4

Control of Power and Frequency

4.1 Introduction

Although to a certain extent the control of power and frequency is interrelated to the control of reactive power and voltage, it is hoped that by dealing with power and frequency separately from voltage control, a better appreciation of the operation of power systems will be obtained. In a large interconnected system, many generation stations, large and small, are synchronously connected and hence all have the same frequency. The following remarks refer mainly to networks in which the control of power is carried out by the decisions and actions of engineers, as opposed to systems in which the control and allocation of load to machines is effected completely automatically. The latter are sometimes based on a continuous load-flow calculation by analogue or digital computers. The allocation of the required power among the generators has to be decided before the advent of the load, which must therefore be predicted. An analysis is made of the loads experienced over the same period in previous years, account is also taken of the value of the load immediately previous to the period under study and of the weather forecast. The probable load to be expected, having been decided, is allocated to the various turbine-generators.

A daily load cycle is shown in Figure 1.2. It is seen that the rate of rise of the load is very high and varies between 2 MW/min per 1000 MW of peak demand in the early hours and about 8 MW/min per 1000 MW of peak demand between 07.00 and 08.00 hours. Hence the ability of machines to increase their output quickly from zero to full load is important. It is extremely unlikely that the output of the machines at any instant will exactly equal the load on the system. If the output is higher than the demand the machines will tend to increase in speed and the frequency will rise, and vice versa. Hence the frequency is not a constant quantity but continuously varies; these variations are normally small and have no noticeable effect on most consumers. The frequency is continuously monitored against standard time-sources and when long-term tendencies to rise or fall are noticed, the control engineers take appropriate action by regulating the generator outputs.

Should the total generation available be insufficient to meet the demand, the frequency will fall. If the frequency falls by more than 1 Hz the reduced speed of power station pumps, fans, etc., may reduce the station output and a serious situation arises. In this type of situation, although the reduction of frequency will cause a reduction in power demand, voltage must be reduced, and if this is not sufficient then loads will have to be disconnected and continue to be disconnected until the frequency rises to a reasonable level. All utilities have a scheme of planned load shedding based on under-frequency relays set to reduce loads in blocks to prevent complete shut-down in extreme emergencies.

When a permanent increase in load occurs on the system, the speed and frequency of all the interconnected generators fall, since the increased energy requirement is met from the kinetic energy of the machines. This causes an increase in steam or water admitted to the turbines due to the operation of the governors and hence a new load balance is obtained. Initially, the boilers have a thermal reserve by means of which sudden changes can be supplied until the new firing rate has been established. Modern gas turbines have an overload capability for a few minutes which can be usefully exploited in emergency situations.

4.2 The Turbine Governor

In the following discussion of governor mechanisms, both steam and water turbines will be considered together and points of difference indicated where required. A simplified schematic diagram of a traditional governor system is shown in Figure 4.1. The sensing device, which is sensitive to change of speed,

