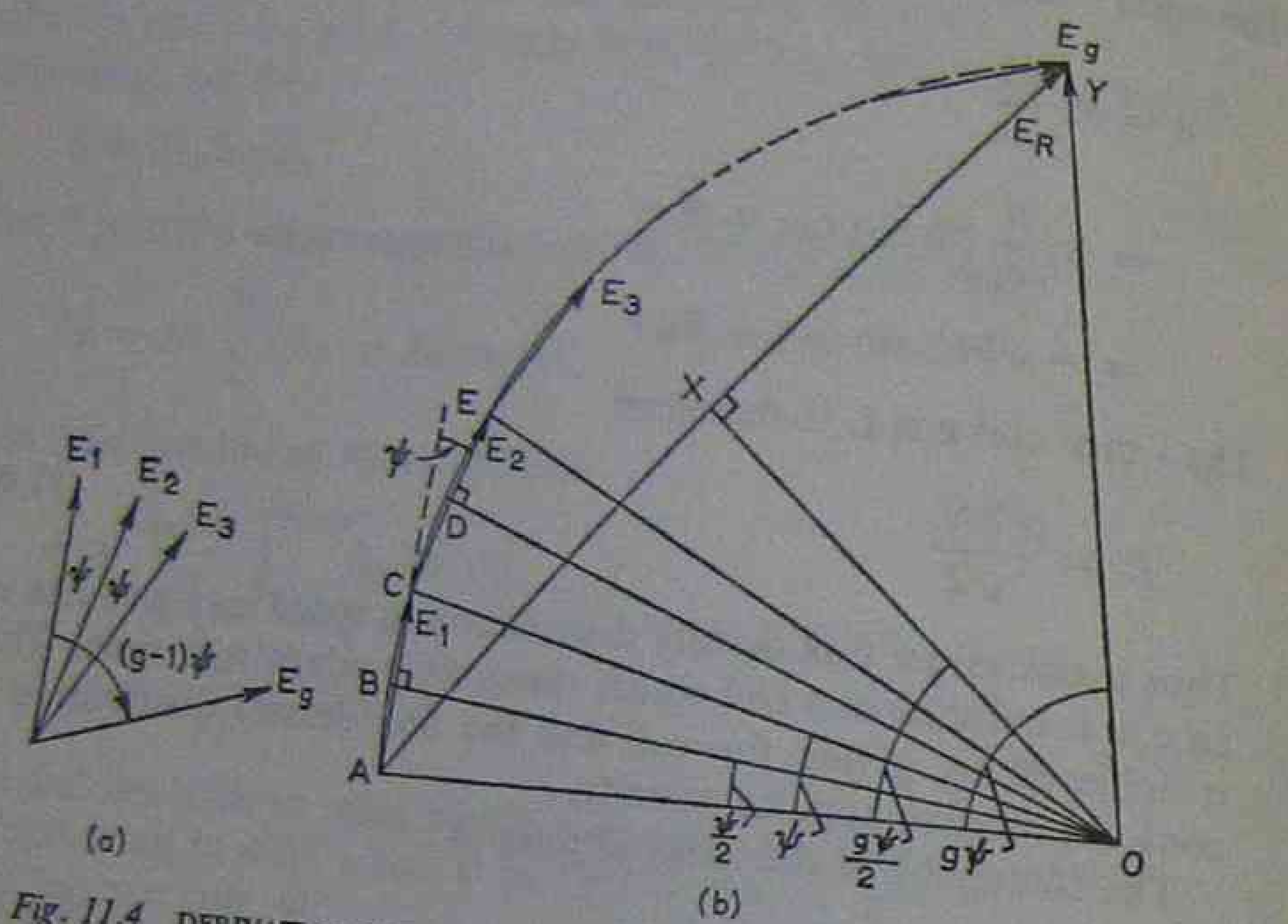



*Fig. 11.4* DERIVATION OF DISTRIBUTION FACTOR
(a) Complexor diagram of slot e.m.f.s
(b) Resultant of slot e.m.f.s

The e.m.f. complexors $E_1, E_2, E_3 \ldots E_g$ are placed end to end in order in Fig. 11.4(*b*). The resultant complexor $E_R$, represents the complexor sum of the e.m.f.s of the $g$ coils connected in series.

Since the complexors $E_1, E_2, E_3 \ldots E_g$ are all of the same length and are displaced from one another by the same angle, they must be successive chords of the circle whose centre is O in Fig. 11.4(*b*). The complexor sum AY may be found as follows.

Join OA, OC, OE, etc., draw the perpendicular bisectors of each chord (i.e. OB, OD, etc.) and also the perpendicular bisector OX of the chord AY.

In the triangle AOX,

$$\text{AX} = \text{AO} \sin \text{AOX} = \text{AO} \sin g\frac{\psi}{2}$$



Therefore

$$\text{AY} = 2\text{AO} \sin g\frac{\psi}{2}$$

In the triangle AOB,

$$\text{AB} = \text{AO} \sin \text{AOB} = \text{AO} \sin \frac{\psi}{2}$$

$$\text{AC} = 2\text{AB} = 2\text{AO} \sin \frac{\psi}{2}$$

Therefore

$$\frac{\text{AY}}{\text{AC}} = \frac{E_R}{E_1} = \frac{\sin g\frac{\psi}{2}}{\sin\frac{\psi}{2}}$$

Thus the *distribution factor* is

$$K_d = \frac{\text{Complexor sum of coil e.m.f.s}}{\text{Arithmetic sum of coil e.m.f.s}}$$

$$= \frac{E_R}{gE_1} = \frac{\sin g\frac{\psi}{2}}{g \sin\frac{\psi}{2}} \tag{11.12}$$

The product $g\psi$ represents the electrical angle over which the conductors of one phase are spread under any one pole and is referred to as the *phase spread.* In a 3-phase single-layer winding each phase has two phase spreads under each pole pair. Therefore, for a single-layer 3-phase winding,

$$g\psi = \frac{360}{2 \times 3} = 60°_e \quad \text{or} \quad \pi/3 \text{ electrical radians}$$

Clearly the highest value which the distribution factor $K_d$ can have is unity, corresponding to a situation where there is one coil per pole pair and phase. A lower limit for the value of $K_d$ also exists. Thus, if the number of separate slots $g$ in the phase spread $g\psi$ is considered to increase without limit, then

$$\psi \to 0 \quad \text{and} \quad \sin\frac{\psi}{2} \to \frac{\psi}{2}$$

A 3-phase winding with a phase spread of $60°_e$ is said to be *narrow spread.*

For a narrow-spread 3-phase winding ($g\psi = \pi/3$),

$$\lim_{\psi \to 0} K_d = \frac{\sin \frac{g\psi}{2}}{g\frac{\psi}{2}} = \frac{\sin \pi/6}{\pi/6} = \frac{3}{\pi} \qquad (11.13)$$

A winding having this limiting condition is called a *uniform winding*, and in such winding the phase spreads may be thought of as current sheets with the effect of the slotting eliminated.

The lower limit of $K_d$ for a 3-phase narrow-spread winding ($3/\pi = 0{\cdot}955$), corresponding to a very large number of slots per pole and phase, shows that the distribution of the winding will have little effect on the magnitude of the fundamental e.m.f. per phase.

Ideally the flux density distribution linking the winding should be sinusoidal. In practice this ideal is not usually achieved; the air-gap flux density distribution is then of the form

$$B = B_{m1} \sin \theta_e + B_{m3} \sin (3\theta_e + \epsilon_3) + \ldots B_{mn} \sin (n\theta_e + \epsilon_n) \qquad (11.14)$$

In this expression the first term on the right-hand side is called the *fundamental space distribution*. The other terms are referred to as *space harmonics*. The $n$th space harmonic goes through $n$ cycles of variation for one cycle of variation of the fundamental. Only odd space harmonics are present since the flux density distribution repeats itself under each pole and is therefore symmetrical.

Just as the fundamental flux density gives rise to a fundamental e.m.f. induced in a coil, so the $n$th space harmonic in the flux density distribution will give rise to an $n$th time harmonic in the coil e.m.f. The distribution factor for the $n$th harmonic is

$$K_{dn} = \frac{\sin \frac{gn\psi}{2}}{g \sin \frac{n\psi}{2}} \qquad (11.15)$$

Although the distribution of the winding has little effect on the magnitude of the fundamental, it may cause considerable reduction in the magnitude of harmonic e.m.f.s compared with those occurring in a winding for which $g = 1$, i.e. one coil per pole pair and phase.

## 11.7 Coil-span Factor

The e.m.f. equation of Section 11.5 has been deduced on the assumption of full-pitch coils, i.e. coils whose sides are separated by one



pole pitch. As has been pointed out, the coils in double-layer windings are often made either slightly more or slightly less than a pole pitch. Fig. 11.5 illustrates coils with various pitches.

If the coil has a pitch of exactly one pole pitch, it will at some instant link the entire flux of a rotor pole. If the coil pitch is less than one pole pitch, it will never link the entire flux of a rotor pole and the maximum coil e.m.f. will be reduced. If the coil pitch is greater than one pole pitch, the coil must always be linking flux

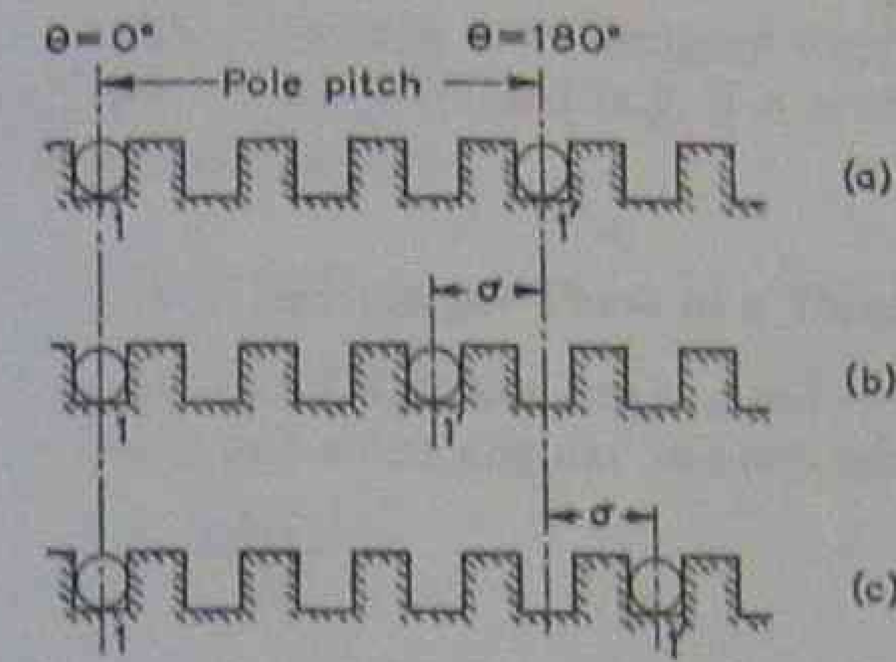



*Fig. 11.5* COIL SPANS

(*a*) Full pitch
(*b*) Short pitch
(*c*) Over-full pitch

from at least two adjacent rotor poles so that the net flux linked will be less than the flux of one pole and the maximum coil e.m.f. will again be reduced.

The factor by which the e.m.f. per coil is reduced is called the *coil span factor*, $K_s$:

$$K_s = \frac{\text{E.M.F. in the short or long coil}}{\text{E.M.F. in a full-pitched coil}} \qquad (11.16)$$

The magnitude of the coil span factor may most readily be obtained by considering the e.m.f. induced in each coil side, namely

$$e = Blv \text{ volts}$$

where $B$ = air-gap flux density, $l$ = active conductor length and $v$ = conductor velocity at right angles to the direction of $B$.

This e.m.f. will have the same waveform as the flux density in the air-gap, since $l$ and $v$ are constant, and hence if the flux density is sinusoidally distributed the e.m.f. in each conductor will be sinusoidal so that the resultant coil e.m.f. will also be sinusoidal. If the pitch is short or long by an electrical angle $\sigma$, then, assuming a sinusoidal flux density distribution, the e.m.f.s in each side of the coil will differ in phase by $\sigma$ but will have the same r.m.s. value. The resultant coil e.m.f. will be the complexor sum of the e.m.f.s in each coil side, as shown in Fig. 11.6.

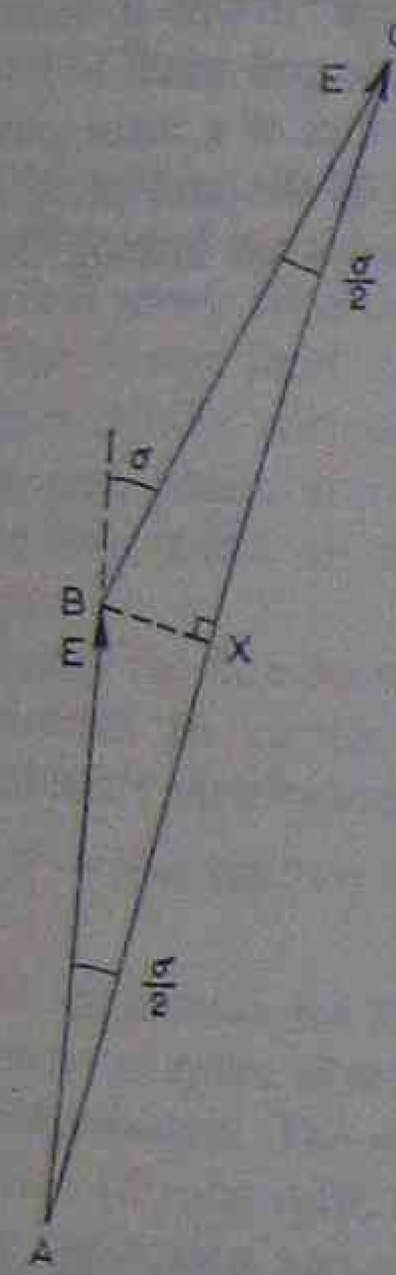



*Fig. 11.6* DERIVATION OF COIL SPAN FACTOR

$$\text{Resultant e.m.f.} = \text{AC} = 2\text{AB}\cos\frac{\sigma}{2}$$

$$\text{E.M.F. for a full-pitch coil} = 2\text{AB}$$

Therefore

$$K_s = \frac{2\text{AB}\cos\frac{\sigma}{2}}{2\text{AB}} = \cos\frac{\sigma}{2} \tag{11.17}$$

If the flux density distribution contains space harmonics, the coil span factor for the $n$th harmonic e.m.f. is

$$K_{sn} = \cos\frac{n\sigma}{2} \tag{11.18}$$

All single-layer windings are effectively made up of full-pitch coils, but double-layer windings usually have short-pitched or



short-chorded coils. The $n$th harmonic coil e.m.f. is reduced to zero if the *chording angle*, $\sigma$, is such that

$$\cos\frac{n\sigma}{2} = 0$$

or

$$\frac{n\sigma}{2} = 90°_e \tag{11.19}$$

This enables windings to be designed which will not permit specified harmonics to be generated (e.g. if $\sigma = 60°_e$ there can be no third-harmonic generation).

## 11.8 E.M.F. Induced per Phase of a Three-phase Winding

Following eqn. (11.9) the r.m.s. e.m.f. induced in a full-pitch coil of $N_c$ turns due to its angular velocity relative to the pole system is

$$E_c = \frac{\omega\Phi N_c}{\sqrt{2}} \tag{11.9}$$

For a coil-span factor, $K_s$, due to chording,

$$E_c = K_s\frac{\omega\Phi N_c}{\sqrt{2}}$$

Further, if there are $g$ coils in a phase group under a pole pair the resultant complexor sum is

$$E_g = K_d g E_c = K_d K_s g\frac{\omega\Phi N_c}{\sqrt{2}}$$

Assuming that the e.m.f.s of coil groups of the same phase under successive pole pairs are in phase and connected in series, the e.m.f. per phase is

$$E_p = pEg = pK_dK_sg\frac{\omega\Phi N_c}{\sqrt{2}}$$

or

$$E_p = K_dK_s\frac{\omega\Phi N_p}{\sqrt{2}} \tag{11.20}$$

where the number of turns per phase, $N_p$, is $pgN_c$.

This equation is sometimes written in the form

$$E_p = 4{\cdot}44\ K_dK_sf\Phi N_p \tag{11.21}$$

since $\omega = 2\pi f$ and $2\pi/\sqrt{2} = 4{\cdot}44$.

Sometimes the conductors per phase rather than the turns per phase are specified, in which case eqn. (11.21) becomes

$$E_p = 2{\cdot}22\, K_s K_d f \Phi Z_p \tag{11.22}$$

since $N_p = Z_p/2$.

The line voltage will depend on whether the winding is star or delta connected.

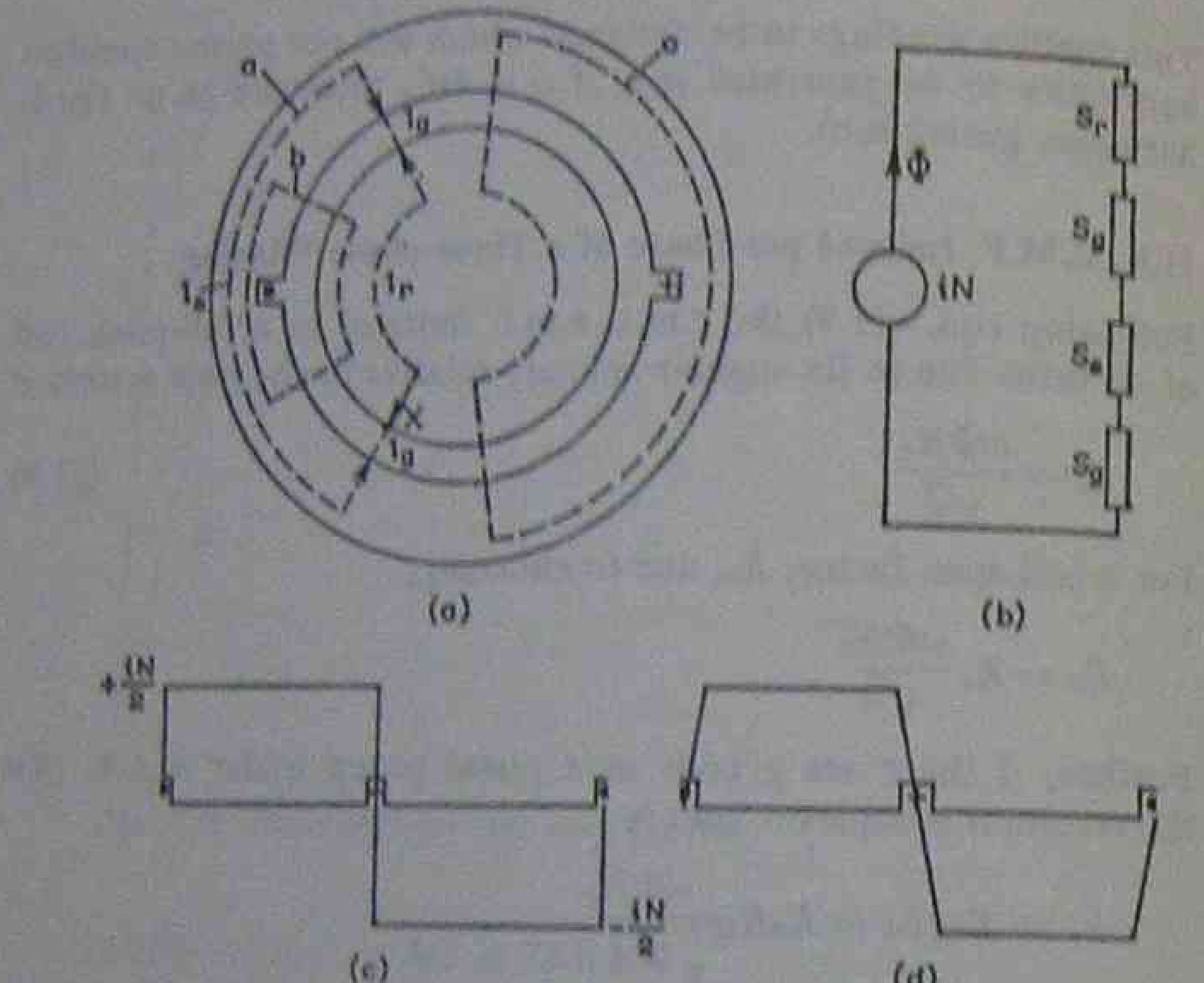



*Fig. 11.7* M.M.F. DUE TO A FULL-PITCH COIL

## 11.9 M.M.F. due to a Full-pitch Coil

Fig. 11.7(*a*) shows a stator and rotor separated by a uniform air-gap, i.e. one whose radial length $l_g$ is constant. The stator has two diametrically opposite slots in which one stator coil of $N$ turns carries a current $i$. The slot opening is assumed to be very small compared with the internal circumference of the stator.

Consider the closed path $a$ of Fig. 11.7(*a*) and let distances be measured from point X on this path. Now,

$$\text{M.M.F.} = iN = \oint H\,dl$$



Assuming that the reluctance of the rotor and stator core paths is zero, and that the magnetic field strength in the gap, $H_g$, is constant along the radial length $l_g$, then

$$iN = 2H_g l_g$$

or

$$H_g l_g = \frac{iN}{2} \tag{11.23}$$

Fig. 11.7(*b*) shows the equivalent magnetic circuit with path reluctances $S_r$ (rotor), $S_s$ (stator), $S_g$ (air-gap). It will readily be confirmed that, if $S_r = 0$ and $S_s = 0$, the magnetic potential difference across each of the two equal reluctances $S_g$ is $\frac{1}{2}iN$.

It is clear that, adhering to the assumptions of zero reluctance in the rotor and stator core and constant field strength in the air gap, the same result as that of eqn. (11.23) is obtained for other paths of integration such as $b$ or $c$ in Fig. 11.7(*a*). Indeed the magnetic potential drop across the air gap is $\frac{1}{2}iN$ at all points. The magnetic potential drop across the air-gap is, for the direction of coil current chosen, directed from rotor to stator for the upper half of the stator, and from stator to rotor for the lower half of the stator in this case.

Fig. 11.7(*c*) shows a graph of air-gap magnetic potential difference plotted to a base of the developed stator surface. The magnetic potential difference has been arbitrarily assumed positive when it is directed from rotor to stator and shown above the datum line. It is therefore taken to be negative when directed from stator to rotor and shown below the datum line. The magnetic potential difference is shown as changing abruptly from $+\frac{1}{2}iN$ to $-\frac{1}{2}iN$ opposite the slot opening. This corresponds to the situation where the slot is extremely thin.

Although Fig. 11.7(*c*) is properly described as showing the variation of air-gap magnetic potential difference to a base of the developed stator surface, such a diagram is often called an m.m.f. wave diagram, and the quantity $\frac{1}{2}iN$ is often called the m.m.f. per pole.

Where the width of the slot opening is not negligible the m.m.f. wave for a coil may be taken to be trapezoidal as shown in Fig. 11.7(*d*).

## 11.10 M.M.F. due to One Phase of a Three-phase Winding

Fig. 11.8(*a*) shows the coil for a double pole pitch of one phase of a 3-phase concentric winding of the type illustrated in Fig. 11.2. The position of the stator slots and coils is indicated on a developed diagram of the stator slotting. The start of the red phase winding is shown with a current emerging from the start end of the winding. This is taken as a conventionally positive current for generator

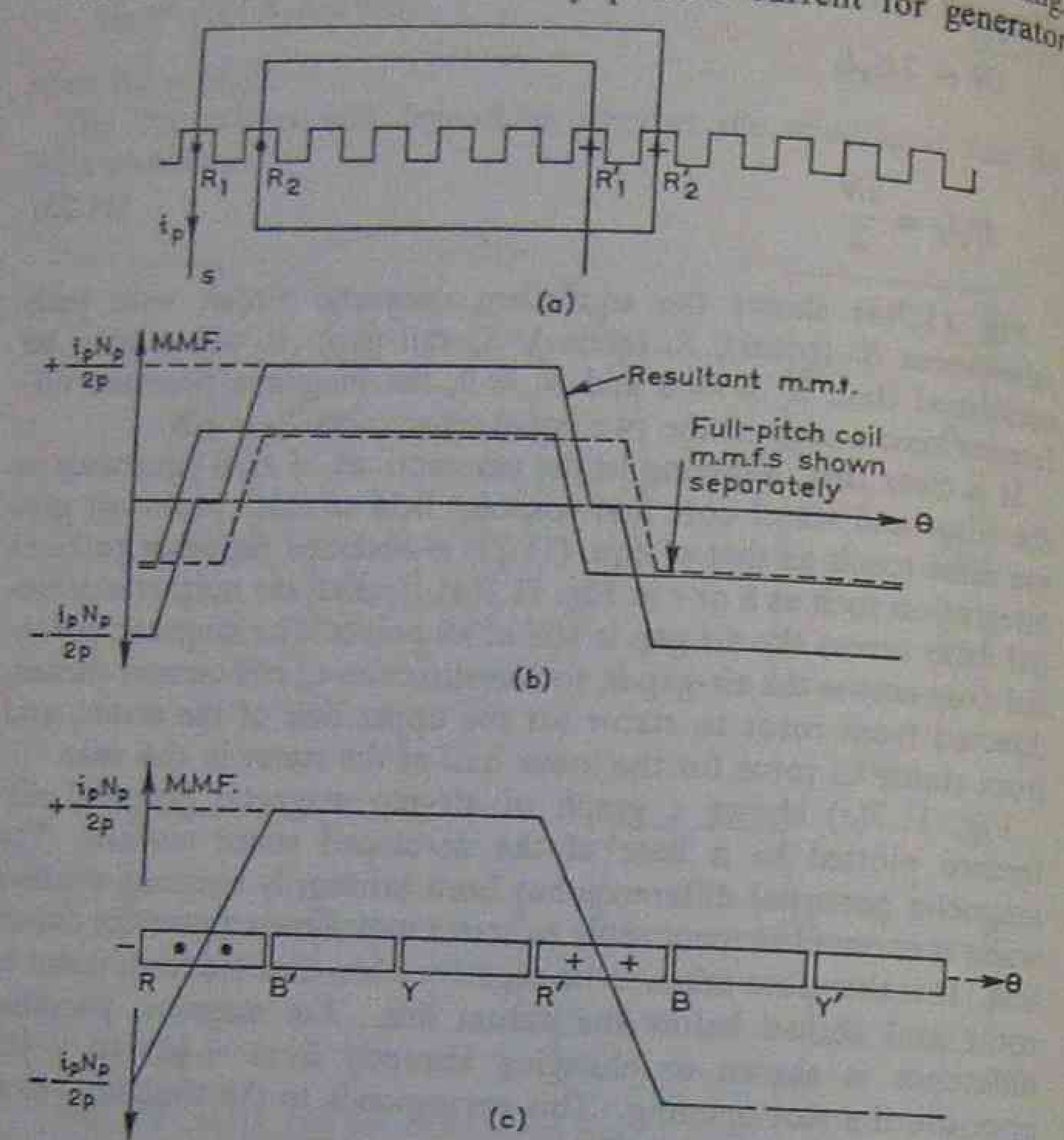



*Fig. 11.8* M.M.F. DUE TO ONE PHASE OF A 3-PHASE WINDING

action. Conventionally positive current for motor action enters the start end of the winding.

As has been explained previously, the concentric coil arrangement shown is equivalent to an arrangement of two full-pitch coils where the coil sides in slots $R_1$ and $R_1'$ and the coil sides in $R_2$ and $R_2'$ are joined. Fig. 11.8(*b*) shows the m.m.f. for each of such coils separately and also their resultant obtained by adding together the two separate m.m.f. waves. The convention regarding positive m.m.f. explained in the previous section has been adhered to.

The resultant m.m.f. shown in Fig. 11.8(*b*) is stepped, owing to the effect of the discrete coils. Fig. 11.8(*c*) shows the m.m.f. per phase



when the effect of discrete coils is ignored. The rectangular blocks represent the phase spreads, and these are considered to extend over both the regions previously occupied by slots and by teeth. The phase spread containing the start of the phase winding is identified by the unprimed letter R. The other phase spread of the same phase is marked R′. The phase current is considered to be uniformly distributed in the block representing the phase spread. Such a winding is a uniformly distributed winding as described in Section 11.6, and the m.m.f. per phase for such a winding is of the trapezoidal shape shown.

The maximum value of the m.m.f. wave at any instant is the m.m.f. per pole for the phase considered. For $N_p$ total turns per phase the m.m.f. per phase and pole is $i_pN_p/2p$.

If a sinusoidal alternating current $i_p = I_{pm} \sin \omega t$ flows in the phase winding, the maximum value of the m.m.f. wave will vary sinusoidally:

$$\frac{i_pN_p}{2p} = \frac{I_{pm}N_p \sin \omega t}{2p}$$

In subsequent work the m.m.f. due to uniform windings only will be considered.

## 11.11 M.M.F. due to a Three-phase Winding (graphical treatment)

Fig. 11.9 shows the m.m.f.s for each phase of a 3-phase winding carrying balanced 3-phase currents for two different instants in the current cycle. The resultant m.m.f., due to the combined action of the separate phases, is also shown in each diagram.

Fig. 11.9(*a*) is drawn for the instant when the instantaneous currents in the three phases are

$$i_r = I_{pm}$$
$$i_y = -\tfrac{1}{2}I_{pm}$$
$$i_b = -\tfrac{1}{2}I_{pm}$$

The current in the red phase is positive, so according to the convention for positive current explained in Section 11.10, phase spread R has the current direction indicated by a dot and phase spread R′ has the current direction indicated by a cross. The red phase m.m.f., $F_r$, therefore has the trapezoidal distribution shown having a maximum value of

$$\frac{i_pN_p}{2p} = \frac{I_{pm}N_p}{2p} = F_{pm}$$

where $F_{pm}$ is the maximum m.m.f. per phase and pole.

The currents in the yellow and blue phases are both negative so that the Y and B phase spreads have crosses, and the phase spreads Y′ and B′ have dots, to show the current direction. The m.m.f.

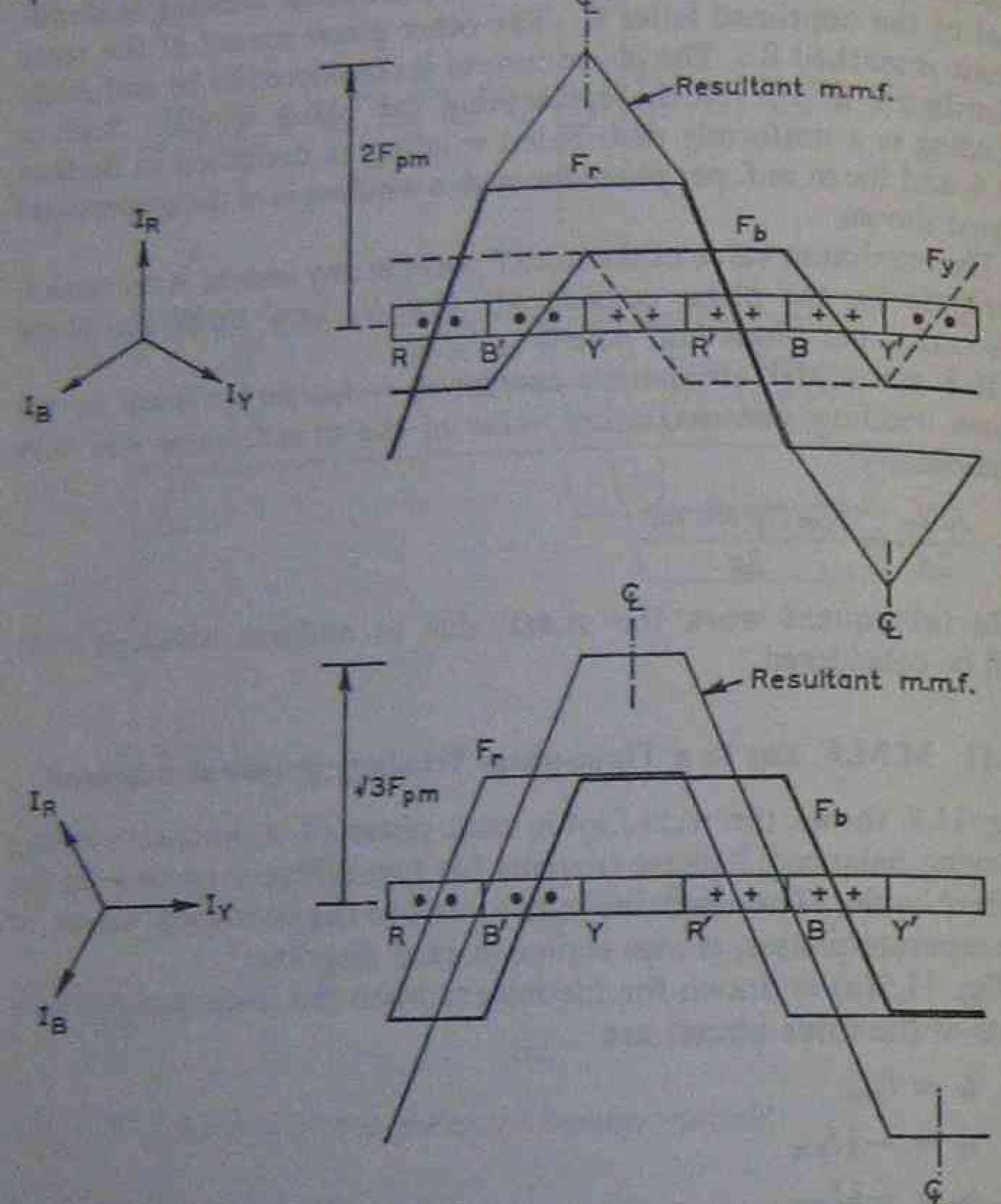



*Fig. 11.9* M.M.F. DUE TO A 3-PHASE WINDING (GRAPHICAL TREATMENT)

waves for these phases, $F_y$ and $F_b$, are also trapezoidal and at the instant shown in the diagram have a maximum value of

$$\frac{i_p N_p}{2p} = \frac{1}{2}\,\frac{I_{pm} N_p}{2p} = \tfrac{1}{2} F_{pm}$$

The resultant stator m.m.f., $F_A$, is obtained by finding the sum of the separate phase m.m.f.s, $F_r$, $F_y$ and $F_b$. This resultant m.m.f. has a maximum value of $2F_{pm}$.



Fig. 11.9(*b*) has been drawn for an instant $\frac{1}{12}$th of a cycle later than Fig. 11.9(*a*). The instantaneous phase currents are

$$i_r = \frac{\sqrt{3}}{2} I_{pm}$$

$$i_y = 0$$

$$i_b = -\frac{\sqrt{3}}{2} I_{pm}$$

The red and blue phase m.m.f.s occupy the same positions as in Fig. 11.9(*a*), but the maximum value of the red phase m.m.f., $F_r$, has fallen to $(\sqrt{3}/2)F_{pm}$, whereas the maximum value of the blue phase m.m.f., $F_b$, has risen to this value. Since the yellow phase current is zero, the yellow phase m.m.f., $F_y$, is zero.

The resultant stator m.m.f. is the sum of $F_r$ and $F_b$, and has a maximum value of $\sqrt{3}F_{pm}$.

Comparing the resultant m.m.f.s of Figs. 11.9(*a*) and (*b*) it will be seen that the centre-line of the resultant m.m.f. has moved $30°_e$ in the $+\theta$ direction, and that the shape of the distribution has become trapezoidal. The maximum value of the resultant m.m.f. has fallen slightly from $2F_{pm}$ to $\sqrt{3}_{pm}$.

After the next $\frac{1}{12}$th cycle the waveshape will be found to be the same as in Fig. 11.9(*a*), but displaced a further $30°_e$ round the armature. Hence the following points may be noted.

1. The m.m.f. wave is continually changing shape between the limits of the peaked wave of Fig. 11.9(*a*) and the flat-topped wave of Fig. 11.9(*b*).
2. The wave may be approximated to by a sinusoidal wave of constant maximum value. It is shown in Section 11.12 that this value is $\frac{18}{\pi^2} F_{pm}$.
3. The m.m.f. wave moves past the coils as the alternating currents vary throughout their cycle.
4. The m.m.f. wave moves by $\frac{1}{12}$th of one pole pair in $\frac{1}{12}$th cycle, i.e. the m.m.f. wave moves through one pole pair in one cycle.

If the frequency of the 3-phase currents is $f$ and the speed of rotation of the field is $n$ revolutions per second,

$$\text{Time to move through 1 pole pair} = \frac{1}{f} = \frac{1}{np}$$

Therefore

$$n = \frac{f}{p} = n_0$$

i.e. the field rotates at synchronous speed as defined by eqn. (11.10).

Summarizing these points it may be said that a 3-phase current in a 3-phase winding produces a rotating magnetic field in the air-gap of the machine, the speed of rotation being the synchronous speed for the frequency of the currents and the number of pole pairs in the machine.

The production of the rotating field is the significant difference between a 3-phase and a single-phase machine. Due to its rotating field a 3-phase machine gives a constant, non-pulsating torque in a direction independent of any subsidiary gear or auxiliary windings.

**EXAMPLE 11.1** Compare the e.m.f.s at 50 Hz of the following 20-pole alternator windings wound in identical stators having 180 slots:

(*a*) a single-phase winding with 5 adjacents slots per pole wound, the remaining slots being unwound,
(*b*) a single-phase winding with all slots wound,
(*c*) a 3-phase star-connected winding with all slots wound.

All the coils in each phase are connected in series, and each slot accommodates 6 conductors. The total flux per pole is 0·025 Wb.

Assuming a single-layer winding with full-pitch coils there will be 6 turns per coil and the coil span factor will be unity.

There are 9 slots per pole, and thus the slot pitch, $\psi$, is given by

$$\psi = \frac{180}{9} = 20°_e$$

(*a*) Number of coils per pole pair and phase, $g = 5$

$$\text{Distribution factor} = \frac{\sin 5 \times \frac{20°}{2}}{5 \sin \frac{20°}{2}} = \frac{0{\cdot}766}{0{\cdot}868} = 0{\cdot}883$$

$$\text{E.M.F. per phase} = 4{\cdot}44 K_d K_s f \Phi N_p \qquad (11.21)$$
$$= 4{\cdot}44 \times 0{\cdot}883 \times 1 \times 50 \times 0{\cdot}025 \times 5 \times 6 \times 10$$
$$= \underline{\underline{1{,}470\ \text{V}}}$$

(*b*) Number of coils per pole pair and phase, $g = 9$

$$\text{Distribution factor} = \frac{\sin 9 \times \frac{20°}{2}}{9 \sin \frac{20°}{2}} = \frac{1}{1{\cdot}563} = 0{\cdot}64$$



$$\text{E.M.F. per phase} = 4{\cdot}44 K_d K_s f \Phi N_p$$
$$= 4{\cdot}44 \times 0{\cdot}64 \times 1 \times 50 \times 0{\cdot}025 \times 9 \times 6 \times 10$$
$$= \underline{\underline{1{,}920\ \text{V}}}$$

(*c*) Number of coils per pole pair and phase, $g = 3$

$$\text{Distribution factor} = \frac{\sin 3 \times \frac{20°}{2}}{3 \sin \frac{20°}{2}} = \frac{0{\cdot}5}{0{\cdot}521} = 0{\cdot}96$$

$$\text{E.M.F. per phase} = 4{\cdot}44 K_d K_s f \Phi N_p$$
$$= 4{\cdot}44 \times 0{\cdot}96 \times 1 \times 50 \times 0{\cdot}025 \times 3 \times 6 \times 10$$
$$= \underline{\underline{960\ \text{V}}}$$

(Line voltage for star connexion = 1,600 V)

Comparing (*a*) and (*b*) it will be seen that the e.m.f. in case (*b*) is only 30 per cent greater than that in case (*a*), while the amount of winding material is 80 per cent greater.

The winding losses for the same current would also be 80 per cent greater for case (*b*). Thus it is common practice to omit some coils in each pole pair in a single-phase winding.

Supposing that with the above e.m.f.s there is a current of $I$ amperes in the coils.

In case (*a*), armature power = $1{,}470I$ watts
In case (*b*), armature power = $1{,}920I$ watts
In case (*c*), armature power = $\sqrt{3} \times 1{,}660I = 2{,}800I$ watts

Comparing (*b*) and (*c*) above it will be realized that for the same frame size with the same winding and core losses the output from a 3-phase machine is about 1·5 times greater than that from a single-phase machine.

## 11.12 M.M.F. due to a Three-phase winding (analytical treatment)

In Fig. 11.10 the m.m.f. due to one phase acting separately is shown as a trapezoidal wave. This trapezoidal wave can be represented by using an appropriate Fourier series consisting of a fundamental and a series of space harmonics. In the analysis below all space harmonics are neglected, and the m.m.f. due to each phase acting separately is assumed to be of sinusoidal form having a maximum value equal to the maximum value of the fundamental in the Fourier series.

In Fig. 11.10 the axis $\theta = 0$ is the centre-line of the positive half-wave of the m.m.f., $F_r'$, due to the red phase only when the red phase carries conventionally positive current (i.e. emerging from the start end of the winding). The Fourier series of a trapezoidal wave having this origin is

$$F(\theta) = \frac{8A}{\pi(\pi - 2\beta)} \sum_n \frac{1}{n^2} \cos n\beta \cos n\theta \quad (n \text{ is odd}) \qquad (11.24)$$

where $A$ and $\beta$ are as indicated on Fig. 11.10.

$A$ is the maximum value of the trapezoid, and since the red phase is excited by alternating current, this maximum value varies sinusoidally with time so that

$$A = F_{pm} \cos \omega t \qquad \left(F_{pm} = \frac{I_{pm} N_p}{2p}\right)$$

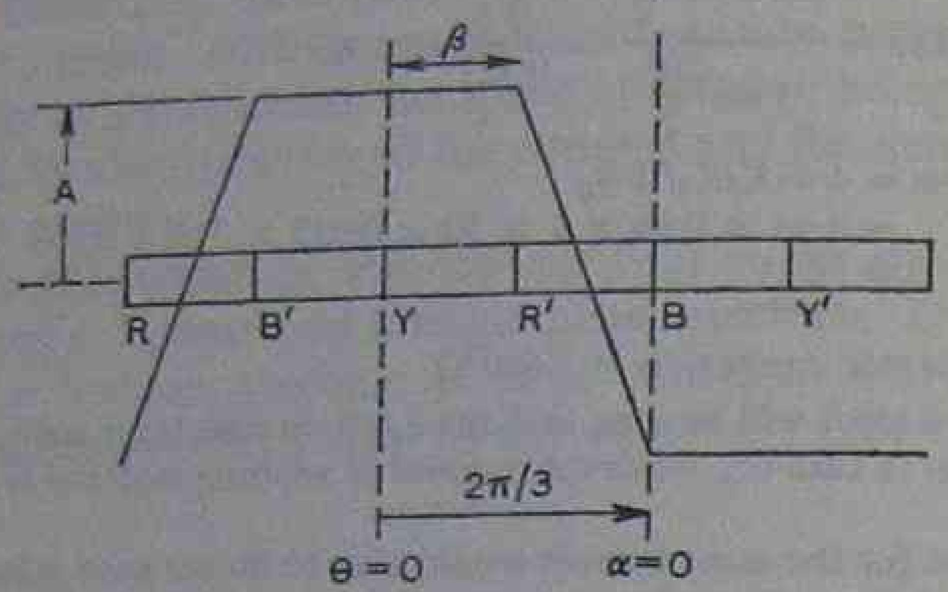



*Fig. 11.10* M.M.F. DUE TO A 3-PHASE WINDING (ANALYTICAL TREATMENT)

The angle $\beta$ corresponds to a phase spread, so that $\beta = \pi/3$. Substituting in eqn. (11.24), the red phase m.m.f. at any time $t$ and at any position $\theta$ is

$$F_r' = \frac{8F_{pm} \cos \omega t}{\pi(\pi - 2\pi/3)} \cos \frac{\pi}{3} \cos \theta$$

$$= \frac{12F_{pm}}{\pi^2} \cos \omega t \cos \theta \tag{11.25}$$

In Fig. 11.10 the axis $\alpha = 0$ is the centre-line of the positive half-wave of the m.m.f. due to the yellow phase. Since the yellow phase current lags in time behind the red phase current by $2\pi/3$ radians, the variation of the maximum value of the yellow phase m.m.f. will lag behind that of the red phase m.m.f. by the same amount.