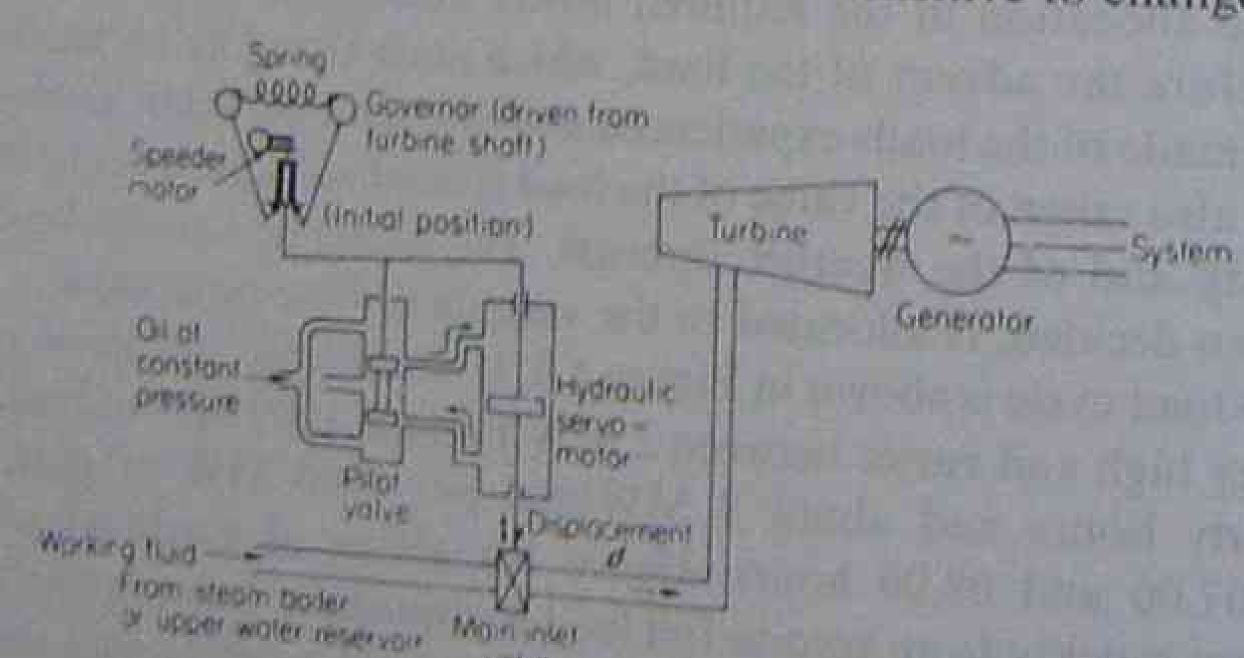

Figure 4.1 Governor control system employing the Watt governor as sensing device and a hydraulic servo-system to operate main supply valve. Speeder-motor gear determines the initial setting of the governor position

is the time-honoured Watt centrifugal governor. In this, two weights move radially outwards as their speed of rotation increases and thus move a sleeve on the central spindle. This sleeve movement is transmitted by a lever mechanism to the pilot-valve piston and hence the servo-motor is operated. A dead band is present in this mechanism, i.e. the speed must change by a certain amount before the valve commences to operate, because of friction and mechanical backlash. The time taken for the main steam valve to move due to delays in the hydraulic pilot-valve and servo-motor systems is appreciable, 0.2-0.3 s. The governor characteristic for a large steam turboalternator is shown in Figure 4.2 and it is seen that there is a 4 per cent drop in speed between no load and full load. Because of the requirement for high response speed, low dead band, and accuracy in speed and load control, the mechanical governor has been replaced in large modern turbogenerators by electrohydraulic governing. The method normally used to measure the speed is based on a toothed wheel (on shaft) and magnetic-probe pickup. The use of electronic controls requires an electrohydraulic conversion stage, using conventional servo-valves.

An important feature of the governor system is the mechanism by means of which the governor sleeve and hence the main-valve positions can be changed and adjusted quite apart from when actuated by the speed changes. This is accomplished by the speed changer, or 'speeder motor', as it is sometimes termed. The effect of this adjustment is the production of a family of parallel characteristics, as shown in Figure 4.3. Hence the power output of the generator at a given speed may be adjusted at will and this is of extreme importance when operating at optimum economy.

The torque of the turbine may be considered to be approximately proportional to the displacement d of the main inlet valve. Also, the expression for the change in torque with speed may be expressed approximately by the equation

$$T = T_0(1 - kN) \tag{4.1}$$

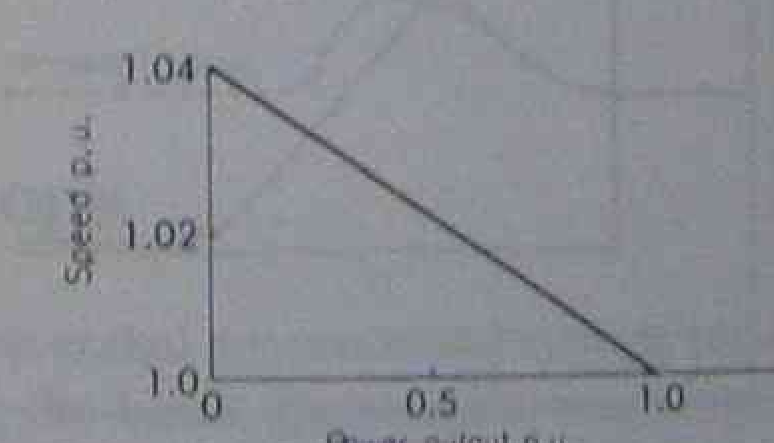

Figure 4.2 Idealized governor characteristic of a turboalternator with 4 per cent droop from zero to full load

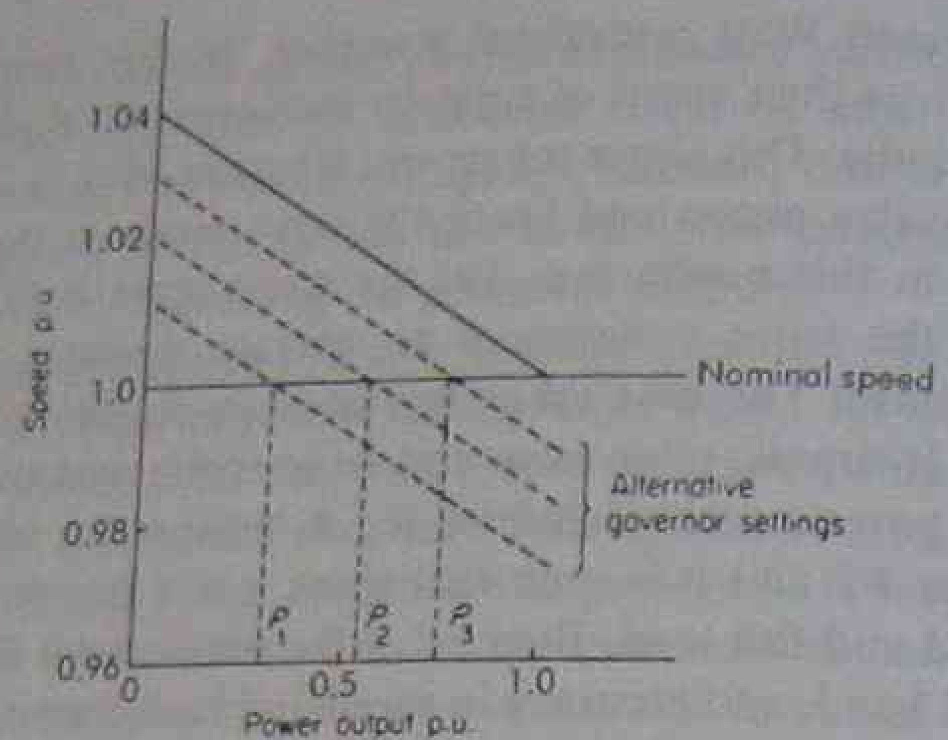

Figure 4.3 Effect of speeder gear on governor characteristics P_1, P_2, and P_3 = outputs at various settings but at same speed

where T_0 is the torque at speed N_0 and T the torque at speed N; k is a constant for the governor system. As the torque depends on both the main-valve position and the speed, $T = f(d, N)$.

There is a time delay between the occurrence of a load change and the new operating conditions. This is due not only to the governor mechanism but also to the fact that the new flow rate of steam or water must accelerate or decelerate the rotor in order to attain the new speed. In Figure 4.4 typical curves are shown for a turbogenerator which has a sudden decrease in the electrical power required, pehaps due to an external power network fault, and hence the retarding torque on the turbine shaft is suddenly much smaller. In the ungoverned