The yellow phase m.m.f. is

$$F_y' = \frac{8F_{pm} \cos (\omega t - 2\pi/3)}{\pi(\pi - 2\pi/3)} \cos \frac{\pi}{3} \cos \alpha$$

Evidently

$$\theta = \alpha + 2\pi/3 \qquad \text{so that} \qquad \alpha = \theta - 2\pi/3$$

and

$$F_y' = \frac{12F_{pm}}{\pi^2} \cos \left(\omega t - \frac{2\pi}{3}\right) \cos \left(\theta - \frac{2\pi}{3}\right) \tag{11.26}$$





Similarly the blue phase m.m.f. is

$$F_b' = \frac{12F_{pm}}{\pi^2} \cos \left(\omega t + \frac{2\pi}{3}\right) \cos \left(\theta + \frac{2\pi}{3}\right) \tag{11.27}$$

The resultant m.m.f. of the 3-phase winding is

$$F_A' = F_r' + F_y' + F_b'$$

$$= \frac{12F_{pm}}{\pi^2} \begin{Bmatrix} \cos \omega t \cos \theta + \cos (\omega t - 2\pi/3) \cos (\theta - 2\pi/3) \\ + \cos (\omega t + 2\pi/3) \cos (\theta + 2\pi/3) \end{Bmatrix}$$

On simplifying this gives

$$F_A' = \frac{18}{\pi^2} F_{pm} \cos (\theta - \omega t) \tag{11.28}$$

This is the fundamental in the space distribution of m.m.f. It has a maximum value at $\theta - \omega t = 0$, i.e. when $\theta = \omega t$. That is, the position of the maximum value travels in the $+\theta$ direction with an angular velocity $\omega$ radians per second.

Thus an m.m.f. of the form $F = F_m \cos (\theta - \omega t)$ represents an m.m.f. wave cosinusoidally distributed in space and travelling in the $+\theta$ direction at $\omega$ radians per second. The above equation is therefore the equation of a *travelling wave* and is referred to as a *retarded function*.

## 11.13 Three-phase Rotating Field Torques

Consider a rotating field, derived from the rotor, say, linking a 3-phase winding in which a 3-phase current is flowing; i.e. the rotating field is due to the m.m.f. of rotating poles, not to the 3-phase current. Suppose the speed of the rotating field is such that the frequency of the e.m.f. induced in the 3-phase winding is the same as that of the currents in the 3-phase winding. Unless this is the case there will be no mean torque since the direction of the torque will be alternating.

Let $E_{ph}$ = R.M.S. value of e.m.f. induced in each phase of 3-phase winding

$\phi$ = Phase angle between induced e.m.f. and winding current

$I_{ph}$ = R.M.S. phase current

The machine may be acting as either a generator or a motor:

Mean phase power = $E_{ph} I_{ph} \cos \phi$ watts

This, by the law of conservation of energy and neglecting losses, must be the mechanical power required to drive the rotor if the machine is acting as a generator, or the mechanical power developed if the machine is acting as a motor.

$$\text{Total mechanical power developed} = 3E_{ph}I_{ph}\cos\phi = 2\pi n_0 T$$

where $T$ is the total torque developed (newton-metres) and $n_0$ is the speed of the rotor ($f/p$ revolutions per second). Therefore

$$T = \frac{3E_{ph}I_{ph}\cos\phi}{2\pi n_0} \tag{11.29}$$

$$= \frac{3}{2\pi\dfrac{f}{p}}\,\frac{2\pi f N_{ph}\Phi_m K_d K_s}{\sqrt{2}}\,I_{ph}\cos\phi$$

$$= \frac{3p}{\sqrt{2}} N_{ph}\Phi_m K_d K_s I_{ph}\cos\phi \quad \text{newton-metres} \tag{11.30}$$

When the machine is motoring, the torque will act on the rotor in the direction of rotation and react on the stator in the opposite direction. These directions will interchange when the machine is generating.

## 11.14 Non-pulsating Nature of the Torque in a Three-phase Machine

It has been shown that the 3-phase currents in the stator of a 3-phase machine produce a magnetic field of effectively constant amplitude rotating round the air-gap at synchronous speed. The torque developed is due to the magnetic forces between the rotor poles and the rotating field, so that so long as the rotor poles move at synchronous speed there will be a constant magnetic force between stator and rotor. Hence the 3-phase machine will develop a constant torque which does not pulsate in magnitude. (Note that this differs from the case of the single-phase machine.)

The above conclusion may also be derived by considering that the total power delivered to a balanced 3-phase load is non-pulsating, so that, if the load is a machine which is running at a constant speed, the torque developed must also be non-pulsating. On this basis the single-phase machine has a pulsating torque since the power supplied pulsates at twice the supply frequency.

## PROBLEMS

**11.1** Derive an expression for the e.m.f. induced in a full-pitched coil in an alternator winding, assuming a sinusoidal distribution of flux in the air-gap.



Show how the voltage of a group of such coils, connected in series, may be found.

Calculate the speed and open-circuit line and phase voltages of a 4-pole 3-phase 50 Hz star-connected alternator with 36 slots and 30 conductors per slot. The flux per pole is 0·0496 Wb, sinusoidally distributed.

*Ans.* 1,500 rev/min, 3,300 V, 1,910 V.

**11.2** Derive the expression for the voltage in a group of $m$ full-pitch coils each having an electrical displacement of $\psi$.

An 8-pole 3-phase star-connected alternator has 9 slots per pole and 12 conductors per slot. Calculate the necessary flux per pole to generate 1,500 V at 50 Hz on open-circuit. The coil span is one pole pitch.

With the same flux per pole and speed, what would be the e.m.f. when the armature is wound as a single-phase alternator using two-thirds of the slots? *(H.N.C.)*

*Ans.* 0·0283 Wb; 1,480 V.

**11.3** A 6-pole machine has an armature of 90 slots and 8 conductors per slot and revolves at 1,000 rev/min, the flux per pole being $5 \times 10^{-2}$ Wb. Calculate the e.m.f. generated (*a*) as a d.c. machine if the winding is lap-connected; (*b*) as a 3-phase star-connected machine if the winding factor is 0·96 and all the conductors in each phase are in series. Deduce the expression used in each case. *(L.U.)*

*Ans.* 600 V, 2,200 V.

**11.4** Derive an expression for the e.m.f. induced in each phase of a single-layer distributed polyphase winding assuming the flux density distribution to be sinusoidal.

A 4-pole 3-phase 50 Hz star-connected alternator has a single-layer armature winding in 36 slots with 30 conductor per slot. The flux per pole is 0·05 Wb. Determine the speed of rotation. Draw, to scale, the complexor diagram of the phase e.m.f.s,

1. when the phase windings are symmetrically star-connected,
2. when the phase windings are asymmetrical star-connected, the yellow phase winding being reversed with respect to the red and blue phase windings.

Give the numerical values of all the line voltages in each case. The phase sequence for the symmetrical star connection is RYB.

*Ans.* 1,500 rev/min; $V_{RY} = V_{YB} = B_{BR} = 3{,}320$ V; $V_{RY} = 1{,}920$ V; $V_{YB} = 1{,}920$ V; $V_{BR} = 3{,}320$ V.

**11.5** An 8-pole rotor, excited to give a steady flux per pole of 0·01 Wb, is rotated at 1,200 rev/min in a stator containing 72 slots. Two 100-turn coils A and B are accommodated in the stator slotting as follows:

Coil A. Coil sides lie in slots 1 and 11,
Coil B. Coil sides lie in slots 2 and 10.

Calculate the resultant e.m.f. of the two coils when they are joined (*a*) in series aiding and (*b*) in series opposing. Assume the flux density distribution to be sinusoidal.

*Ans.* 700 V; 0 V.

**11.6** A rotor having a d.c. excited field winding is rotated at $n$ revolutions per second in a stator having uniformly distributed slots. If the air-gap flux density is sinusoidally distributed, derive an expression for the e.m.f. induced in a coil of $N$ turns the sides of which lie in any two slots.

In such an arrangement the flux per pole is 0·01 Wb, the rotor speed is 1,800 rev/min, the stator has 36 slots and the rotor has 4 poles. Calculate the frequency and r.m.s. value of the induced e.m.f. in the coil if the coil sides lie in slot 1 and slot 9 and the coil has 100 turns.

An exactly similar coil is now placed with its coil sides in slots adjacent to the first coil. Determine the resultant e.m.f.s when the coils are connected in series.

*Ans* 60 Hz; 263 V; 518 V or 91·3 V.

# Chapter 12

# THE THREE-PHASE SYNCHRONOUS MACHINE

A synchronous machine is an a.c. machine in which the rotor moves at a speed which bears a constant relationship to the frequency of the current in the armature winding. As a motor, the shaft speed must remain constant irrespective of the load, provided that the supply frequency remains constant. As a generator, the speed must remain constant if the frequency of the output is not to vary. The field of a synchronous machine is a steady one. In very small machines this field may be produced by permanent magnets, but in most cases the field is excited by a direct current obtained from an auxiliary generator which is mechanically coupled to the shaft of the main machine.

## 12.1 Types of Synchronous Machine

The armature or main winding of a synchronous machine may be on either the stator or the rotor. The difficulties of passing relatively large currents at high voltages across moving contacts have made the stator-wound armature the common choice for large machines. When the armature winding is on the rotor, the stator carries a salient-pole field winding excited by direct current and very similar to the stator of a d.c. machine (Fig. 12.1($a$)). Stator-wound armature machines fall into two classes: ($a$) salient-pole rotor machines, and ($b$) non-salient-pole, or cylindrical-rotor, machines

(Fig. 12.1(*b*) and (*c*)). The salient-pole machine has concentrated field windings and generally is cheaper than the cylindrical-rotor machine when the speed is low (less than 1,500 rev/min). Salient-pole alternators are generally used when the prime mover is a water

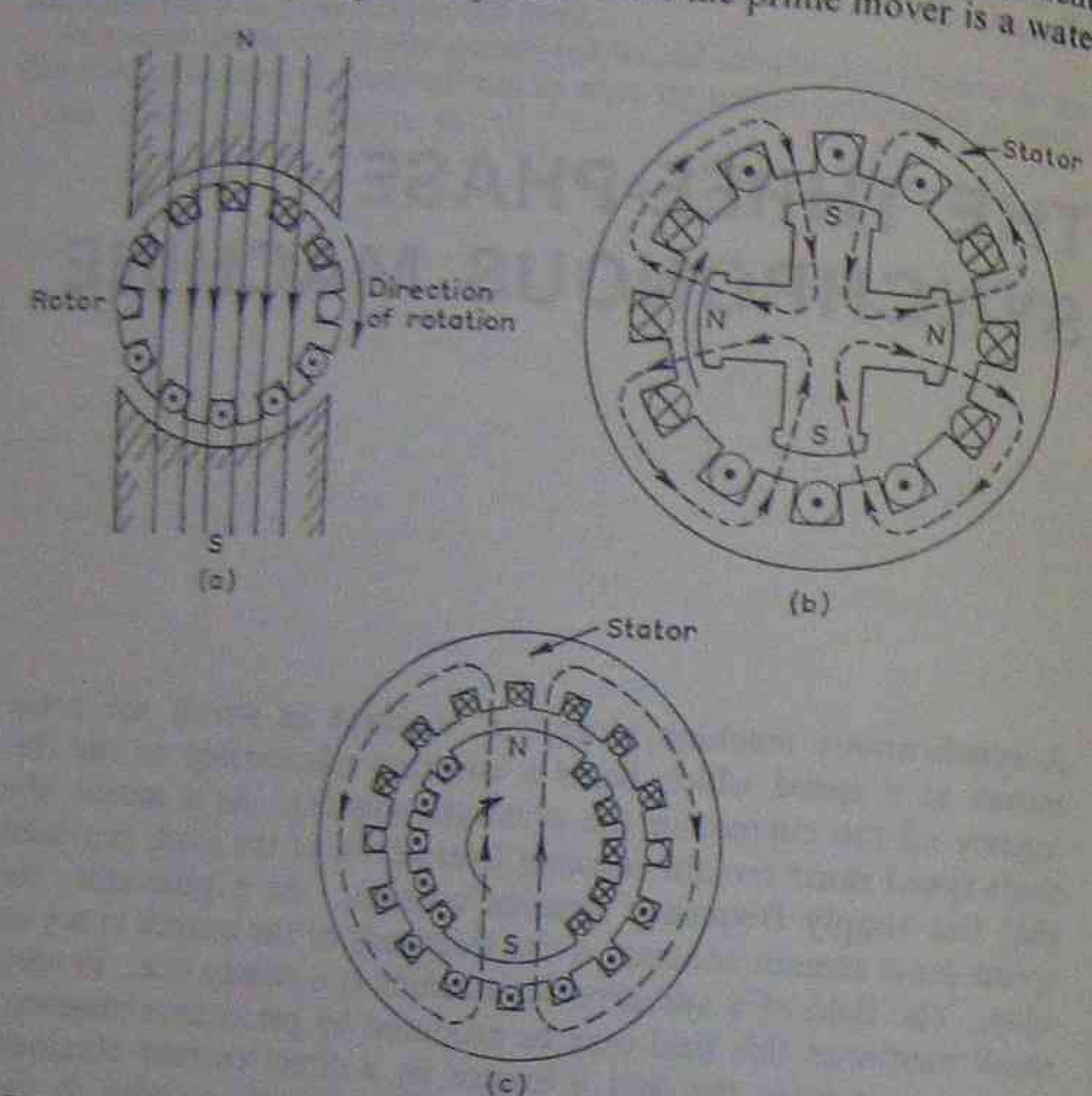



*Fig. 12.1* SYNCHRONOUS MACHINES
(*a*) Armature on rotor
(*b*) Salient-pole rotor (4 poles)
(*c*) Cylindrical rotor (2 poles)

turbine or a reciprocating engine. The cylindrical rotor has a distributed winding in the rotor slots and is most suitable for high speeds: steam-turbine-driven alternators are generally high-speed machines (3,000 rev/min) and have the cylindrical-rotor construction.

For an alternator to generate a 50 Hz voltage the speed $n_0$ will be given by eqn. (11.10) as

$$n_0 = \frac{60 \times 50}{p} = \frac{3{,}000}{p} \quad \text{rev/min}$$

where $p$ is the number of pole-pairs on the machine. $n_0$ would also



be the speed of a synchronous motor with $p$ pole-pairs operating from a 50 Hz supply.

## 12.2 M.M.F. Wave Diagrams of the Synchronous Generator

The operation of a synchronous machine may be understood by a consideration of its m.m.f. waves. There are three m.m.f. waves to be considered: that due to the field winding, $F_F$, which is separately excited with direct current; that due to the 3-phase armature winding, $F_A$; and their resultant, $F_R$.

In the first stages of the explanation the following assumptions will be made.

1. Magnetic saturation is absent so that the machine is linear.
2. The field and armature m.m.f.s are sinusoidally distributed.
3. The air-gap is uniform, i.e. the machine does not exhibit saliency on either side of the air-gap.
4. The reluctance of the magnetic paths in the stator and rotor is negligible.
5. The armature-winding leakage inductance and resistance are negligible.

The last assumption will be removed at a convenient stage in the development. The machine considered will be a cylindrical-rotor machine with the 3-phase winding on the stator and the d.c.-excited field winding on the rotor. The generating mode of action will first be considered.

The method of drawing the m.m.f. waves will be that used in Chapter 11, which should be read in conjunction with this chapter. The same conventions for positive m.m.f. and positive current are adopted. Positive current is assumed to emerge from the start ends of the phase windings, so that the unprimed phase spreads R,Y,B are dotted when the current is positive, and the primed phase spreads R′Y′B′ are crossed when the current is positive. Positive e.m.f. may now be defined in the same way. Positive m.m.f. is assumed to be directed from rotor to stator and is shown above the $\theta$ axis in the m.m.f. wave diagrams. These are shown superimposed on a representation of a double pole-pitch of the stator winding and of the rotor.

### GENERATING-MODE OPERATION ON OPEN-CIRCUIT

Fig. 12.2(*a*) shows the relevant m.m.f. wave for the synchronous generator on open-circuit. The instant chosen in the e.m.f. cycle, indicated by the complexor diagram, is such that

$$E_R = E_{pm} \qquad E_Y = -\tfrac{1}{2}E_{pm} \qquad E_B = -\tfrac{1}{2}E_{pm}$$

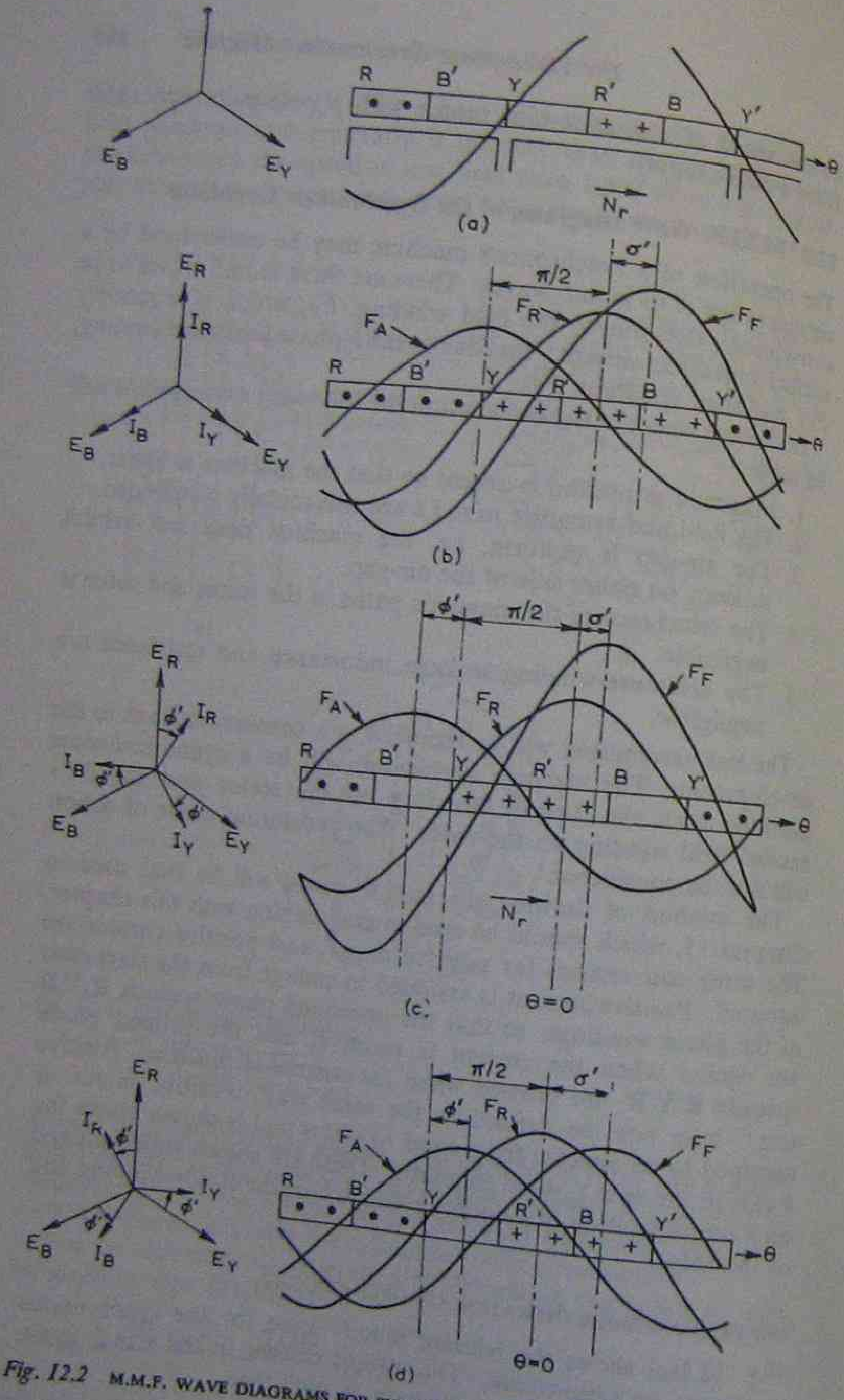



*Fig. 12.2* M.M.F. WAVE DIAGRAMS FOR SYNCHRONOUS GENERATOR





The phase spreads of the red phase, $R$ and $R'$, have been dotted and crossed in accordance with the convention for positive e.m.f. The rotor rotates in the direction shown; the direction of the stator conductors relative to the rotor is in the opposite direction, and this latter direction must be used in applying the right-hand rule to find the direction of the field flux and m.m.f. This is as shown in Fig. 12.2($a$), the maximum rotor m.m.f.s occurring opposite the centres of the red phase spreads, since this phase has maximum e.m.f. induced in it. The field m.m.f., $F_F$, shown in Fig. 12.2($a$) is also the resultant m.m.f., $F_R$, since on open-circuit there is no armature current and consequently $F_A$ is zero at all times and at all points in the air-gap.

The field m.m.f. is stationary with respect to the rotor winding, which is excited with direct current but moves, with the rotor, at synchronous speed past the stator winding.

### GENERATING-MODE OPERATION AT UNITY POWER FACTOR

Fig. 12.2($b$) represents the m.m.f. waves when the armature winding is supplying current at unity power factor.

To obtain comparability between Figs. 12.2($a$) and ($b$), both diagrams have been drawn for the same instant in the e.m.f. cycle, and the magnitudes of the e.m.f.s are the same in each case as indicated by the e.m.f. complexor diagram. Since the e.m.f. is caused by the resultant m.m.f., $F_R$, this will have the same magnitude and position in Fig. 12.2($b$) as it has in Fig. 12.2($a$).

However, since in this case armature current flows, there will be an armature m.m.f., $F_A$. The resultant m.m.f., $F_R$, is the sum of $F_A$ and $F_F$, so in this case $F_F$ and $F_R$ are different.

The instant in the current cycle is such that

$$i_R = I_{pm} \qquad i_Y = -\tfrac{1}{2}I_{pm} \qquad i_B = -\tfrac{1}{2}I_{pm}$$

This is the instant in the 3-phase cycle for which Fig. 11.9($a$) was drawn. The armature m.m.f. in Fig. 12.2($b$), therefore, is in the same position as the armature m.m.f. in Fig. 11.9($a$) and lags behind the resultant m.m.f. wave by $\pi/2$ radians, but the space harmonics which give the armature m.m.f. wave its distinctive peaked shape are ignored, and this m.m.f. is shown as a sine distributed wave. The armature m.m.f. moves at synchronous speed, so that the m.m.f.s $F_A$ and $F_F$ and their resultant $F_R$ all move at the same speed and in the same direction under steady conditions.

At any time and at any point in the air-gap,

$$F_R = F_A + F_F \tag{12.1}$$

Therefore $F_A = F_R - F_F$.

The field m.m.f., $F_F$, in Fig. 12.2(*b*) is therefore obtained by point-by-point subtraction of the resultant and armature m.m.f. waves. Comparing the field m.m.f. at Fig. 12.2(*b*) with that for Fig. 12.2(*a*) two changes may be observed:

1. To maintain the e.m.f. constant, the separate excitation has had to be increased in value as shown by the higher maximum value of $F_F$. The effect of armature m.m.f. is therefore the same as that of an internal voltage drop.
2. The axis of the field m.m.f. has been displaced by an angle $\sigma'$ in the direction of rotation, and as a result a torque is exerted on the rotor in the direction opposite to that of rotation. The rotor must be driven by a prime mover against this torque, so that machine absorbs mechanical energy and is therefore able to deliver electrical energy.