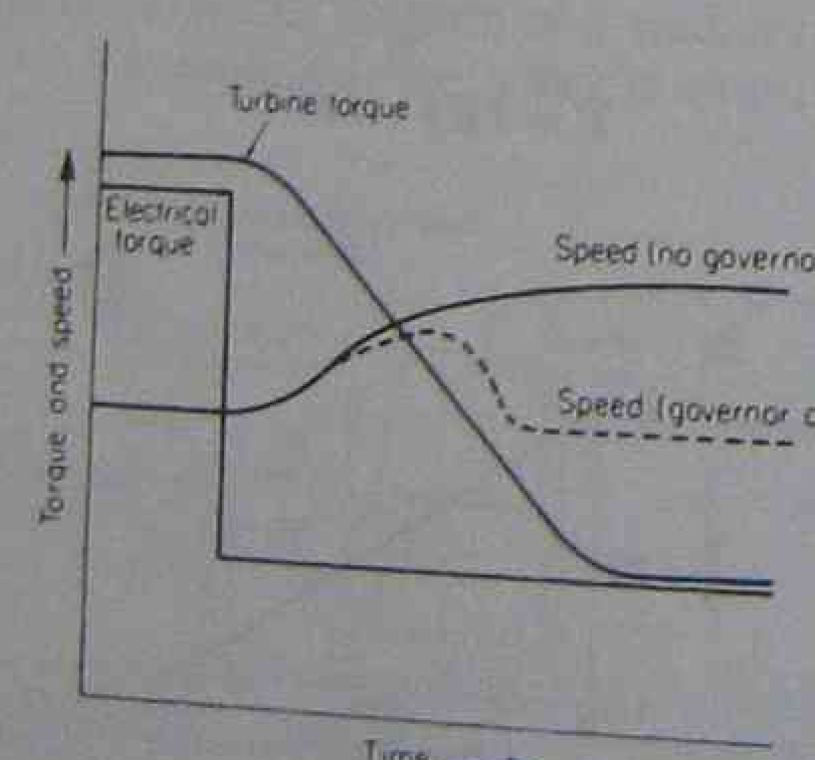

Figure 4.4 Graphs of turbine torque, electrical torque, and speed against time when the load on a generator suddenly falls

case the considerable time-lag between the load change and the attainment of the new steady speed is obvious. With the regulated or governed machine, due to the dead band in the governor mechanism, the speed–time curve starts to rise, the valve then operates, and the fluid supply is adjusted. It is possible for damped oscillations to be set up after the load change; this will be discussed in Chapter 8.

An important factor regarding turbines is the possibility of overspeeding, when the load on the shaft is lost completely, with possible drastic mechanical breakdown. To avoid this, special valves are incorporated to automatically cut off the energy supply to the turbine. In a turbogenerator normally running at 3000 r.p.m. this overspeed protection operates at about 3300 r.p.m.

Example 4.1

An isolated 75 MVA synchronous generator feeds its own load and operates initially at no-load at 3000 r.p.m., 50 Hz. A 20 MW load is suddenly applied and the steam valves to the turbine commence to open after 0.5 s due to the time-lag in the governor system. Calculate the frequency to which the generated voltage drops before the steam flow meets the new load. The stored energy for the machine is 4 kW-s per kVA of generator capacity.

Solution

For this machine the stored energy at 3000 r.p.m.

$$= 4 \times 75\,000 = 300\,000 \text{ kW–s}$$

Before the steam valves start to open the machine loses $20\,000 \times 0.5 = 10\,000$ kW-s of the stored energy in order to supply the load.

The stored energy $\propto$ (speed)^2. Therefore the new frequency

$$= \sqrt{\frac{300\,000 - 10\,000}{300\,000}} \times 50 \text{ Hz}$$

$$= 49.2 \text{ Hz}$$

4.3 Control Loops

The machine and its associated governor and voltage-regulator control systems may be represented by the block diagram shown in Figure 4.5. The nature of the voltage control loop has been discussed already in Chapter 3; accurate machine representation is required, involving two-axis expressions, as indicated in Chapter 2.