### GENERATING-MODE OPERATION AT POWER FACTORS OTHER THAN UNITY

Fig. 12.2(*c*) shows the m.m.f. waves for the same instant in the e.m.f. cycle but with the phase currents lagging behind their respective e.m.f.s by a phase angle $\phi'$. As compared with its position in Fig. 12.2(*b*), therefore, the armature m.m.f. wave is displaced by an angle $\phi'$ in the direction opposite to that of rotation as compared with its unity-power-factor position. This change in the relative position of the armature m.m.f. wave brings it more into opposition with the field m.m.f., so that the latter must be further increased to maintain the resultant m.m.f. and e.m.f. constant.

Fig. 12.2(*d*) shows the m.m.f. waves for the same instant in the e.m.f. cycle but with the phase currents leading their respective e.m.f.s by a phase angle $\phi'$. As compared with its position for unity power factor, the armature m.m.f. wave is displaced in the direction of rotation by the angle $\phi'$. In this case the change in the relative position of the armature m.m.f. wave gives it a component which aids the field m.m.f., which must then be reduced for a constant resultant m.m.f. and e.m.f.

If the power factor is zero lagging, the armature and field m.m.f. waves are in direct opposition, whereas if the power factor is zero leading, the armature and field m.m.f. waves are directly aiding.

### MOTORING-MODE OPERATION

Positive current in a motor conventionally enters the positive terminal and circulates in the windings in the direction opposite to that in which the e.m.f. acts. In Fig. 12.3 the phase of the current with



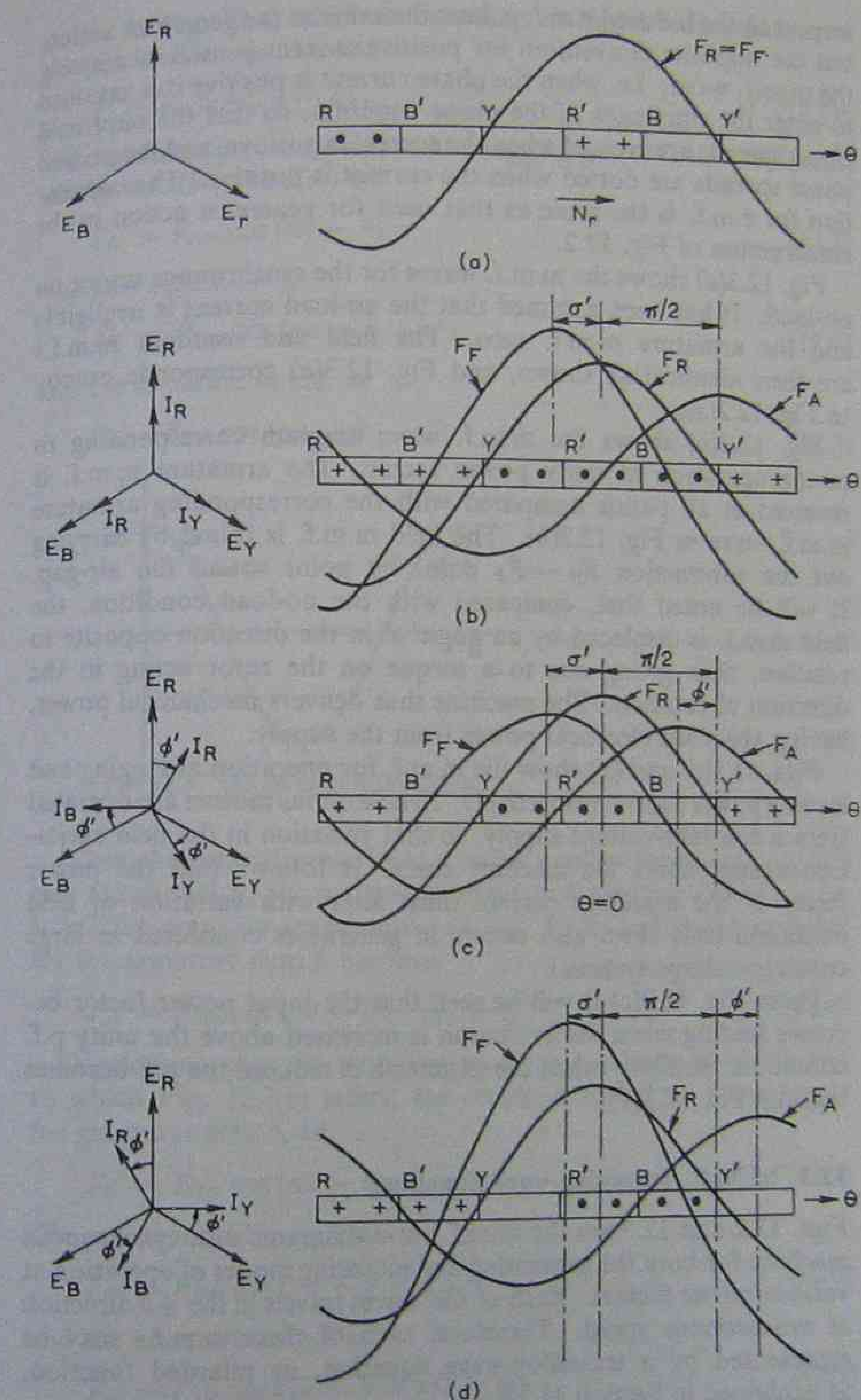



*Fig. 12.3* M.M.F. WAVE DIAGRAMS FOR SYNCHRONOUS MOTOR

respect to the induced e.m.f. is kept the same as for generator action, but the opposite convention for positive current is used in drawing the m.m.f. wave; i.e. when the phase current is positive it is assumed to enter the start ends of the phase windings, so that the unprimed phase spreads are crossed when the current is positive, and the primed phase spreads are dotted when the current is positive. The convention for e.m.f. is the same as that used for generator action in the construction of Fig. 12.2.

Fig. 12.3(*a*) shows the m.m.f. waves for the synchronous motor on no-load. It has been assumed that the no-load current is negligible and the armature m.m.f. zero. The field and resultant m.m.f.s are then identical as shown, and Fig. 12.3(*a*) corresponds exactly to Fig. 12.2(*a*).

Fig. 12.3(*b*) shows the m.m.f. wave diagram corresponding to motor operation at unity power factor. The armature m.m.f. is reversed at all points compared with the corresponding armature m.m.f. wave in Fig. 12.2(*b*). The field m.m.f. is found by carrying out the subtraction $F_R - F_A$ point by point round the air-gap. It will be noted that, compared with the no-load condition, the field m.m.f. is displaced by an angle $\sigma'$ in the direction opposite to rotation, thus giving rise to a torque on the rotor acting in the direction of rotation. The machine thus delivers mechanical power, having absorbed electrical power from the supply.

Figs. 12.3(*c*) and (*d*) show the m.m.f. for operation at lagging and leading power factors respectively. Synchronous motors are operated from a constant-voltage supply, so that variation in the field excitation cannot affect the machine e.m.f. It follows that the power factor of the armature current must alter with variation of field excitation (this effect also occurs in generators connected in large constant-voltage systems).

From Fig. 12.3(*c*) it will be seen that the input power factor becomes leading when the excitation is increased above the unity p.f. condition. Similarly when the excitation is reduced the p.f. becomes lagging (Fig. 12.3(*d*)).

## 12.3 M.M.F. Travelling-wave Equations

Figs. 12.2 and 12.3 are the m.m.f. wave diagrams of a synchronous machine for both the generating and motoring modes of operation at various power factors. Each of the waves travels in the $+\theta$ direction at synchronous speed. Therefore, each of these m.m.f.s may be represented by a travelling-wave equation, or retarded function, as explained in Section 11.12.

For example, the equation of the cosinusoidally distributed m.m.f.



shown in Fig. 12.4, which is travelling in the $+\theta$ direction at $\omega$ electrical radians per second is

$$F' = F_m \cos(\omega t - \theta) \tag{12.2}$$ *

Considering generator action at a lagging power factor, the resultant m.m.f. (Fig. 12.2(*c*)) is

$$F_R' = F_{Rm} \cos(\omega t - \theta) \tag{12.3}$$

The field m.m.f. is

$$F_F' = F_{Fm} \cos(\omega t - \theta + \sigma') \tag{12.4}$$

and the armature m.m.f. is

$$F_A' = F_{Am} \cos(\omega t - \theta - \pi/2 - \phi') \tag{12.5}$$

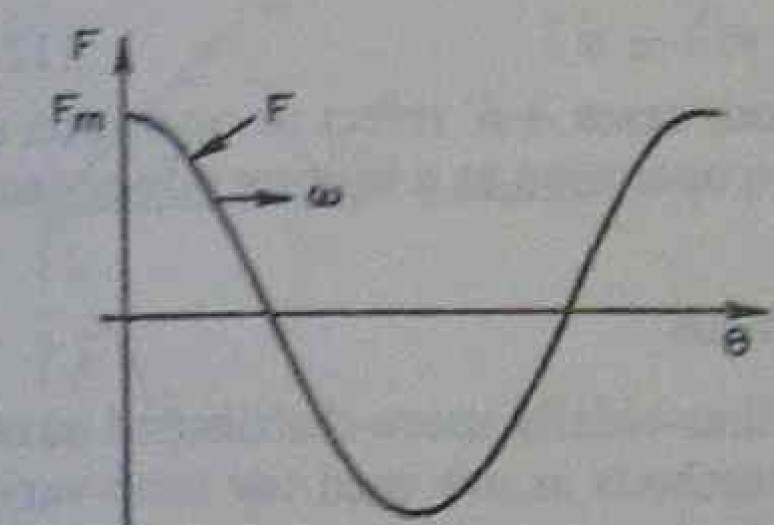



*Fig. 12.4* TRAVELLING-WAVE M.M.F.

When the generator works at a leading power factor, eqns. (12.3) and (12.4) still apply for the resultant and separate field m.m.f.s. It will be seen, by examination of Fig. 12.2(*d*), that the equation for the armature m.m.f. becomes

$$F_A' = F_{Am} \cos(\omega t - \theta - \pi/2 + \phi') \tag{12.6}$$

Considering now the motoring mode at a lagging power factor, to which Fig. 12.3(*c*) refers, the resultant m.m.f. is the same as for generator action, i.e.

$$F_R' = F_{Rm} \cos(\omega t - \theta) \tag{12.7}$$

The separate field m.m.f. is

$$F_F' = F_{Fm} \cos(\omega t - \theta - \sigma') \tag{12.8}$$

and the armature m.m.f. is

$$F_A' = F_{Am} \cos(\omega t - \theta + \pi/2 - \phi') \tag{12.9}$$

* A prime (′) is used to indicate instantaneous values of m.m.f.

When the power factor is leading, the armature m.m.f. becomes

$$F_A' = F_{Am} \cos(\omega t - \theta + \pi/2 + \phi') \tag{12.10}$$

These results may be summarized as follows: The resultant m.m.f. for any mode of operation is

$$F_R' = F_{Rm} \cos(\omega t - \theta) \tag{12.3}$$

The separate field m.m.f. is

$$F_F' = F_{Fm} \cos(\omega t - \theta \pm \sigma') \tag{12.8}$$

where $+\sigma'$ is used for the generating mode and $-\sigma'$ for the motoring mode.

The armature m.m.f. is

$$F_A' = F_{Am} \cos(\omega t - \theta - \pi/2 \pm \phi') \tag{12.11}$$

for the generating mode and

$$F_A' = F_{Am} \cos(\omega t - \theta + \pi/2 \pm \phi') \tag{12.12}$$

for the motoring mode. For both cases $+\phi'$ refers to operation at a leading power factor and $-\phi'$ to operation at a lagging power factor.

## 12.4 M.M.F. Complexor Diagrams

The sum (or difference) of two sinusoidally space-distributed m.m.f.s may be found using the same methods as are used for time-varying sinusoidal quantities. Although the sinusoidally space-distributed m.m.f.s of the synchronous machine are all travelling at synchronous speed (in a stator-wound machine), they may still be dealt with by means of complexor diagrams, since, under steady-state conditions, the relative positions of the waves do not alter.

The m.m.f. complexor diagrams may be deduced either directly from the m.m.f. wave diagrams of Figs. 12.2 and 12.3 or from the travelling-wave equations derived in Section 12.3.

Adopting the latter method for generator mode operation at a lagging power the travelling-wave equations are

$$F_R' = F_{Rm} \cos(\omega t - \theta) \tag{12.2}$$

$$F_F' = F_{Fm} \cos(\omega t - \theta + \sigma') \tag{12.4}$$

$$F_A' = F_{Am} \cos(\omega t - \theta - \pi/2 - \phi') \tag{12.5}$$

At any particular point in the air-gap denoted by $\theta = \theta_0$ the m.m.f.s are

$$F_R' = F_{Rm} \cos(\omega t - \theta_0) \tag{12.13}$$

$$F_F' = F_{Fm} \cos(\omega t - \theta_0 + \sigma') \tag{12.14}$$

$$F_A' = F_{Am} \cos(\omega t - \theta_0 - \pi/2 - \phi') \tag{12.15}$$



Since $\theta_0$ is a particular value of $\theta$ and therefore not a variable, the above equations represent, not travelling waves, but quantities varying sinusoidally with time.

The corresponding complexor diagram is shown in Fig. 12.5(*a*), where the m.m.fs. $F_R'$, $F_F'$ and $F_A'$ are represented by the complexors

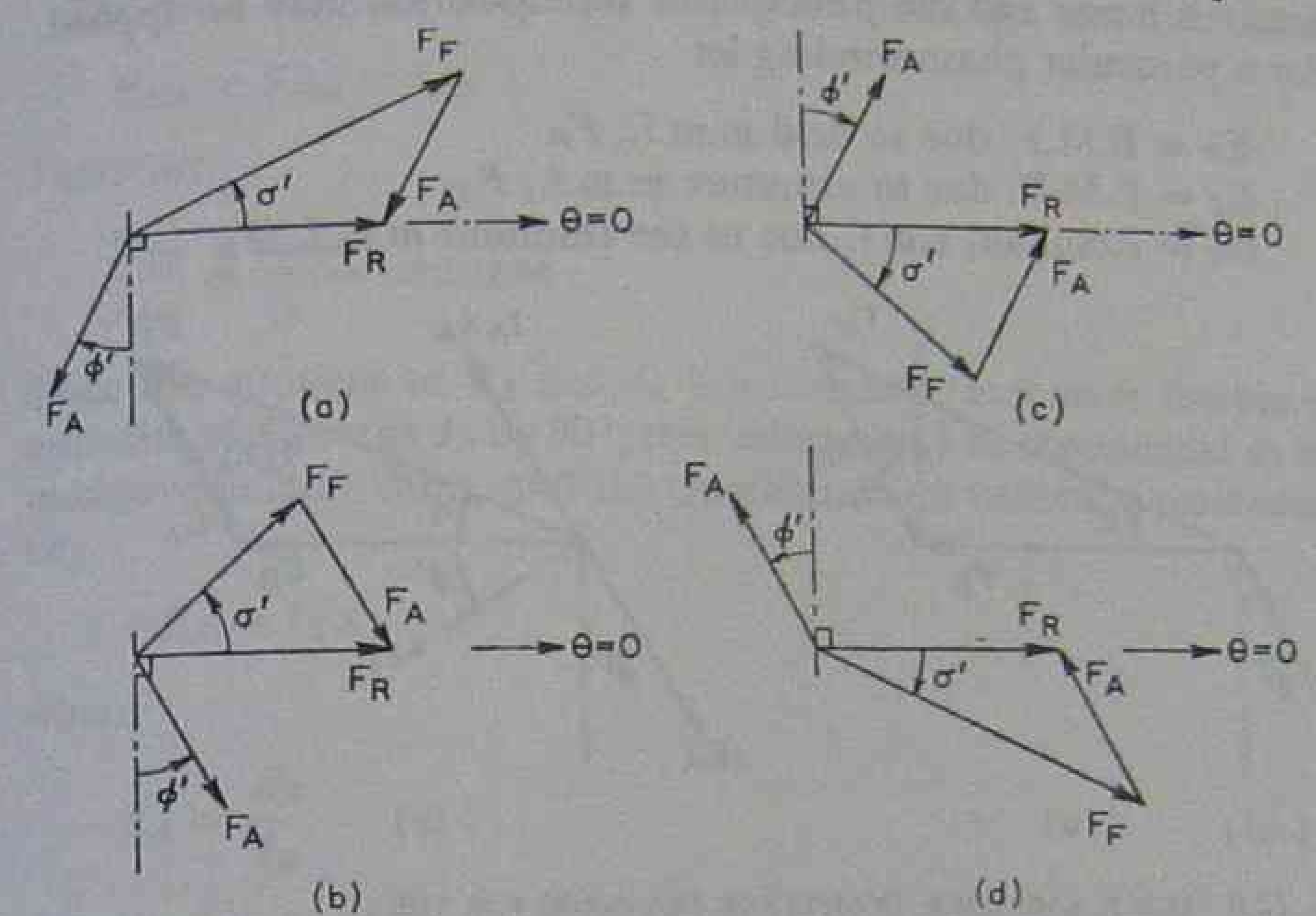



*Fig. 12.5* M.M.F. COMPLEXOR DIAGRAMS FOR THE SYNCHRONOUS MACHINE

(*a*) Generator operation at a lagging power factor
(*b*) Generator operation at a leading power factor
(*c*) Motor operation at a lagging power factor
(*d*) Motor operation at a leading power factor

$\boldsymbol{F_R}$, $\boldsymbol{F_F}$ and $\boldsymbol{F_A}$. For simplicity the diagram has been drawn for the air-gap position $\theta_0 = 0$. The chain-dotted line indicates the unity-power-factor position of the m.m.f. $\boldsymbol{F_A}$. Figs. 12.5(*b*), (*c*) and (*d*) are similar diagrams for different power factors and different operating modes.

## 12.5 E.M.F. Complexor Diagram

Assuming that the reluctance of the magnetic paths in the stator and rotor is negligible and that the air-gap is uniform, the sinusoidally distributed travelling-wave m.m.f.s $F_F$ and $F_A$ and their resultant $F_R$ may be assumed to give rise to separate sinusoidally distributed flux densities. Each of these flux density distributions will travel at synchronous speed, its maximum value occurring at the same place as that of the corresponding m.m.f. and travelling with it at synchronous speed. The relative motion between these flux density distributions and the phase windings will induce e.m.f.s in the windings.

Since only the air-gap reluctance is taken into account, the magnetic circuit is linear and the principle of superposition may be applied. For a particular phase winding let

$E_F$ = E.M.F. due to field m.m.f., $F_F$
$E_A$ = E.M.F. due to armature m.m.f., $F_A$
$E_R$ = Resultant e.m.f. due to the resultant m.m.f., $F_R$

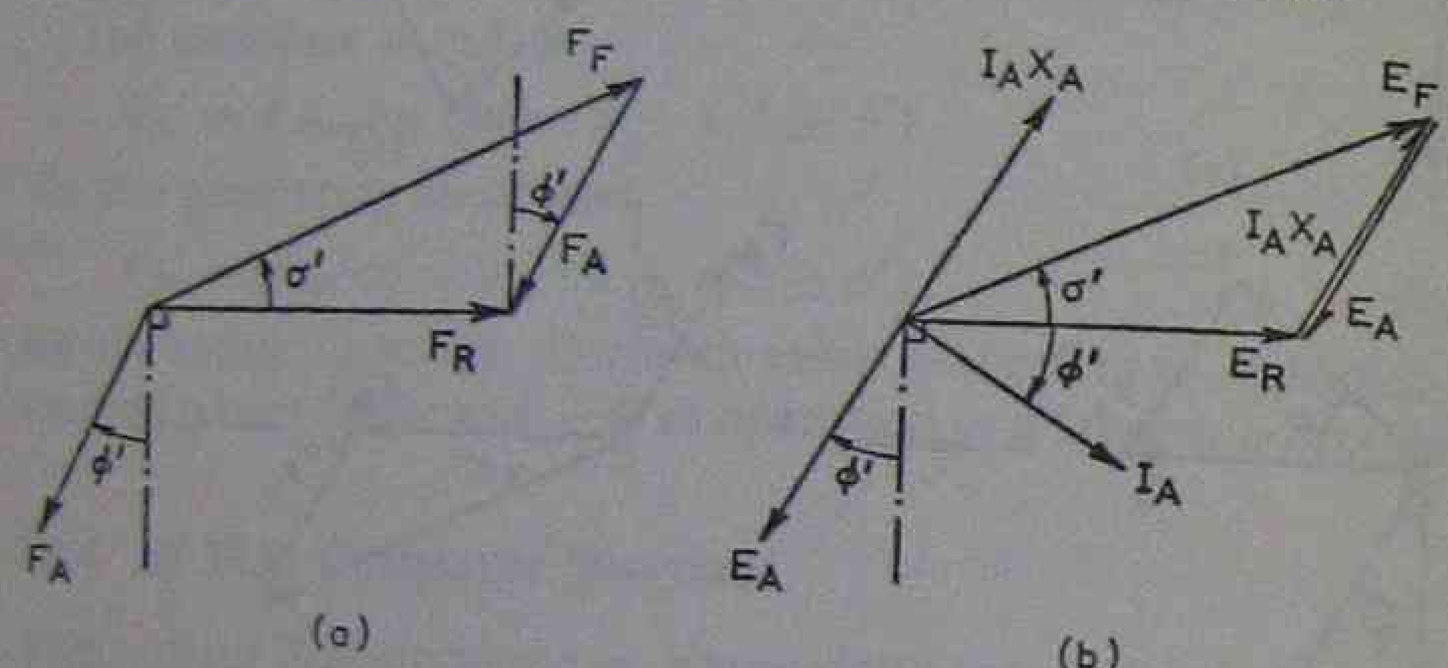



*Fig. 12.6* M.M.F. AND E.M.F. COMPLEXOR DIAGRAMS FOR THE SYNCHRONOUS GENERATOR

The relative phase angles of $E_F$, $E_A$ and $E_R$ will be the same as those of $F_F$, $F_A$ and $F_R$.

The m.m.f. complexor diagram, as explained in Section 12.4, is drawn for a particular position round the air-gap $\theta_0 = 0$.