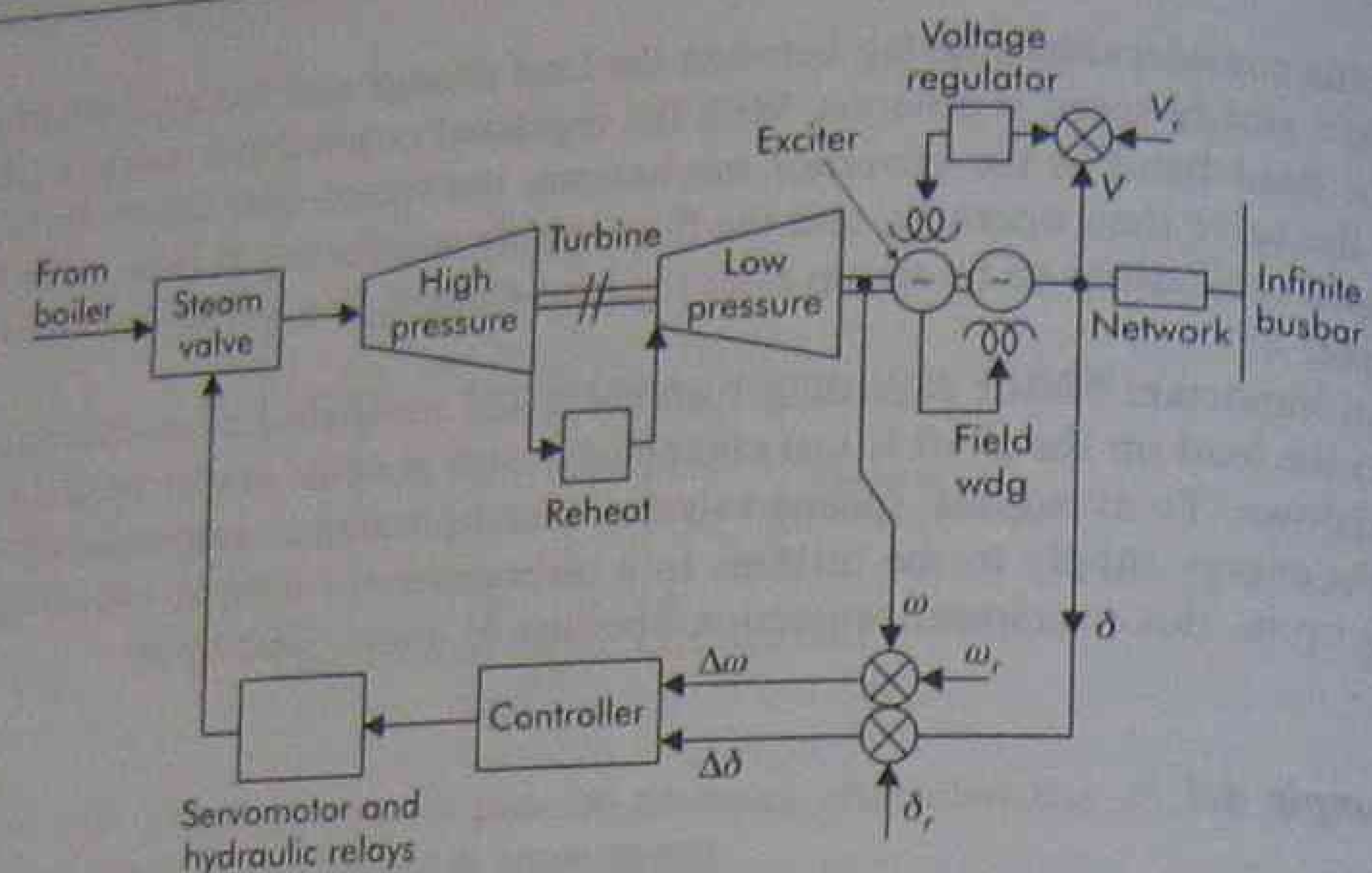

Figure 4.5 Block diagram of complete turboalternator control systems. The governor system is more complicated that that shown in Figure 4.1 owing to the inclusion of the load-angle δ in the control loop. Suffixes r refer to reference quantities and Δ to the error quantities. The controller modifies the error signal from the governor by taking into account the load angle

Two factors have a large influence on the dynamic response of the prime mover: (1) entrained steam between the inlet valves and the first stage of the turbine (in large machines this can be sufficient to cause loss of synchronism after the valves have closed); (2) the storage action in the reheater which causes the output of the low-pressure turbine to lag behind that of the high-pressure side. The transfer function

$$\frac{\text{prime mover torque}}{\text{valve opening}}$$

accounting for both these effects is

$$\frac{G_1 G_2}{(1+\tau_i s)(1+\tau_r s)} \tag{4.2}$$

where

G_1 = entrained steam constant;
G_2 = reheated gain constant;
τ_i = entrained steam time constant;
τ_r = reheater time constant.