The m.m.f. and e.m.f. complexor diagrams are shown in Figs. 12.6(*a*) and (*b*) respectively for the case of a generator working at a lagging power factor. The resultant e.m.f. $E_R$ is shown as the complexor sum of $E_F$ and $E_A$:

$$E_R = E_F + E_A \tag{12.16}$$

For a fixed value of separate excitation, $E_F$ will be constant. An examination of Fig. 12.6(*b*) reveals that any change of either the armature current or the load power factor would alter the resultant e.m.f. $E_R$. It is more convenient to work with a constant-voltage source in an equivalent circuit, so that $E_F$ is customarily regarded as the e.m.f. since it does not alter with load and is also the terminal voltage on open-circuit (when $E_A = 0$). The effect of the armature m.m.f. is treated, not as a contribution to the available e.m.f. but



as an internal voltage drop, and the phase opposite of $E_A$ is subtracted from $E_F$.

Examination of Fig. 12.6(*b*) shows that $E_A$ will always lag $I_A$ by 90° irrespective of the power-factor angle. The phase opposite of $E_A$, namely $-E_A$, leads $I_A$ by 90° for all conditions.

The peak value of the e.m.f. due to the armature m.m.f. is $E_{Am}$:

$$E_{Am} \propto F_{Am} \propto I_{Am}$$

Therefore

$$\frac{E_{Am}}{I_{Am}} = \frac{E_A}{I_A} = \text{constant}$$

Since the quotient of $E_A$ and $I_A$ is a constant, and since the phase opposite of $E_A$ leads $I_A$ by 90°, this voltage may be represented as an inductive voltage drop, and the quotient as an inductive reactance, i.e.

$$-E_A = I_A X_A$$

where

$$X_A = \frac{E_A}{I_A} \tag{12.17}$$

Substituting for $E_A$ in eqn. (12.16) and rearranging,

$$E_F = E_R + I_A X_A \tag{12.18}$$

This complexor summation is shown in Fig. 12.6(*b*).

In Section 12.7 an expression for $X_A$ is found in terms of the physical dimensions of the machine.

### 12.6 Equivalent Circuit of the Synchronous Machine

The preceding section has shown that the equivalent circuit of a synchronous machine must contain a voltage source $E_F$ which is constant for a constant excitation current $I_F$ and a series-connected reactance $X_A$. In addition, an actual machine winding will have resistance $R$ and (in the same way as a transformer) leakage reactance $X_L$.

Fig. 12.7(*a*) shows the full equivalent circuit of the synchronous machine in which the current flows in the conventionally positive direction for generator-mode operation (a source), i.e. emerging from the positive terminal. Applying Kirchhoff's law to this circuit,

$$E_F = V + IR + jIX_L + jIX_A \tag{12.19}$$

Fig. 12.7($b$) is the corresponding complexor diagram. The resultant e.m.f. $E_R$ is shown for the sake of completeness but will be omitted in subsequent diagrams.

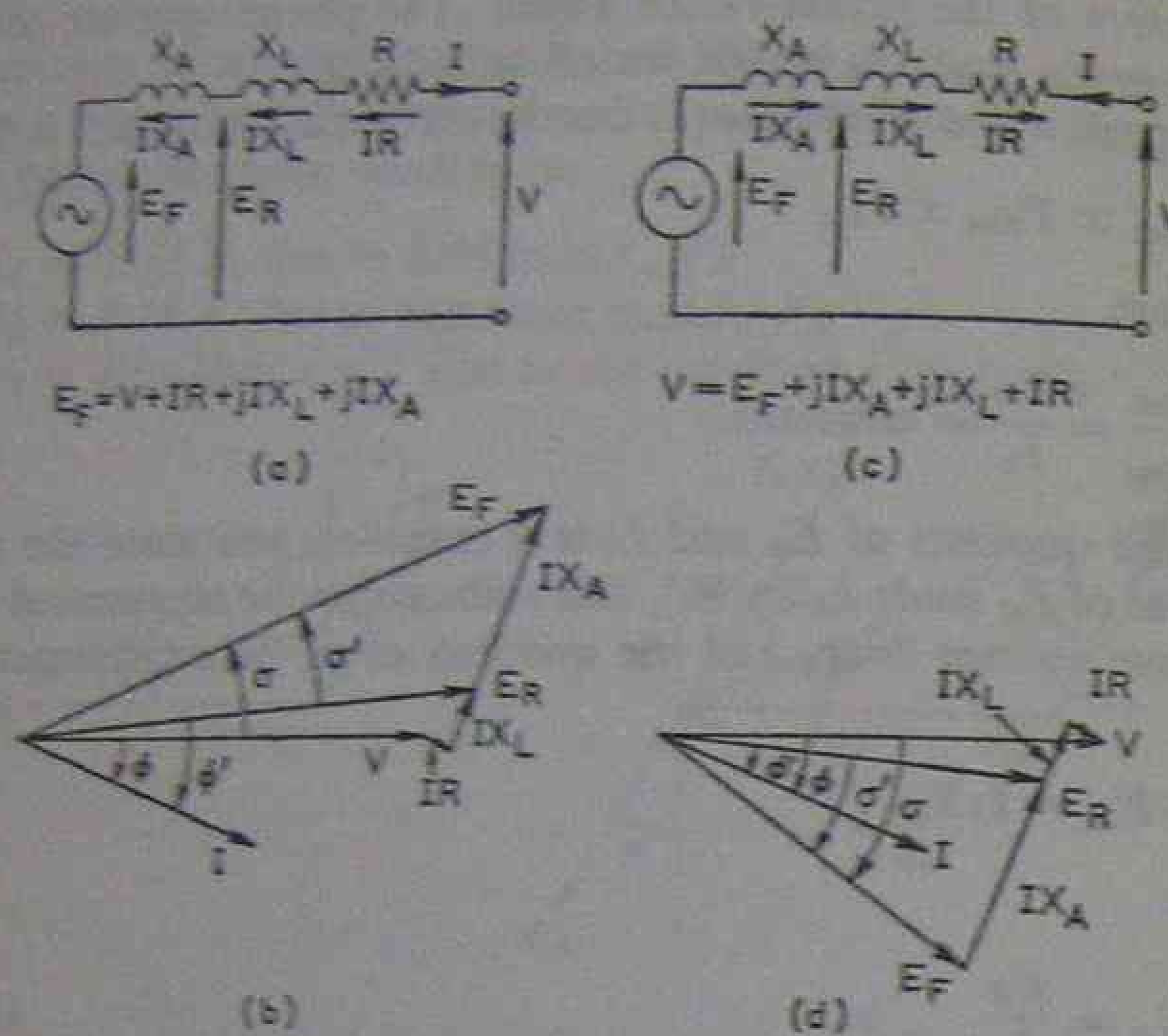



*Fig. 12.7* EQUIVALENT CIRCUITS AND FULL COMPLEXOR DIAGRAMS FOR THE SYNCHRONOUS MACHINE
($a$), ($b$) Generator ($c$), ($d$) Motor

Eqn. (12.19) may be rewritten as

$$E_F = V + IZ_s \tag{12.20}$$

where $Z_s$ is the *synchronous impedance*.

$$Z_s = R + j(X_L + X_A) \tag{12.21}$$

or

$$Z_s = R + jX_s \tag{12.22}$$

where $X_s$ is the *synchronous reactance*:

$$X_s = X_L + X_A \tag{12.23}$$

In polar form the synchronous impedance is

$$Z_s = Z_s\underline{/\psi} \tag{12.24}$$

where

$$\psi = \tan^{-1}\frac{X_s}{R} \tag{12.25}$$

and

$$Z_s = \sqrt{(R^2 + X_s^2)} \tag{12.26}$$



Frequently in synchronous machines $X_s \gg R$, in which case eqn. (12.26) becomes

$$Z_s = X_s\underline{/90^\circ} = jX_s \tag{12.27}$$

Fig. 12.7($c$) shows the full equivalent circuit of the synchronous motor, in which the current flows in the conventionally positive direction for motor-mode operation (a load), i.e. entering the position terminal. Applying Kirchhoff's law to this circuit gives

$$V = E_F + jIX_A + jIX_s + IR \tag{12.28}$$

Fig. 12.7($d$) is the corresponding complexor diagram.

**EXAMPLE 12.1** A 3-phase 11·8kV 75MVA, 50Hz 2-pole star-connected synchronous generator requires a separate field m.m.f. having a maximum value of $3{\cdot}0 \times 10^4$ At/pole to give normal rated voltage on open-circuit. The flux per pole on open-circuit is approximately 5·3Wb.

Determine ($a$) the maximum armature m.m.f. per pole corresponding to rated full-load current, and ($b$) the synchronous reactance if the leakage reactance of the armature winding is 0·18Ω. Find also the p.u. value of the synchronous reactance.

Neglect the effect of space harmonics in the field and armature m.m.f.s. Assume the flux per pole to be proportional to the m.m.f. and the armature winding to be uniform and narrow spread.

The e.m.f. per phase, from eqn. (11.20), is

$$E_p = \frac{K_d K_s \omega \Phi N_p}{\sqrt{2}}$$

and the distribution factor for a uniform narrow-spread winding is

$$K_d = \frac{3}{\pi} = 0{\cdot}955 \tag{11.13}$$

$$E_p = \frac{11{\cdot}8 \times 10^3}{\sqrt{3}} = 6{,}800\,\text{V}$$

Taking $K_s = 1$, the number of turns per phase is

$$N_p = \frac{\sqrt{2}E_p}{K_s K_d \omega \Phi} = \frac{2 \times 6{,}800}{0{\cdot}955 \times 2\pi 50 \times 5{\cdot}3} = 6{\cdot}05$$

Since the number of turns per phase must be an integer, take $N_p = 6$.

The maximum armature m.m.f., from eqn. (11.28), is

$$F_{Am} = \frac{18 F_{pm}}{\pi^2} = \frac{18}{\pi^2}\frac{\sqrt{2} I_p N_p}{2}$$

$$\text{Rated current per phase} = \frac{75 \times 10^6}{3 \times 6{,}800} = 3{,}680\,\text{A}$$

so that

$$F_{Am} = \frac{18}{\pi^2} \times \sqrt{2} \times \frac{3{,}680 \times 6}{2} = \underline{\underline{2{\cdot}85 \times 10^4\,\text{At/pole}}}$$

Since the flux per pole is proportional to the m.m.f.,

$$\frac{E_a}{E_f} = \frac{F_{am}}{F_{fm}}$$

$$E_a = 6{,}900 \times \frac{245 \times 10^3}{340 \times 10^3} = 6{,}450\,\text{V}$$

$$X_a = \frac{E_a}{I_a} = \frac{6{,}450}{3{,}680} = 1{\cdot}75\,\Omega$$

$$X_s = X_a + X_l = 1{\cdot}75 + 0{\cdot}18 = \underline{\underline{1{\cdot}93\,\Omega}}$$

Taking rated phase voltage and current as bases,

$$X_{spu} = \frac{3{,}680 \times 1{\cdot}93}{6{,}900} = \underline{\underline{1{\cdot}04\,\text{p.u.}}}$$

## 12.7 Synchronous Reactance in Terms of Main Dimensions

It is assumed that (*a*) magnetic saturation is absent; (*b*) the armature m.m.f. is sinusoidally distributed; (*c*) the air-gap is uniform; and (*d*) the reluctance of the magnetic paths in the stator and rotor is negligible.

Let $D$ = Internal stator diameter
$L$ = effective stator (or core) length
$l_g$ = radial gap length

The synchronous reactance $X_s$ is

$$X_s = X_l + X_a \tag{12.23}$$

The reactance $X_a$ is

$$X_a = \frac{E_a}{I_a} \tag{12.17}$$

where $E_a$ is the armature e.m.f. per phase due to the armature m.m.f. $F_a$, and $I_a$ is the armature current per phase.

The value of $E_a$ may be found by using eqn. (11.20), which gives the e.m.f. per phase of a polyphase winding:

$$E_a = K_d K_s \frac{\omega \Phi_a N_p}{\sqrt{2}} \tag{12.30}$$

where the distribution factor for a narrow-spread uniform winding is, from eqn. (11.13), $K_d = 3/\pi$; the coil span factor, $k_s = 1$; and $\Phi_a$ is the flux per pole due to the armature m.m.f. $F_a$.



From eqn. (11.28), and assuming that the armature m.m.f. is sinusoidally distributed,

Maximum armature m.m.f. per pole: $F_{am} = \dfrac{18F_{pm}}{\pi^2} = \dfrac{18}{\pi^2}\dfrac{\sqrt{2}I_a N_p}{2p}$

Maximum air-gap field strength: $H_{gm} = \dfrac{F_{am}}{l_g} = \dfrac{1}{l_g}\dfrac{18}{\pi^2}\dfrac{\sqrt{2}I_a N_p}{2p}$

Maximum air-gap flux density: $B_{gm} = \mu_0 H_{gm} = \dfrac{\mu_0}{l_g}\dfrac{18}{\pi^2}\dfrac{\sqrt{2}I_a N_p}{2p}$

Since the air-gap flux density is sinusoidally distributed,

Average air-gap flux density: $B_{av} = \dfrac{2}{\pi} B_{gm}$

Flux per pole due to armature m.m.f.: $\Phi_a = B_{av} \times \text{Pole area}$

$$= \frac{2}{\pi}\frac{\mu_0}{l_g}\frac{18}{\pi^2}\frac{\sqrt{2}\,I_a N_p}{2p}\frac{\pi D L}{2p}$$

Substituting for $K_d$, $K_s$ and $\Phi_a$ in eqn. (12.30), and then substituting the resulting expression for $E_a$ in eqn. (12.17),

$$X_a = \frac{E_a}{I_a} = \frac{\dfrac{3}{\pi}\,\omega\,\dfrac{2}{\pi}\dfrac{\mu_0}{l_g}\dfrac{18}{\pi^2}\dfrac{\pi D L}{2p}\dfrac{\sqrt{2}I_a N_p}{2p}N_p}{\sqrt{2}I_a}$$

$$= \omega\left(\frac{18}{\pi^2}\right)^2 \frac{\mu_0}{3l_g}\pi D L\left(\frac{N_p}{2p}\right)^2 \tag{12.31}$$

To obtain the synchronous reactance, an allowance for the leakage reactance $X_l$ must be added to $X_a$. The leakage reactance is mainly due to (*a*) slot leakage flux, which links individual slots and is not, therefore, part of the main flux, and (*b*) end-turn leakage flux, which links the end turns of the stator winding following mainly air paths. The evaluation of these leakage fluxes, particularly the latter, presents some difficulties and is beyond the scope of the present volume.

The e.m.f. per phase, $E_a$, is due to the armature current itself and is therefore an e.m.f. of self-induction. The reactance $X_a$ is therefore a *magnetizing reactance*.

Eqn. (12.31) shows that the value of this reactance may be reduced by increasing the gap length $l_g$.

Looked at in another way, for a given machine rating the rated armature current is fixed as is also, as a consequence, the maximum armature m.m.f. per pole. This fixed value of armature m.m.f. has a progressively smaller effect as the air-gap is lengthened.

EXAMPLE 12.2 A 3-phase 13·8kV 100MVA 50Hz 2-pole star-connected cylindrical-rotor synchronous generator has an internal stator diameter of 1·08m and an effective core length of 4·6m. The machine has a synchronous reactance of 2p.u. and a leakage reactance of 0·16p.u. The average flux density over the pole area is approximately 0·6Wb/m². Estimate the gap length.

Assume that the radial air-gap is constant and the armature winding uniform. Neglect the reluctance of the iron core and the space harmonics in the armature m.m.f.

With the above assumptions the reactance $X_A$ is

$$X_A = \omega \left(\frac{18}{\pi^2}\right)^2 \frac{\mu_0}{3l_g} \pi DL \left(\frac{N_p}{2p}\right)^2 \tag{12.31}$$

Base voltage, $V_B = V_p = \dfrac{13{\cdot}8 \times 10^3}{\sqrt{3}} = 7{,}960\,\text{V}$

Base current, $I_B = \dfrac{\text{VA/phase}}{V_B} = \dfrac{100 \times 10^6}{3 \times 7{,}960} = 4{,}180\,\text{A}$

Base impedance, $Z_B = \dfrac{V_B}{I_B} = \dfrac{7{,}960}{4{,}180} = 1{\cdot}91\,\Omega$

$X_{Apu} = X_{spu} - X_{Lpu} = 2{\cdot}00 - 0{\cdot}16 = 1{\cdot}84\,\text{p.u.}$

$X_A = X_{Apu}Z_B = 1{\cdot}84 \times 1{\cdot}91 = 3{\cdot}52\,\Omega$

Flux per pole, $B_{av} \times$ Pole area $= B_{av}\dfrac{\pi DL}{2} = \dfrac{0{\cdot}6 \times \pi \times 1{\cdot}08 \times 4{\cdot}6}{2}$

$= 4{\cdot}68\,\text{Wb}$

$$E_p = K_d K_s \frac{\omega \Phi N_p}{\sqrt{2}} \tag{11.20}$$

For a uniform winding, $K_d = 3/\pi$ and $K_s = 1$, so that

$$N_p = \frac{\sqrt{2}E_p}{K_d K_s \omega \Phi} = \frac{\sqrt{2} \times 7{,}960}{3/\pi \times 2\pi \times 50 \times 4{\cdot}68} = 8{\cdot}02$$

The number of turns per phase must be an integer, say 8. This will require a slightly higher flux per pole and average value of flux density. From eqn. (12.31),

$$l_g = 2\pi \times 50 \times \left(\frac{18}{\pi^2}\right)^2 \times \frac{4\pi \times 10^{-7}}{3 \times 3{\cdot}52} \times \pi \times 1{\cdot}08 \times 4{\cdot}6 \times \left(\frac{8}{2}\right)^2$$

$$= \underline{\underline{3{\cdot}10 \times 10^{-2}\,\text{m}}}$$

## 12.8 Determination of Synchronous Impedance

The ohmic value of the synchronous impedance, at a given value of excitation may be determined by open-circuit and short-circuit tests (Fig. 12.8).



On open-circuit the terminal voltage depends on the field excitation and the magnetic characteristics of the machine. Fig. 12.9 includes a

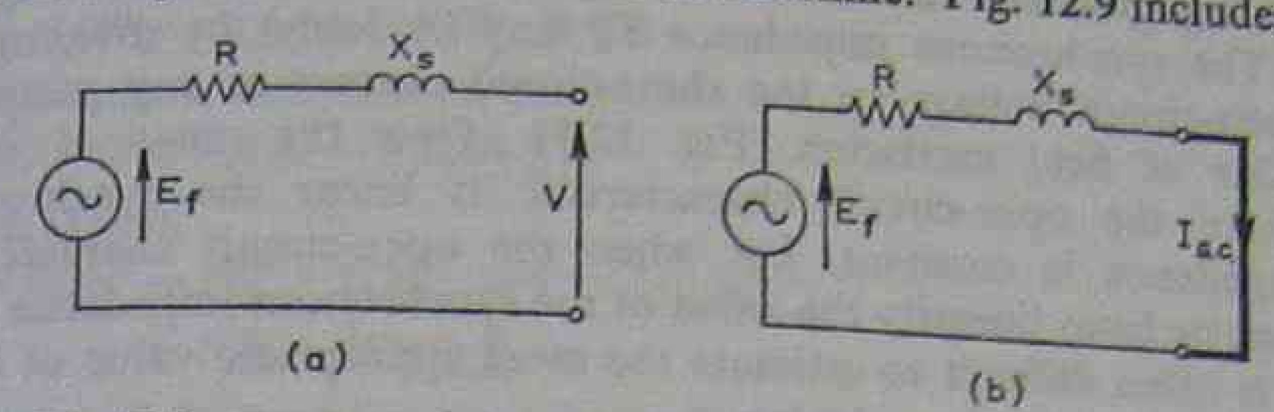



*Fig. 12.8* DETERMINATION OF SYCHRONOUS IMPEDANCE
(*a*) Open-circuit test (*b*) Short-circuit test

typical open-circuit characteristic showing the usual initial linear portion and subsequent saturation portion of a magnetization curve.

On short-circuit the current in an alternator winding will normally lag behind the induced voltage by approximately 90° since the leakage

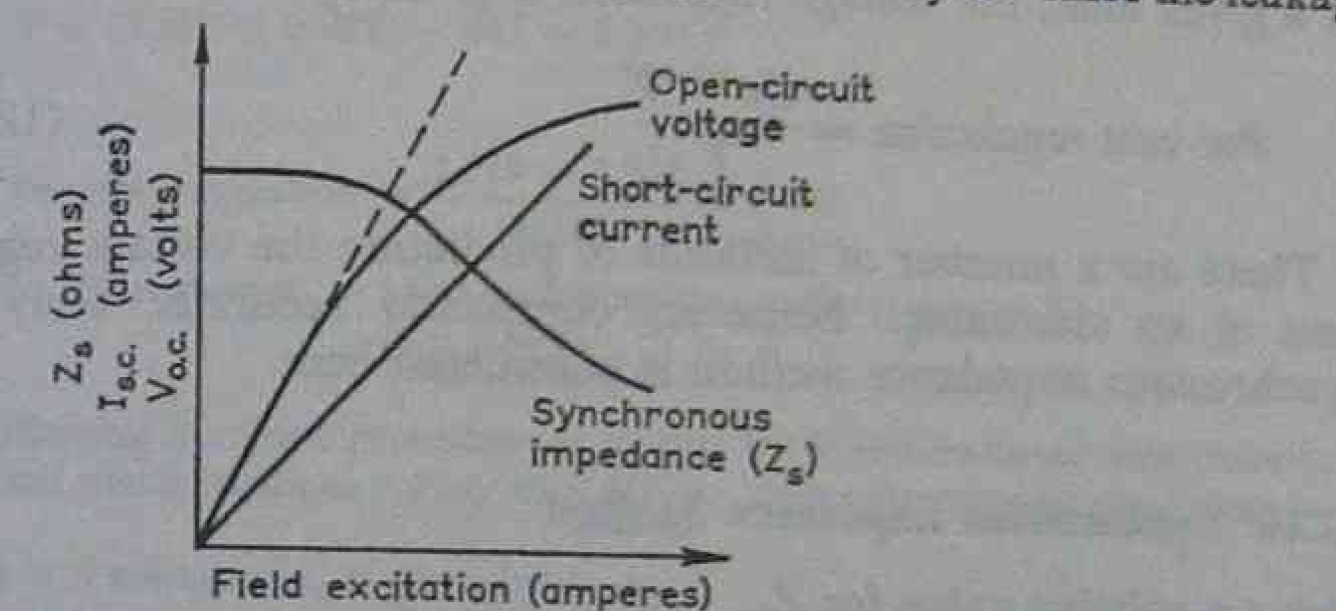



*Fig. 12.9* VARIATION OF SYNCHRONOUS IMPEDANCE WITH EXCITATION

reactance of the winding is normally much greater than the winding resistance. The complexor diagram for short-circuit conditions is shown in Fig. 12.10. It is found that the armature and field m.m.f.s

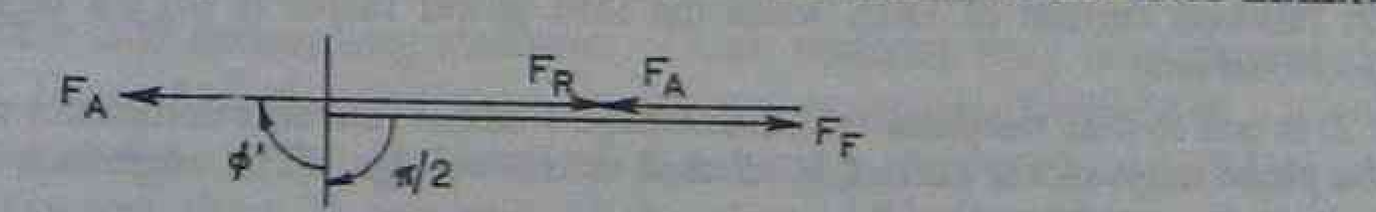



*Fig. 12.10* COMPLEXOR DIAGRAM FOR SHORT-CIRCUIT CONDITIONS

are directly in opposition, so that a surprisingly large excitation is required to give full-load short-circuit current in the windings. The resultant m.m.f. and flux are small since the induced voltage is only required to overcome resistance and leakage reactance voltage drops in the windings. Since the flux is small, saturation effects will be negligible and the short-circuit characteristic is almost straight.

The synchronous impedance $Z_s$ may be found by dividing the open-circuit voltage by the short-circuit current at any particular value of field excitation (Fig. 12.9). Over the range of values where the open-circuit characteristic is linear the synchronous impedance is constant, but when the open-circuit characteristic departs from linearity the value of the synchronous impedance falls. It is often difficult to estimate the most appropriate value of $Z_s$ to use for a particular calculation.

## 12.9 Voltage Regulation

The voltage regulation of an alternator is normally defined as the rise in terminal voltage when a given load is thrown off. Thus, if $E_F$ is the induced voltage on open-circuit and $V$ is the terminal voltage at a given load, the voltage regulation is given by

$$\text{Per-unit regulation} = \frac{E_F - V}{V} \tag{12.32}$$

There are a number of methods of predicting the voltage regulation of an alternator. None are completely accurate. Only the synchronous impedance method is considered here.

## 12.10 Synchronous Impedance Method

Using a suitable value for $Z_s$,

$$E_F = V + IZ_s \tag{12.20}$$

**EXAMPLE 12.3** A 3-phase star-connected alternator has a resistance of 0·5Ω and a synchronous reactance of 5Ω per phase. It is excited to give 6,600 V (line) on open circuit. Determine the terminal voltage and per-unit voltage regulation on full-load current of 130A when the load power factor is (*a*) 0·8 lagging, (*b*) 0·6 leading.

It is best to take the phase terminal voltage $V$ as the reference complexor since the phase angle of the current is referred to this voltage. (The magnitude of $V$ is, however, not known): i.e.

Phase terminal voltage, $V = V\angle 0^\circ$

The magnitude of the e.m.f $E_F$ is known but not its phase with respect to $V$; i.e.

$$E_F = E_F\angle\sigma^\circ = \frac{6{,}600}{\sqrt{3}}\angle\sigma^\circ = 3{,}810\angle\sigma^\circ$$

where $\sigma^\circ$ is the phase of $E_F$ with respect to $V$ as reference.



(*a*) The phase current $I$ lags behind $V$ by a phase angle corresponding to a power factor of 0·8 lagging, i.e.

$$I = 130\angle{-\cos^{-1} 0{\cdot}8} = 130\angle{-36{\cdot}9^\circ}\ \text{A}$$

The synchronous impedance per phase is

$$Z_s = (0{\cdot}5 + j5)\,\Omega = 5{\cdot}02\angle 84{\cdot}3^\circ\,\Omega$$

In eqn. (12.20),

$$3{,}810\angle\sigma^\circ = V\angle 0^\circ + (130\angle{-36{\cdot}9^\circ} \times 5{\cdot}02\angle 84{\cdot}3^\circ)$$
$$= V\angle 0^\circ + 653\angle 47{\cdot}4^\circ$$

Expressing all the terms in rectangular form,

$$3{,}810 \cos\sigma + j\,3{,}810 \sin\sigma = V + j0 + 442 + j482$$

Equating quadrate parts,

$$3{,}810 \sin\sigma = 482$$

whence $\sin\sigma = 0{\cdot}127$ and $\cos\sigma = 0{\cdot}992$

Equating reference parts,

$$3{,}810 \cos\sigma = V + 442$$
$$V = (3{,}810 \times 0{\cdot}992) - 442 = \underline{3{,}340\ \text{V}}$$

and

$$\text{Per-unit regulation} = \frac{3{,}810 - 3{,}340}{3{,}340} = \underline{0{\cdot}141}$$

(*b*) Phase current $= 130$ A at 0·6 leading with respect to $V$

$$= 130\angle{+53{\cdot}1^\circ}$$

Following the same procedure as in part (*a*) it will be found that there is an on-load phase terminal voltage of $\underline{4{,}260\ \text{V}}$. Hence the per-unit regulation, since there is a voltage rise, is given by

$$\frac{3810 - 4260}{4{,}260} = \underline{-0{\cdot}106\ \text{p.u.}}$$

## 12.11 Synchronous Machines connected to Large Supply Systems

In Britain, electrical energy is supplied to consumers from approximately 200 generating stations. These stations vary considerably in size, the installed capacity of the largest exceeding 2,000 MW. About one-quarter of the stations have a rating of less than 50 MW and supply less than 2½ per cent of the electrical energy demanded from the public supply.

The generating stations do not operate as isolated units but are interconnected by the national *grid*, which consists of almost 10,000 miles of main transmission line, for the most part overhead lines operating at 132, 275 and 400 kV. The total generating capacity interconnected by the grid system is over 40,000 MW. The output of any single machine is therefore small compared with the total interconnected capacity. The biggest single generator has a rating of 500 MW. For this reason the performance of a single machine is unlikely to affect appreciably the voltage and frequency of the whole system. A machine connected to such a system, where the capacity of any one machine is small compared with the total interconnected capacity, is often said to be connected to *infinite busbars*. The outstanding electrical characteristics of such busbars are that they are constant-voltage constant-frequency busbars.

When the machine is connected to the infinite busbars the terminal voltage and frequency becomes fixed at the values maintained by the rest of the system. Unless the machine is grossly overloaded or under-excited, no change in the mechanical power supply, load or excitation will alter the terminal voltage or frequency. If the machine is acting as a generator and the mechanical driving power is increased the power output from the machine to the busbars must increase, assuming that the efficiency does not greatly change. In the same way, a decrease of mechanical driving power or the application of a mechanical load (motoring) will produce a decrease in output power or the absorption of power from the busbars.

## 12.12 Synchronizing

The method of connecting an incoming alternator to the live busbars will now be considered. This is called *synchronizing*.

A stationary alternator must not be connected to live busbars, or, since the induced e.m.f. is zero at standstill, a short-circuit will result. The alternator induced e.m.f. will prevent dangerously high switching currents only if the following conditions are almost exactly complied with:

1. The frequency of the induced voltages in the incoming machine must equal the frequency of the voltages of the live busbars.
2. The induced voltages in the incoming machine must equal the live busbar voltages in magnitude and phase.
3. The phase sequence of the busbar voltages and the incoming-machine voltages must be the same.

In modern power stations alternators are synchronized automatically. The principles may be illustrated by the three-lamp method, which, along with a voltmeter, may be used for synchronizing low-power machines.

Fig. 12.11(*a*) is the connexion diagram from which it will be noted that one lamp is connected between corresponding phases while the two others are cross-connected between the other two phases. In the complexor diagram at (*b*) the machine induced voltages are represented by $E_R$, $E_Y$ and $E_B$, while the live supply voltages are



represented by $V_R$, $V_Y$, and $V_B$. The lamp symbols have been added to the complexor diagram to indicate the instantaneous lamp voltages. It will be realized that the speed of rotation of the complexors will correspond to the frequencies of the supply and the machine—if these are the same then the lamp brilliancies will be constant. The speed of the machine should be adjusted until the machine frequency is nearly that of the supply, but exact equality is inconvenient for there would then be, in all probability, a permanent phase difference between corresponding voltages. The machine excitation should now be varied until the two sets of voltages

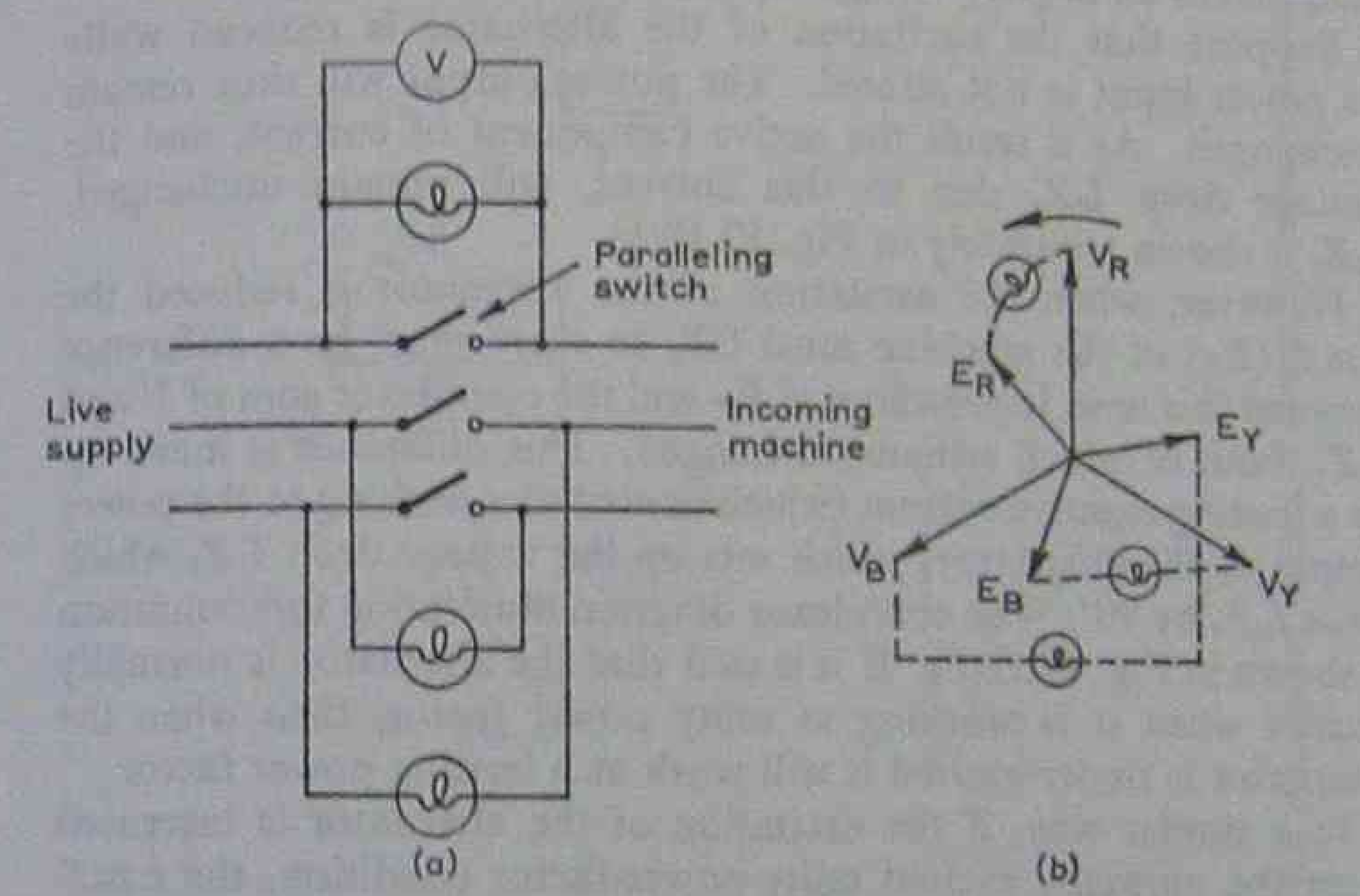



*Fig. 12.11* SYNCHRONIZING BY CROSS-CONNECTED LAMP METHOD

are equal in magnitude. The correct conditions will be obtained at an instant when the straight-connected lamp is dark and the cross-connected lamps are equally bright. If the phase sequence is incorrect no such instant will occur as the cross-connected lamps will, in effect, be straight-connected and all the lamps will be dark simultaneously. In this event the direction of rotation of the incoming machine should be reversed or two lines of the machine should be interchanged. Since the dark range of a lamp extends over a considerable voltage range it is advisable to connect a voltmeter across the straight-connected lamp and to close the paralleling switch when the voltmeter reading is zero. It should be noted that the lamps and the voltmeter must be able to withstand twice the normal phase voltage.

## 12.13 Effect of Variation of Excitation of Synchronous Machine connected to Infinite Busbars

Consider an alternator connected to infinite busbars. Fig. 12.12(*a*) is the complexor diagram of such a machine when operating at unity power factor. The voltage drop $I_aR$ is in phase with $I_a$ and the voltage drop $I_aX_s$ leads $I_a$ by 90°. The complexor sum of $I_aR$ and $I_aX_s$ is $I_aZ_s$, and the e.m.f. ($E_F$) of the machine is obtained by adding $I_aZ_s$ to $V$, the constant busbar voltage. $R$, $X_s$ and $Z_s$ refer to the winding resistance, reactance and impedance respectively. $I_aZ_s$ makes an angle $\psi = \tan^{-1}(X_s/R)$ with $V$.

Suppose that the excitation of the alternator is reduced while its power input is not altered. The power output will thus remain unchanged. As a result the active component of current, and the voltage drop $I_aZ_s$ due to this current, will remain unchanged. $I_aZ_s$ is shown separately in Fig. 12.12(*b*).

However, when the excitation of the alternator is reduced the e.m.f. ($E_F$) of the machine must fall, so there must be a difference between this new, lower value of $E_F$ and the complexor sum of $V$ and $I_aZ_s$, both of which remain unchanged. This difference is made up by a leading reactive current (which contributes nothing to the power output of the alternator) which sets up the voltage drop $I_rZ_s$ which leads $I_aZ_s$ by 90°. The complexor diagram illustrating this condition is shown in Fig. 12.12(*b*). If it is said that the alternator is normally excited when it is working at unity power factor, then when the alternator is under-excited it will work at a leading power factor.

In a similar way, if the excitation of the alternator is increased from the normally excited unity-power-factor condition, the e.m.f. ($E_F$) of the machine will increase, so that there must be a difference between this new, higher value of $E_F$ and the complexor sum of $V$ and $I_aZ_s$, both of which remain unchanged. This difference is made up by a lagging reactive current (which contributes nothing to the power output of the alternator) which sets up the voltage drop $I_rZ_s$ lagging behind $I_aZ_s$ by 90°. The complexor diagram illustrating this condition is shown in Fig. 12.12(*c*). Thus when the alternator is over-excited it will work with a lagging power factor.

Fig. 12.12(*d*), (*e*) and (*f*), give the corresponding diagrams for the synchronous motor connected to infinite busbars. These are essentially similar to those of the alternator. It will be noted that when the motor is under-excited it works with a lagging power factor, whereas the alternator under similar conditions of excitation works with a leading power factor; and that when the synchronous motor is over-excited it works with a leading power factor, whereas the alternator works with a lagging power factor.

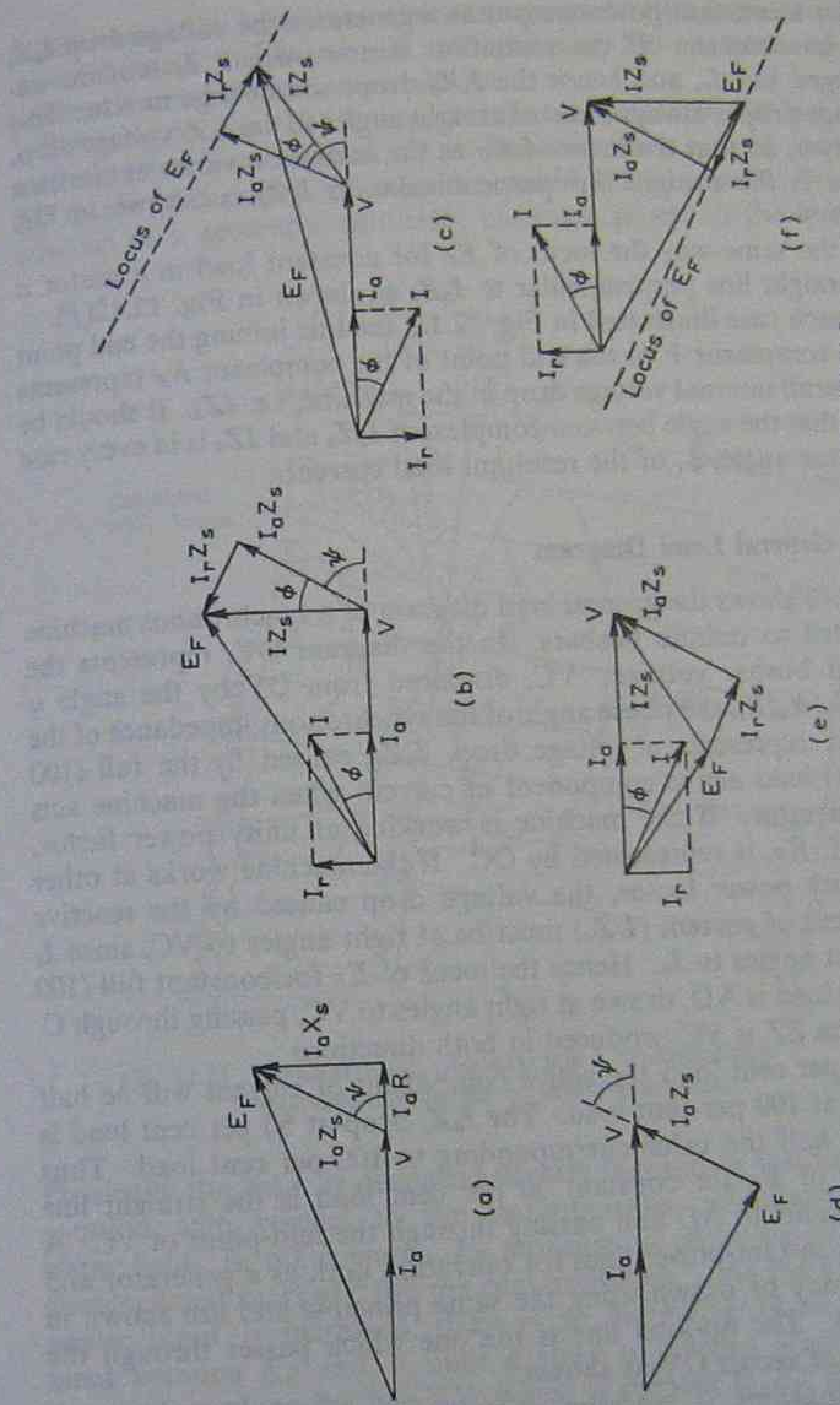


*Fig. 12.12* SYNCHRONOUS GENERATOR DELIVERING CONSTANT POWER TO INFINITE BUSBARS

For a constant power output as a generator the voltage drop $I_aZ_s$ will be constant. If the excitation is now varied, $I_a$ remains unchanged but $I_r$, and hence the $I_rZ_s$ drop, will change in size. This voltage drop is always directed at right angles to the $I_aZ_s$ voltage drop, however, so that the locus of $E_F$ as the excitation varies at constant power is the straight line perpendicular to $I_aZ_s$, as shown in Fig. 12.12(*c*).

In the same way the locus of $E_F$ for constant load in a motor is the straight line perpendicular to $I_aZ_s$ as shown in Fig. 12.12(*f*).

In each case illustrated in Fig. 12.12, the line joining the end point of the complexor $V$ to the end point of the complexor $E_F$ represents the overall internal voltage drop in the machine, i.e. $IZ_s$. It should be noted that the angle between complexors $I_aZ_s$ and $IZ_s$ is in every case the phase angle, $\phi$, of the resultant load current.

## 12.14 General Load Diagram

Fig. 12.13 shows the general load diagram of a synchronous machine connected to infinite busbars. In the diagram OV, represents the constant busbar voltage; VC, displaced from OV by the angle $\psi = \tan^{-1}(X_s/R)$, the phase angle of the synchronous impedance of the machine, represents a voltage drop, $I_aZ_s$, caused by the full (100 per cent) load active component of current when the machine acts as a generator. If the machine is working at unity power factor, the e.m.f. $E_F$, is represented by OC. If the machine works at other than unity power factor, the voltage drop caused by the reactive component of current ($I_rZ_s$) must be at right angles to VC, since $I_r$ is at right angles to $I_a$. Hence the locus of $E_F$ for constant full (100 per cent) load is AD, drawn at right angles to VC, passing through C.

The line ZZ is VC produced in both directions.

At 50 per cent load the active component of current will be half its value at 100 per cent load. The $I_aZ_s$ drop at 50 per cent load is therefore half the value corresponding to 100 per cent load. Thus the locus of $E_F$ for constant 50 per cent load is the straight line drawn parallel to AD and passing through the mid-point of VC. A series of constant-power lines for operation both as a generator and a motor may be drawn using the same principle and are shown in Fig. 12.13. The no-load line is the one which passes through the extremity of vector OV as shown.

If the machine is working at unity power factor there is no reactive current and hence no $I_rZ_s$ voltage drop. The line ZZ therefore represents the locus of $E_F$ for unity power factor at any load. $Z_1Z_1$ is a similar locus for power factor 0·866 leading, and $Z_2Z_2$ for power factor 0·866 lagging.



The locus of $E_F$ for constant excitation is evidently a semicircle with centre O. Loci for 50 per cent, 100 per cent, 150 per cent and 200 per cent normal excitation are shown.

Referring to Fig. 12.13, suppose the machine has 100 per cent excitation and is on no-load; $E_F$ is then coincident with $V$. Suppose the mechanical power input to the machine is increased; it must now act as a generator delivering electrical power to the busbars.