The transfer function relating steam-valve opening d to changes in speed ω due to the governor feedback loop is

$$\frac{\Delta d}{\Delta \omega}(s) = \frac{G_3 G_4 G_5}{(1+\tau_g s)(1+\tau_1 s)(1+\tau_2 s)} \tag{4.3}$$

where

τ_g = governor-relay time constant
τ_1 = primary-relay time constant;
τ_2 = secondary-valve-relay time constant;
$(G_3 G_4 G_5)$ = constants relating system-valve lift to speed change.

By a consideration of the transfer function of the synchronous generator with the above expressions the dynamic response of the overall system may be obtained.

4.4 Division of Load Between Generators

The use of the speed changer enables the steam input and electrical power output at a given frequency to be changed as required. The effect of this on two machines can be seen in Figure 4.6. The output of each machine is not therefore determined by the governor characteristics but can be varied by the operating personnel to meet economical and other considerations. The governor characteristics only completely decide the outputs of the machines when a sudden change in load occurs or when machines are allowed to vary their outputs according to speed within a prescribed range in order to keep the frequency constant. This latter mode of operation is known as *free-governor action*.

It has been shown in Chapter 2 that the voltage difference between the two ends of an interconnector of total impedance $R + jX$ is given by

$$\Delta V = E - V = \frac{RP + XQ}{V}$$

Also the angle between the voltage phasors (i.e. the transmission angle) δ is given by