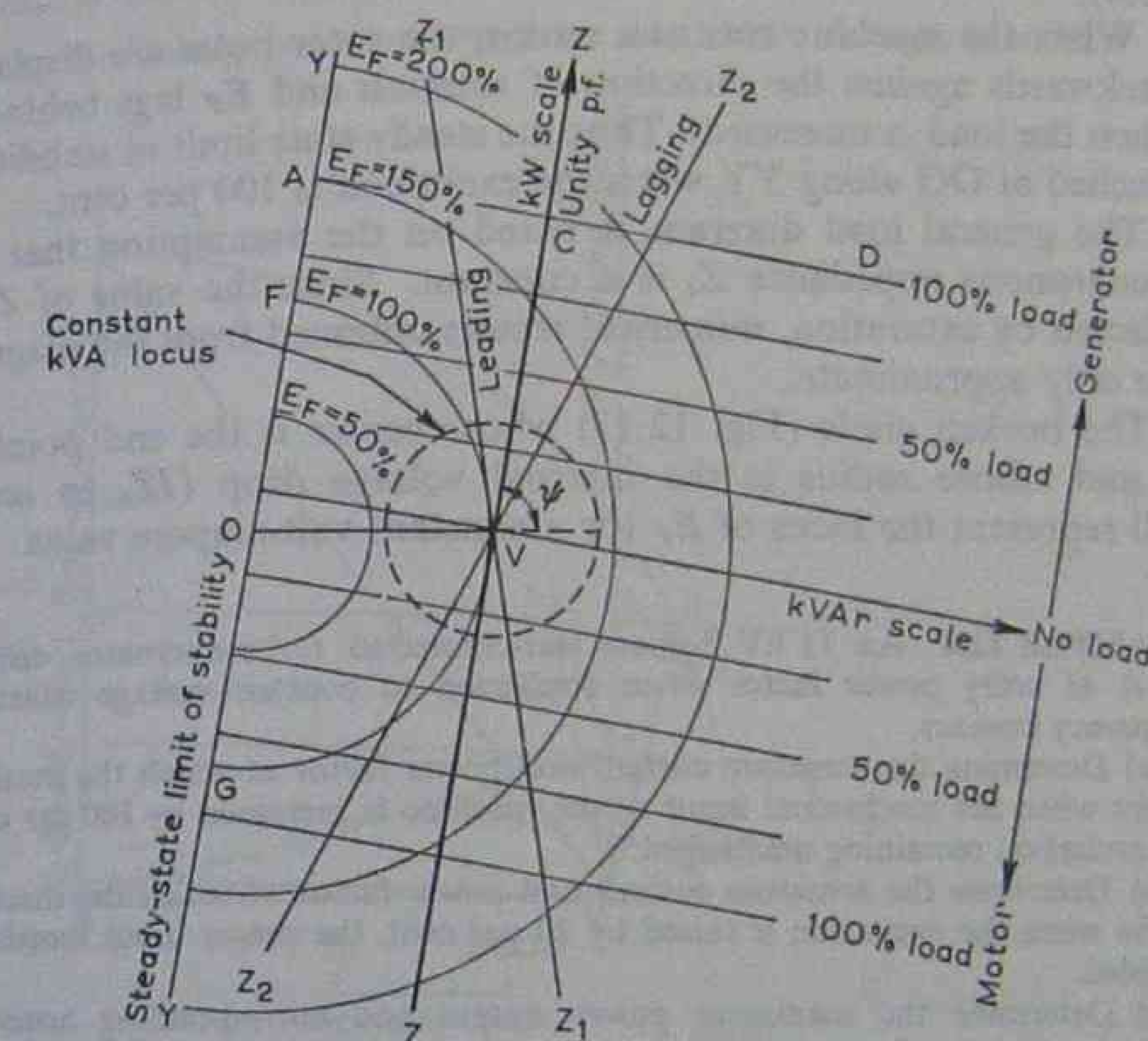



*Fig. 12.13* GENERAL LOAD DIAGRAM FOR A SYNCHRONOUS MACHINE CONNECTED TO INFINITE BUSBARS

Physically the rotor is displaced slightly forward in the direction of rotation with respect to the instantaneous pole-centres of the stator field. In other words, $E_F$ advances in phase on $V$ along the circular locus marked "100 per cent excitation". As the mechanical power input is increased, there is an increasing phase displacement between $E_F$ and $V$ until a stage is reached where $E_F$ has the position along the line YY, which is parallel to ZZ through O. If the mechanical power input were further increased $E_F$ would tend to swing beyond YY. However, the extremity of $E_F$ would then approach the 50 per cent load line instead of moving further away from it and the electrical power fed to the busbars would tend to decrease. The excess mechanical energy fed into the machine must then be absorbed by the machine increasing its speed and breaking from synchronism with the constant-frequency system. YY therefore represents the steady-state limit of stability of the machine. It gives the maximum load which the machine may deliver for a given excitation when the load is very gradually applied. For suddenly applied loads the limit is somewhat lower (see Section 15.6).

When the machine acts as a motor, the rotor poles are displaced backwards against the direction of rotation and $E_F$ lags behind $V$ when the load is increased. Thus the steady-state limit of stability is reached at OG along YY when the excitation is 100 per cent.

The general load diagram is based on the assumption that the synchronous impedance $Z_s$ is a constant. Since the value of $Z_s$ is affected by saturation, numerical results obtained from the diagram are only approximate.

The broken circle (Fig. 12.13) whose centre is the end point of $V$ and whose radius is the internal voltage drop ($IZ_s$ to scale) will represent the locus of $E_F$ for a constant volt-ampere value.

**EXAMPLE 12.4** An 11 kV 3-phase star-connected turbo-alternator delivers 200 A at unity power factor when connected to constant-voltage constant-frequency busbars.

(a) Determine the armature current and power factor at which the machine works when the mechanical input to the machine is increased by 100 per cent, the excitation remaining unchanged.

(b) Determine the armature current and power factor at which the machine works when the excitation is raised by 20 per cent, the power input remaining doubled.

(c) Determine the maximum power output and corresponding armature current and power factor at this new value of excitation, i.e. as in (b).

The armature resistance is 0·4Ω/phase and the synchronous reactance is 8Ω/phase.

Assume that the efficiency remains constant.

The problem is best solved graphically; an analytical solution is tedious in this case where the armature resistance is not neglected. The graphical solution is shown in Fig. 12.14.

Using phase values,

$$V_{ph} = \frac{11{,}000}{\sqrt{3}} = 6{,}350\text{V}$$

OV is drawn as reference complexor 6,350 V in length to a suitable scale.

$$Z_s = 0{\cdot}4 + j8 = 8{\cdot}00\angle 87{\cdot}1^\circ\,\Omega$$

$Vb$ is drawn making an angle of 87·1° with OV as shown. This is the unity-power-factor line.

$$I_aZ_s = 200 \times 8 = 1{,}600\text{V}$$



$Va$ is cut off along the unity-power-factor line 1,600V in length to scale. $Oa$ represents the e.m.f. under the initial conditions stated.

$$E_{F1} = Oa = 6{,}600\text{V to scale}$$

The line at right angles to $Va$ passing through $a$ is the constant-power line corresponding to an active component of 200 A.

(a) When the mechanical power input is doubled the power output must be doubled and therefore the active component of current, $I_a'$, is doubled.

$$I_a'Z_s = 400 \times 8 = 3{,}200\text{V}$$

$Vb$ is cut off along the unity-power-factor line 3,200V in length to scale.

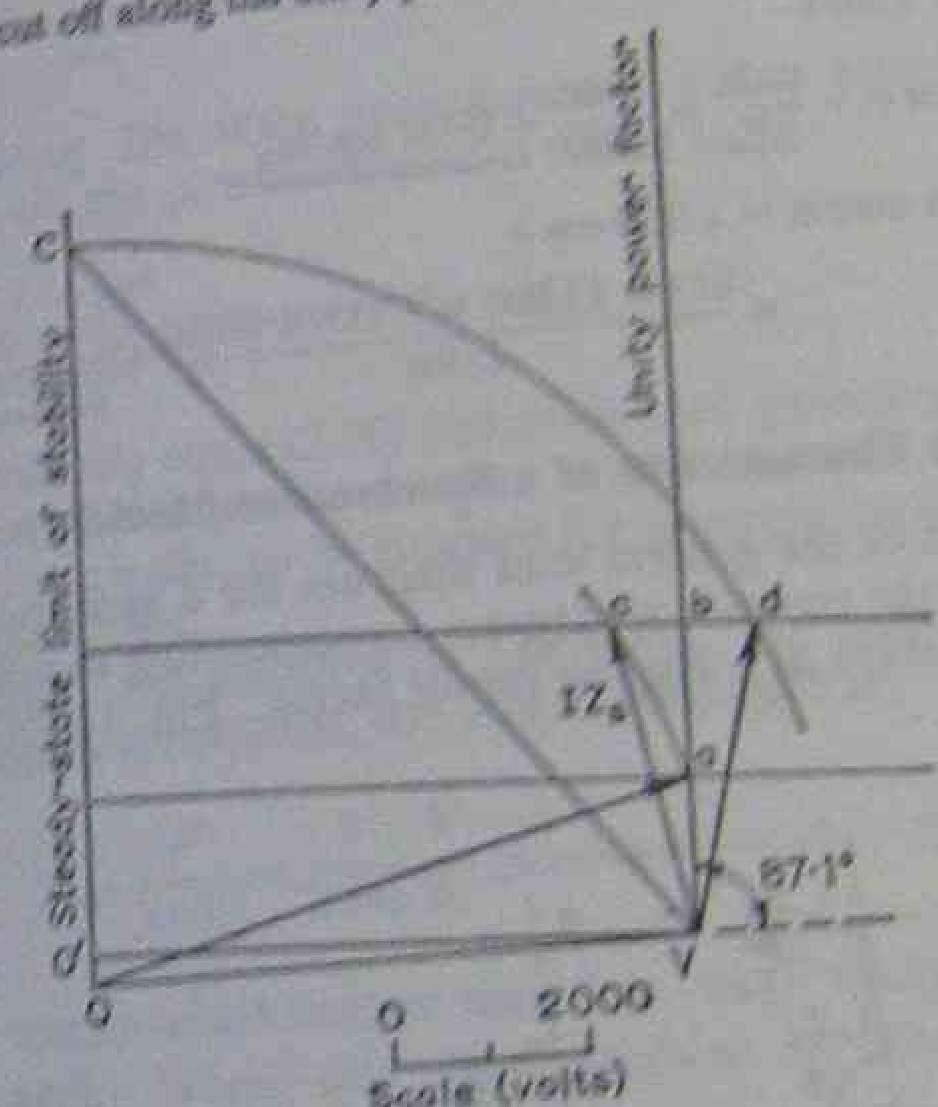



*Fig. 12.14*

A line at right angles to $Vb$ passing through $b$ is drawn. This is the constant-power line. With centre O and radius $Oa$ an arc is drawn to intersect this constant-power line in $c$. Let the resultant current be $I$. Then

$$IZ_s = Vc = 3{,}300\text{V to scale}$$

Thus

$$I = \frac{3{,}300}{8} = \underline{412\text{A}}$$

$$\text{Power factor, } \cos\phi = \frac{I_a'Z_s}{IZ_s} = \frac{3{,}200}{3{,}300} = \underline{0{\cdot}97\text{ leading}}$$

(b) The new value of e.m.f. is 1·2 × 6,600 = 7,920V.

With centre O and radius $E_{F2}$ = 7,920V to scale an arc is struck to cut the constant-power line in $d$, and OC, the steady-state limit of stability line, in C.

$$Vd = 3{,}300\text{V} \quad \text{(by measurement)}$$

New current, $I_3 = \dfrac{3{,}300}{8} = \underline{\underline{412\,\text{A}}}$

Power factor, $\cos\phi = \dfrac{I_a' Z_s}{I_3 Z_s} = \dfrac{3{,}200}{3{,}300} = \underline{\underline{0{\cdot}97 \text{ lagging}}}$

(c) At the steady-state limit of stability for this excitation (point C), the current will be $I_4$, where $I_4 Z_s = \text{CV} = 9{,}900$ V. Thus

$I_4 = \dfrac{9{,}900}{8} = \underline{\underline{1{,}235\,\text{A}}}$

$I_{a4} Z_s = \text{CO}' = 7{,}600$ V

Power factor, $\cos\phi = \dfrac{I_{a4} Z_s}{I_4 Z_s} = \dfrac{7{,}600}{9{,}900} = \underline{\underline{0{\cdot}768 \text{ leading}}}$

Maximum power output $= \sqrt{3} VI \cos\phi$

$$= \frac{\sqrt{3} \times 11{,}000 \times 1{,}235 \times 0{\cdot}768}{1{,}000} = \underline{\underline{18{,}100\,\text{kW}}}$$

## 12.15 Power/Angle Characteristic of a Synchronous Machine

Fig. 12.15($a$) is part of the general load diagram for a synchronous machine and shows the complexor diagram corresponding to generation into infinite busbars at a lagging power factor. Fig. 12.15($b$) is

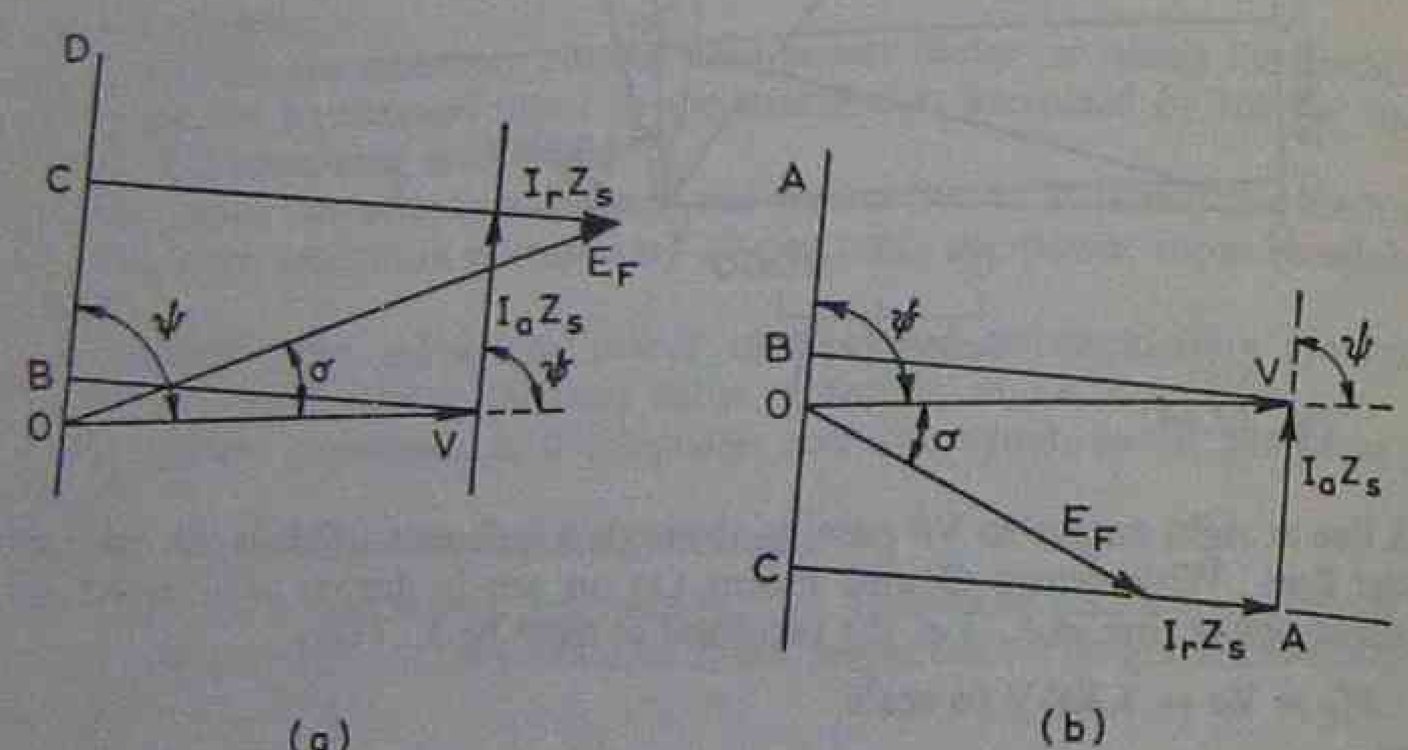



*Fig. 12.15* POWER TRANSFER FOR A SYNCHRONOUS MACHINE
($a$) Generator ($b$) Motor

the corresponding complexor diagram for motor operation also at a lagging power factor. The power transfer is

$$P = 3VI\cos\phi = 3VI_a \qquad (12.33)$$

where $V$ is the phase voltage and $I$ is the phase current.



The projection of the complexors of Fig. 12.15($a$) on the steady-state limit of stability line OD gives

$$I_a Z_s = E_F \cos(\psi - \sigma) - V\cos\psi \qquad (12.34)$$

Substituting the expression for $I_a$ obtained from eqn. (12.34) in eqn. (12.33) gives

$$P = \frac{3V}{Z_s}\{E_F \cos(\psi - \sigma) - V\cos\psi\} \qquad (12.35)$$

Following the same procedure for motor action and using Fig. 12.15 ($b$) the power transfer is found to be

$$P = \frac{3V}{Zs}\{V\cos\psi - E_F\cos(\psi + \sigma)\} \qquad (12.36)$$

Evidently eqn. (12.36) will cover both generator action and motor action if the power transfer $P$ and the load angle $\sigma$ are taken, conventionally, to be positive for generator action and negative for motor action.

Since, for steady-state operation, the speed of a synchronous machine is constant, the torque developed is

$$T = \frac{P}{2\pi n_0} = \frac{3}{2\pi n_0}\frac{V}{Z_s}\{E_F\cos(\psi - \sigma) - V\cos\psi\} \qquad (12.37)$$

In many synchronous machines $X_s \gg R$, in which case $Z_s\underline{/\psi} \approx X_s\underline{/90^\circ}$. When this approximation is permissible eqn. (12.35) becomes

$$P = \frac{3V}{Z_s}\{E_F\cos(90^\circ - \sigma) - V\cos 90^\circ\}$$

$$= \frac{3VE_F}{X_s}\sin\sigma \qquad (12.38)$$

Similarly eqn. (12.37) becomes

$$T = \frac{3}{2\pi n_0}\frac{VE_F}{X_s}\sin\sigma \qquad (12.39)$$

The power/load-angle (or torque/load-angle) characteristic is shown in Fig. 12.16. The dotted parts of this characteristic refer to operation beyond the steady-state limit of stability. Usually stable operation cannot be obtained beyond this limit, so that if the load angle exceeds $\pm 90°$ the operation is dynamic with the machine either

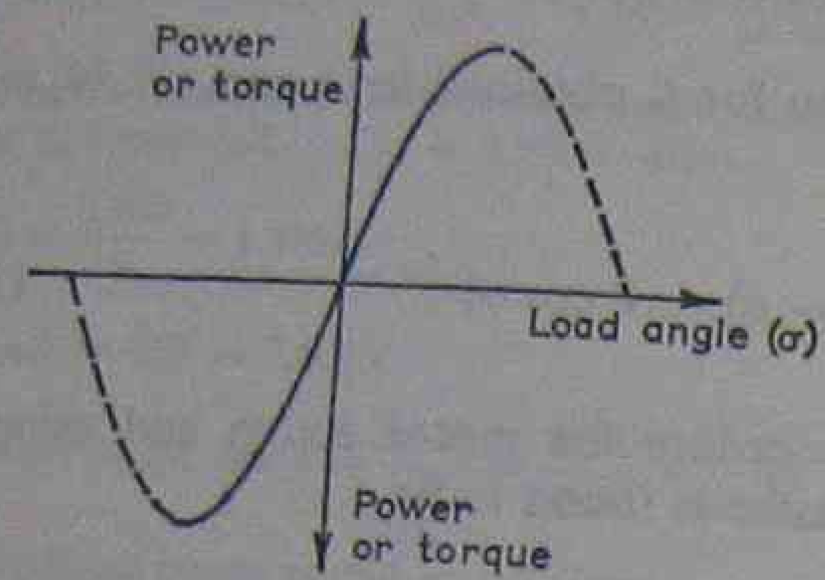



*Fig. 12.16* POWER/LOAD-ANGLE AND TORQUE/LOAD-ANGLE CHARACTERISTICS OF A SYNCHRONOUS MACHINE CONNECTED TO INFINITE BUSBARS

accelerating or decelerating. In this case eqns. (12.38) and (12.39) are only approximately true.

## 12.16 Synchronizing Power and Synchronizing Torque Coefficients

A synchronous machine, whether a generator or a motor, when synchronized to infinite busbars has an inherent tendency to remain synchronized.

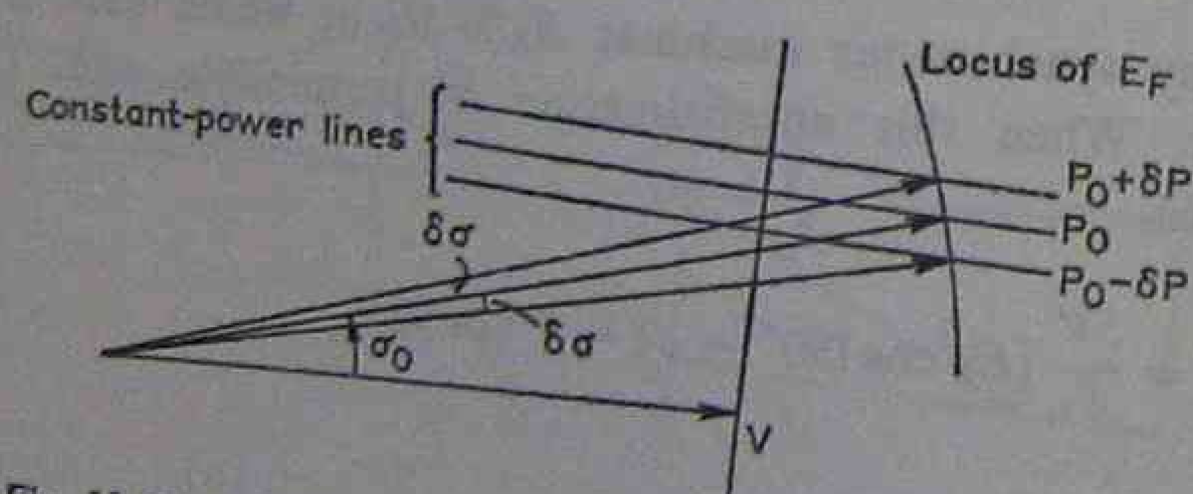



*Fig. 12.17* DETERMINATION OF SYNCHRONIZING POWER COEFFICIENT

In Fig. 12.17, which applies to generator operation at a lagging power factor, the complexor diagram is part of the general load diagram. At a steady load angle $\sigma_0$ the steady power transfer is $P_0$.

Suppose that, due to a transient disturbance, the rotor of the machine accelerates, so that the load angle increases by $\delta\sigma$. This alters the operating point of the machine to a new constant-power



line and the load on the machine increases to $P_0 + \delta P$. Since the steady power input remains unchanged, this additional load retards the machine and brings it back to synchronism.

Similarly, if owing to a transient disturbance, the rotor decelerates so that the load angle decreases, the load on the machine is thereby reduced to $P_0 - \delta P$. This reduction in load causes the rotor to accelerate and the machine is again brought back to synchronism.

Clearly the effectiveness of this inherent correcting action depends on the extent of the change in power transfer for a given change in load angle. A measure of this effectiveness is given by the *synchronizing power coefficient*, which is defined as

$$P_s = \frac{dP}{d\sigma} \tag{12.40}$$

From eqn. (12.35),

$$P = \frac{3V}{Z_s}\{E_F \cos(\psi - \sigma) - V\cos\psi\} \tag{12.35}$$

so that

$$P_s = \frac{dP}{d\sigma} = \frac{3VE_F}{Z_s}\sin(\psi - \sigma) \tag{12.41}$$

Similarly the synchronizing torque coefficient is defined as

$$T_s = \frac{dT}{d\sigma} = \frac{1}{2\pi n_0}\frac{dP}{d\sigma} \tag{12.42}$$

From eqn. (12.42), therefore,

$$T_s = \frac{3}{2\pi n_0}\frac{VE_F}{Z_s}\sin(\psi - \sigma) \tag{12.43}$$

In many synchronous machines $X_s \gg R$, in which case eqns. (12.42) and (12.43) become

$$P_s = \frac{3VE_F}{X_s}\cos\sigma \tag{12.44}$$

$$T_s = \frac{3}{2\pi n_0}\frac{VE_F}{X_s}\cos\sigma \tag{12.45}$$

Eqns. (12.44) and (12.45) show that the restoring action is greatest when $\sigma = 0$, i.e. on no-load. The restoring action is zero when $\sigma = \pm 90°$. At these values of load angle the machine would be at the steady-state limit of stability and in a condition of unstable equilibrium. It is impossible, therefore, to run a machine at the steady-state limit of stability since its ability to resist small changes is zero unless the machine is provided with a special fast-acting excitation system.

**EXAMPLE 12.5** A 2 MVA 3-phase 8-pole alternator is connected to 6,000 V 50 Hz busbars and has a synchronous reactance of 4 Ω per phase. Calculate the synchronizing power and synchronizing torque per mechanical degree of rotor displacement at no-load. (Assume normal excitation.)

The synchronizing power coefficient is

$$P_s = \frac{3VE_p}{X_s}\cos\sigma \tag{12.44}$$

On no-load the load angle $\sigma = 0$.

Since there are 4 pole-pairs, 1 mechanical degree of displacement is equivalent to 4 electrical degrees; therefore

$$P_s = 3 \times \frac{6{,}000}{\sqrt{3}} \times \frac{6{,}000}{\sqrt{3}\times 4} \times \frac{4}{1{,}000} \times \frac{\pi}{180} = \underline{\underline{627\,\text{kW/mech. deg}}}$$

Synchronous speed of alternator, $n_s = \dfrac{f}{p} = 12{\cdot}5$ rev/s

Thus

$$2\pi n_s T_s = 627 \times 10^3$$

and

Synchronizing torque, $T_s = \underline{\underline{8{,}000\ \text{N-m/mech. deg}}}$

## 12.17 Oscillation of Synchronous Machines

In the previous sections, transient accelerations or decelerations of an alternator rotor were assumed in order to investigate the synchronizing power and synchronizing torque. Such transients may be caused by irregularities in the driving torque of the prime mover or, in the case of a motor, by irregularities in the load torque, or by irregularities in other machines connected in parallel, or by sudden changes in load.

Normally the inherent stability of alternators when running in parallel quickly restores the steady-state condition, but if the effect is sufficiently marked, the machine may break from synchronism. Moreover, if the disturbance is cyclic in effect, recurring at regular intervals, it will produce forced oscillations in the machine rotor. If the frequency of this cyclic disturbance approaches the value of the natural frequency of the rotor, when connected to the busbar system, the rotor may be subject to continuous oscillation and may eventually break from synchronism. This continuous oscillation of the rotor (periods of acceleration and deceleration) is sometimes known as *phase swinging* or *hunting*.



Fig. 12.18 shows the torque/load-angle characteristic of a synchronous generator. The steady input torque is $T_0$, corresponding to a steady-state load angle $\sigma_0$. Suppose a transient disturbance occurs such as to make the rotor depart from the steady state by $\sigma'$. Let $\sigma'$ be sufficiently small to assume that the synchronizing

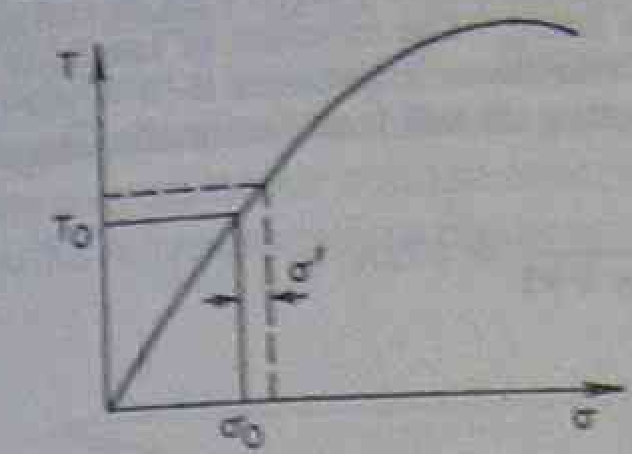



*Fig. 12.18* OSCILLATION OF A SYNCHRONOUS MACHINE CONNECTED TO INFINITE BUSBARS

torque is constant; i.e. the torque/load-angle characteristic is assumed to be linear over the range of $\sigma'$ considered.

Let $T_s$ = Synchronizing torque coefficient (*N-m*/mech. rad)
$\sigma'$ = Load angle deviation from steady-state position (mech. rad)
$J$ = Moment of inertia of rotating system (*kg-m*²)

Assuming that there is no damping,

$$J\frac{d^2\sigma'}{dt^2} = -T_s\sigma' \tag{12.46}$$

The solution of this differential equation is

$$\sigma' = \sigma_m' \sin\left(\sqrt{\frac{T_s}{J}}\,t + \psi\right) \tag{12.47}$$

From eqn. (12.47), the frequency of undamped oscillation is

$$f = \frac{1}{2\pi}\sqrt{\frac{T_s}{J}} \tag{12.48}$$

Synchronous machines intended for operation on infinite busbars are provided with damping windings in order to prevent the sustained oscillations predicted by eqn. (12.48).

In salient-pole machines the damping winding takes the form of a short-circuited cage consisting of copper bars of relatively large cross-section embedded in the rotor pole-face. In cylindrical-rotor machines the solid rotor provides considerable damping, but a cage winding may also be provided. This consists of copper fingers inserted in the rotor slots below the slot wedges and joined together at each end of the rotor. The currents induced in the damping bars give a damping torque which prevents continuous oscillation of the rotor.

**EXAMPLE 12.6** A 3-phase 3·3 kV 2-pole 3,000 rev/min 934 kW synchronous motor has an efficiency of 0·95 p.u. and delivers full-load torque with its excitation adjusted so that the input power factor is unity. The moment of inertia of the motor and its load is 30 kg-m², and its synchronous impedance is $(0 + j11{\cdot}1)\Omega$. Determine the period of undamped oscillation on full-load for small changes in load angle.

$$\text{Input current, } I = \frac{934 \times 10^3}{\sqrt{3} \times 3{\cdot}3 \times 10^3 \times 0{\cdot}95} = 172\,\text{A}$$

Taking the phase voltage as reference,

$$E_F = V - IX_s$$

$$= \frac{3{\cdot}3 \times 10^3}{\sqrt{3}}\underline{/0^\circ} - (172\underline{/0^\circ} \times 11{\cdot}1\underline{/90^\circ}) = 2{,}700\underline{/-45^\circ}\,\text{V}$$

The synchronizing torque coefficient is

$$T_s = \frac{3}{2\pi n_0}\frac{VE_F}{X_s}\cos\sigma \tag{12.45}$$

$$= \frac{3}{2\pi 50} \times \frac{3{\cdot}3 \times 10^3}{\sqrt{3}} \times \frac{2{,}700}{11{\cdot}1} \times 0{\cdot}707 = 3{\cdot}14 \times 10^3\,\text{N-m/rad}$$

The undamped frequency of oscillation is

$$f = \frac{1}{2\pi}\sqrt{\frac{T_s}{J}}$$

The period of oscillation is

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{30}{3{\cdot}14 \times 10^3}} = \underline{0{\cdot}612\,\text{s}}$$

## 12.18 Synchronous Motors

A synchronous motor will not develop a driving torque unless it is running at synchronous speed, since at any other speed the field poles will alternately be acting on the effective N and S poles of the rotating field and only a pulsating torque will be produced. For starting either (*a*) the induction motor principle or (*b*) a separate starting motor must be used. If the latter method is used the machine must be run up to synchronous speed and synchronized as an alternator. To obviate this trouble, synchronous motors are usually started as induction motors, and have a squirrel-cage winding embedded in the rotor pole faces to give the required action. When the machine has run up to almost synchronous speed the d.c. excitation is switched on to the rotor, and it then pulls into synchronism. The induction motor action then ceases (see Chapter 13).



The starting difficulties of a synchronous motor severely limit its usefulness—it may only be used where the load may be reduced for starting and where starting is infrequent. Once started, the motor has the advantage of running at a constant speed with any desired power factor. Typical applications of synchronous motors are the driving of ventilation or pumping machinery where the machines run almost continuously. Synchronous motors are often run with no load to utilize their leading power factor characteristic for power factor correction or voltage control. In these applications the machine is called a synchronous phase modifier.

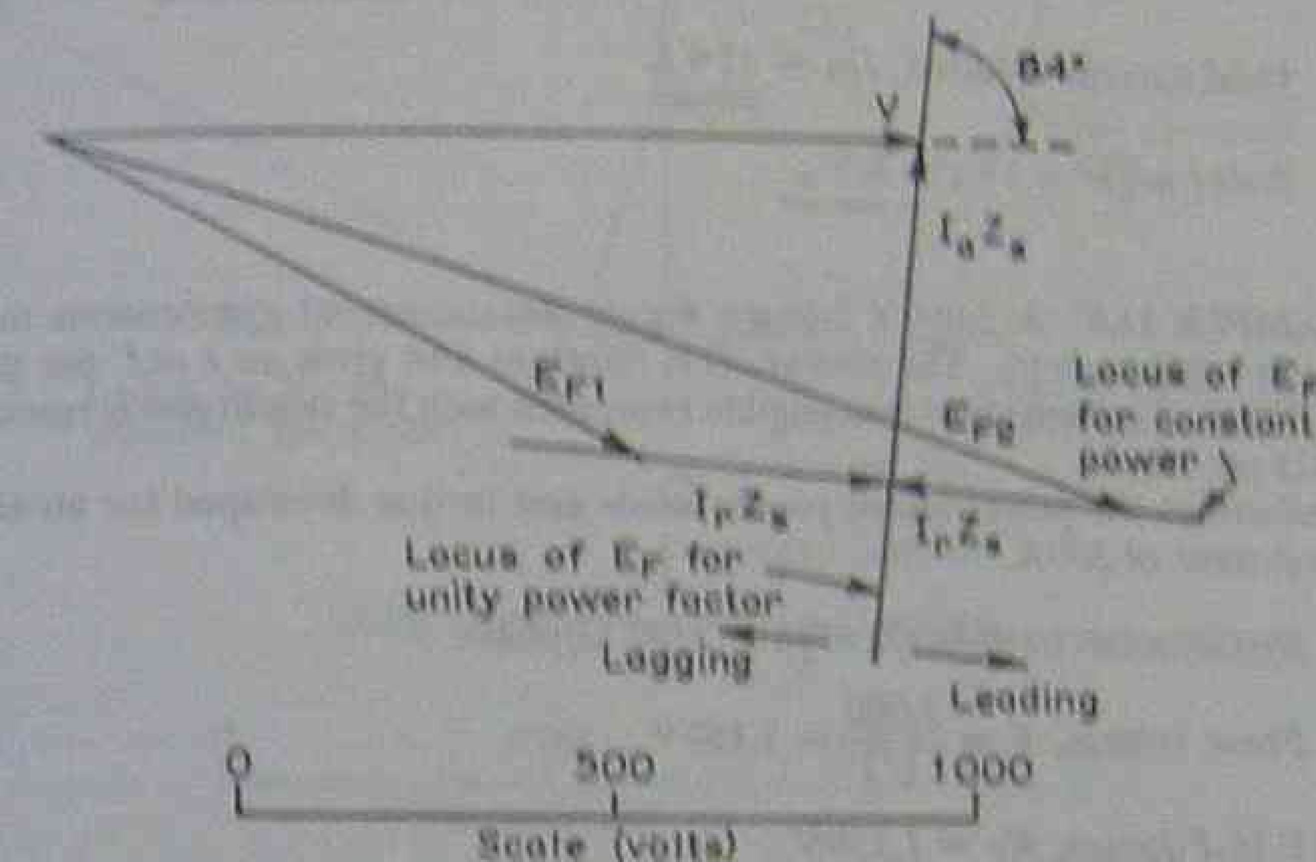



*Fig. 12.19*

**EXAMPLE 12.7** A 2,000 V 3-phase 4-pole star-connected synchronous machine has resistance and synchronous reactance per phase of 0·2Ω and 1·9Ω respectively.

Calculate the e.m.f. and the rotor displacement when the machine acts as a motor with an input of 800 kW at power factors of 0·8 lagging and leading.

If a field current of 40 A is required to produce an e.m.f. per phase equal to rated phase voltage, determine also the field current for each condition.

$$\text{Synchronous impedance, } Z_s = 0{\cdot}2 + j1{\cdot}9 = 1{\cdot}91\underline{/84^\circ}\,\Omega/\text{phase}$$

$$\text{Constant phase terminal voltage, } V = \frac{2{,}000}{\sqrt{3}} = 1{,}150\,\text{V}$$

$$\text{Total phase current in both cases} = \frac{800 \times 10^3}{\sqrt{3} \times 2{,}000 \times 0{\cdot}8} = 288\,\text{A}$$

Active component of current in both cases, $I_a = 288 \times 0{\cdot}8 = 230\,\text{A}$

Reactive component of current in both cases, $I_r = 288 \times 0{\cdot}6 = 173\,\text{A}$

$I_aZ_s = 230 \times 1{\cdot}91 = 440\,\text{V}$

$I_rZ_s = 173 \times 1{\cdot}91 = 330\,\text{V}$

Fig. 12.19 is now drawn to scale for the motoring condition.

At the lagging power factor the excitation voltage is measured from the complexor diagram as $E_{F1} = \underline{880\,\text{V/phase.}}$

$$\text{Field current required, } I_{F1} = 40 \times \frac{880}{1{,}150} = 30{\cdot}5\,\text{A}$$

The rotor displacement is the phase angle between $E_{F1}$ and $V$ with the rotor lagging for motor action as previously described. Therefore at the lagging p.f.

Rotor angle $= 27°_e = \underline{13{\cdot}5°_m}$ for a 4-pole machine

At the leading p.f. the excitation voltage, $E_{F2} = \underline{1{,}520\,\text{V/phase}}$

Field current required, $I_{F2} = \underline{52{\cdot}9\,\text{A}}$

Rotor angle $= 17°_e = \underline{8{\cdot}5°_m}$

**EXAMPLE 12.8** A 2,000V 3-phase 4-pole star-connected synchronous motor runs at 1,500 rev/min. The excitation is constant and gives an e.m.f. per phase of 1,150V. The resistance is negligible compared with the synchronous reactance of 3Ω per phase.

Determine the power input, power factor and torque developed for an armature current of 200A.

Synchronous impedance $= j3 = 3\angle 90°\ \Omega$/phase

$$\text{Phase voltage, } V = \frac{2{,}000}{\sqrt{3}} = 1{,}150\,\text{V}$$

E.M.F./phase, $E_F = 1{,}150\,\text{V}$

$IZ_s = 200 \times 3 = 600\,\text{V}$

In Fig. 12.20 $V$ represents the phase voltage taken as reference complexor.

A circular arc whose radius represents the open-circuit voltage of 1,150V is the locus of $E_F$ for constant excitation.

AB is the locus of $E_F$ for unity power factor operation; in this case AB is perpendicular to $V$ since the phase angle of $Z$ is 90°.

A circle whose radius represents 600V is the locus of $E_F$ for constant kVA operation. For the actual operating conditions $E_F$ must lie at the intersection of the two circles.

From the diagram,

$I_aZ_s = 580\,\text{V}$

Active component of current, $I_a = 193\,\text{A}$

Therefore

$$\text{Total power input} = \frac{3VI_a}{1{,}000} = \frac{3 \times 1{,}150 \times 193}{1{,}000} = \underline{666\,\text{kW}}$$

$$\text{Operating power factor} = \frac{I_a}{I} = \frac{193}{200} = \underline{0{\cdot}96\ \text{lagging}}$$



$$\text{Torque developed, } T = \frac{3VI_a}{2\pi n_s}$$

$$= \frac{3 \times 1150 \times 193}{2\pi \times 1{,}500} \times 60$$

$$= \underline{4{,}250\ \text{N-m}}$$

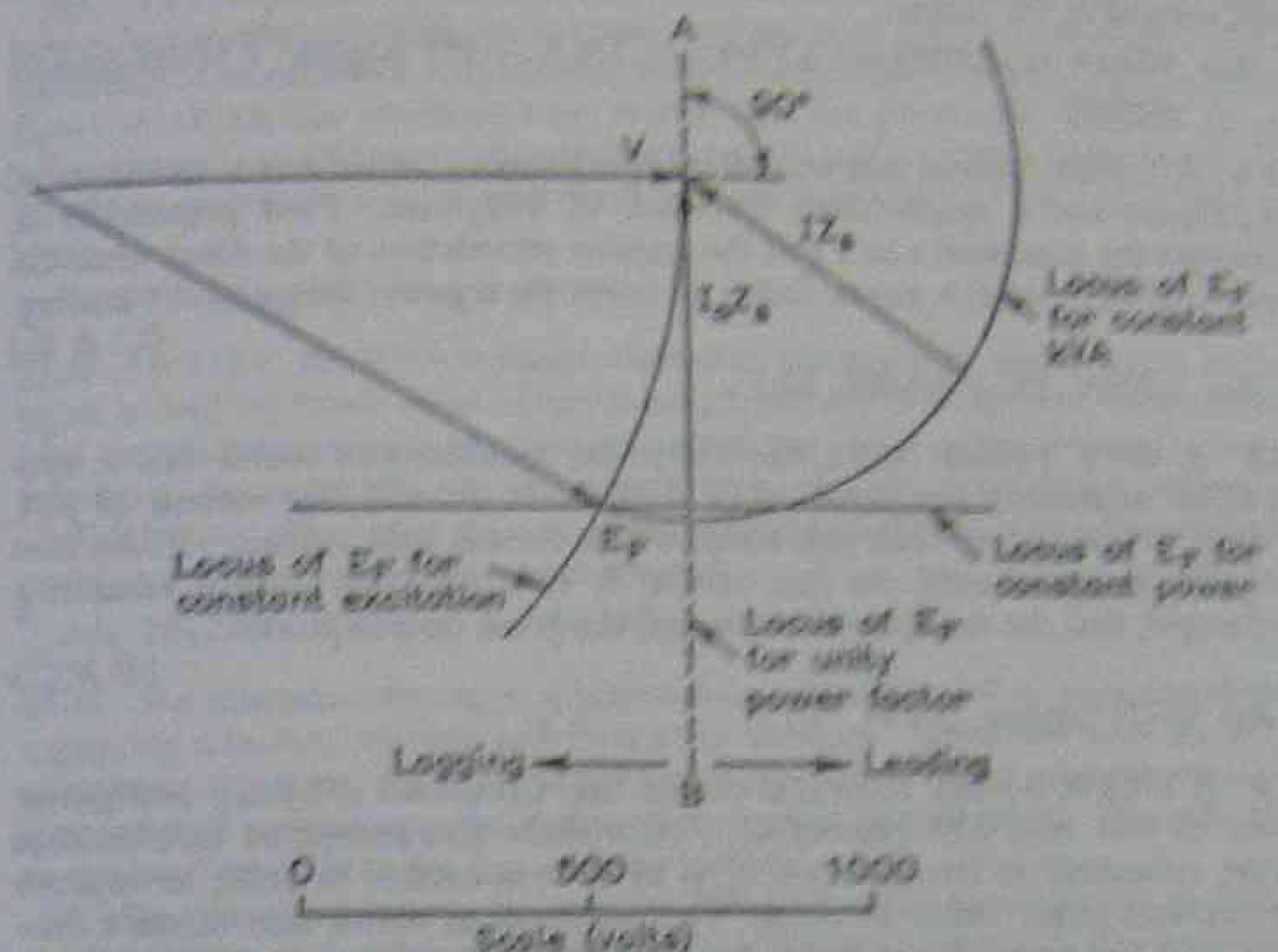



*Fig. 12.20*

## PROBLEMS

**12.1** A 3-phase 11kV star-connected alternator has an effective armature resistance of 1Ω and a synchronous reactance of 20Ω per phase. Calculate the percentage regulation for a load of 1,500kW at p.f.s of (*a*) 0·8 lagging, (*b*) unity, (*c*) 0·8 leading.

*Ans.* 22 per cent, 4·25 per cent, −13·4 per cent.

**12.2** Describe the tests carried out in order that the synchronous impedance of an alternator can be obtained. By means of diagrams show how the synchronous impedance can be used to determine the regulation of an alternator at a particular load and power factor.

A 6,600V 3-phase star-connected alternator has a synchronous impedance of $(0{\cdot}4 + j6)\,\Omega$/phase. Determine the percentage regulation of the machine when supplying a load of 1,000kW at normal voltage and p.f. (i) 0·866 lagging, (ii) unity, (iii) 0·866 leading, giving complexor diagrams in each case. (*H.N.C.*)

*Ans.* 9·7 per cent, 1·84 per cent, −6·03 per cent.

**12.3** Explain, with the aid of complexor diagrams, the effect of varying the excitation of a synchronous motor driving a constant-torque load.