$$\sin^{-1}\frac{\Delta V_q}{E}$$

where

$$\Delta V_q = \frac{XP - RQ}{V}$$

When $X \gg R$, i.e. for most transmission networks,

$$\Delta V_q \propto P \quad \text{and} \quad \Delta V_p \propto Q$$

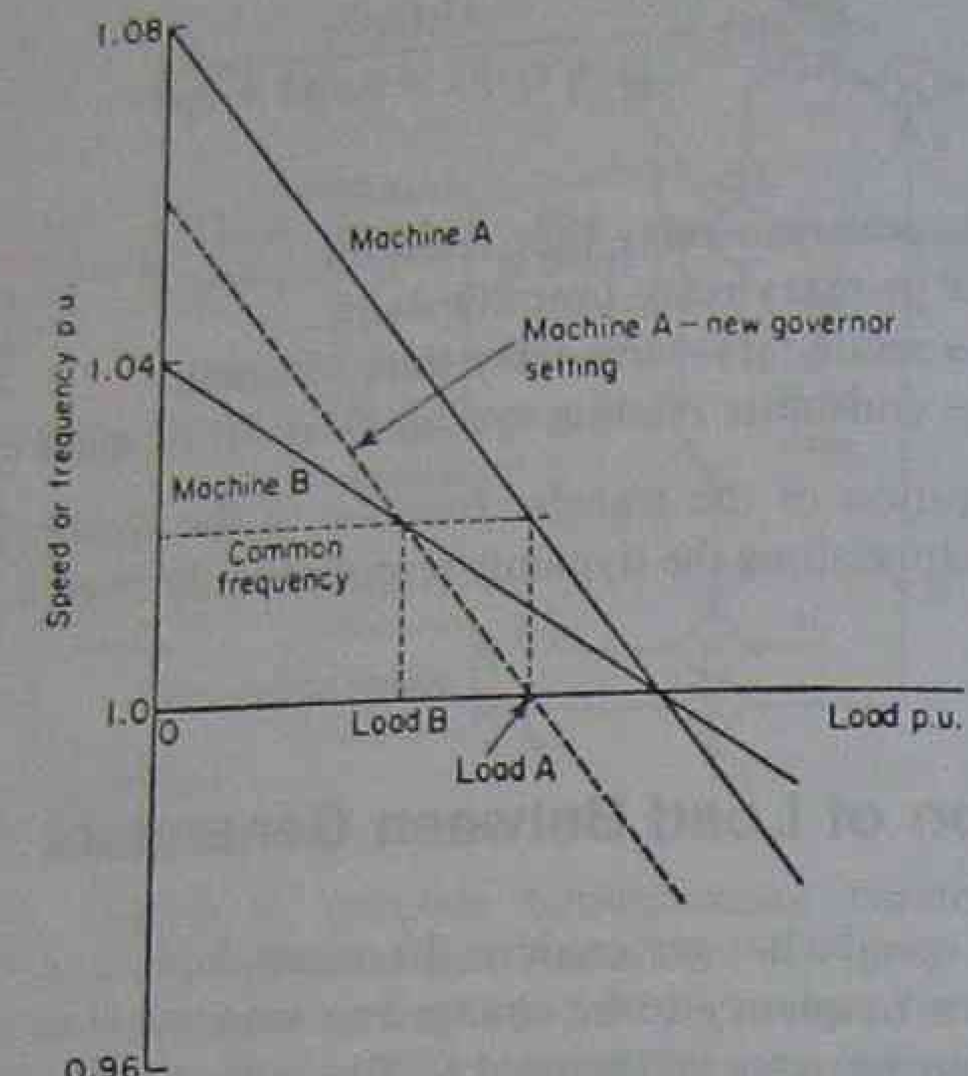

Figure 4.6 Two machines connected to an infinite busbar. The speeder gear of machine A is adjusted so that the machines load equally

Hence, (1) *the flow of power between two nodes is determined largely by the transmission angle;* (2) *the flow of reactive power is determined by the scalar voltage difference between the two nodes.*

These two facts are of fundamental importance to the understanding of the operation of power systems.

The angular advance of G_A (Figure 4.7) is due to a greater relative energy input to turbine A than to B. The provision of this extra steam (or water) to A is possible because of the action of the speeder gear without which the power outputs of A and B would be determined solely by the nominal governor characteristics. The following simple example illustrates these principles.

Example 4.2

Two synchronous generators operate in parallel and supply a total load of 200 MW. The capacities of the machines are 100 MW and 200 MW and both have governor droop characteristics of 4 per cent from no load to full load. Calculate the load taken by each machine, assuming free governor action.

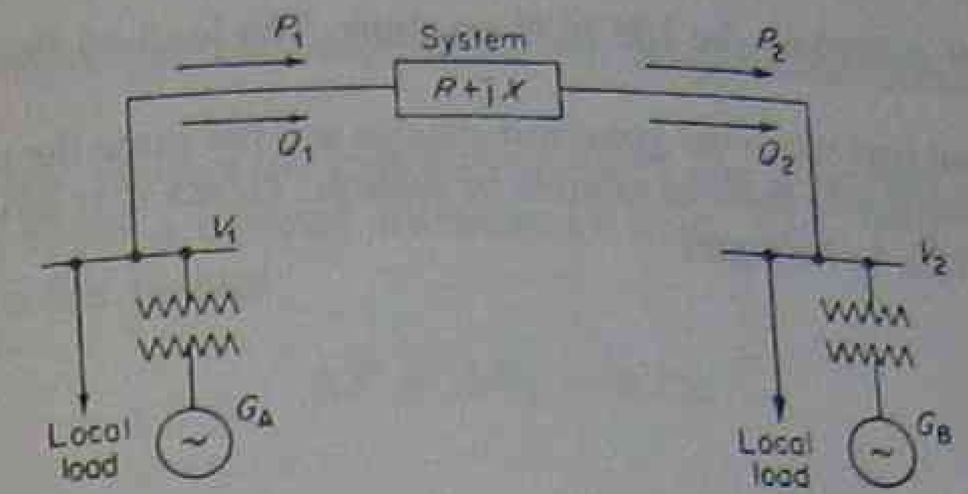

Figure 4.7 Two generating stations linked by an interconnector of impedance $(R+jX)\,\Omega$. The rotor of A is in phase advance of B and $V_1 > V_2$

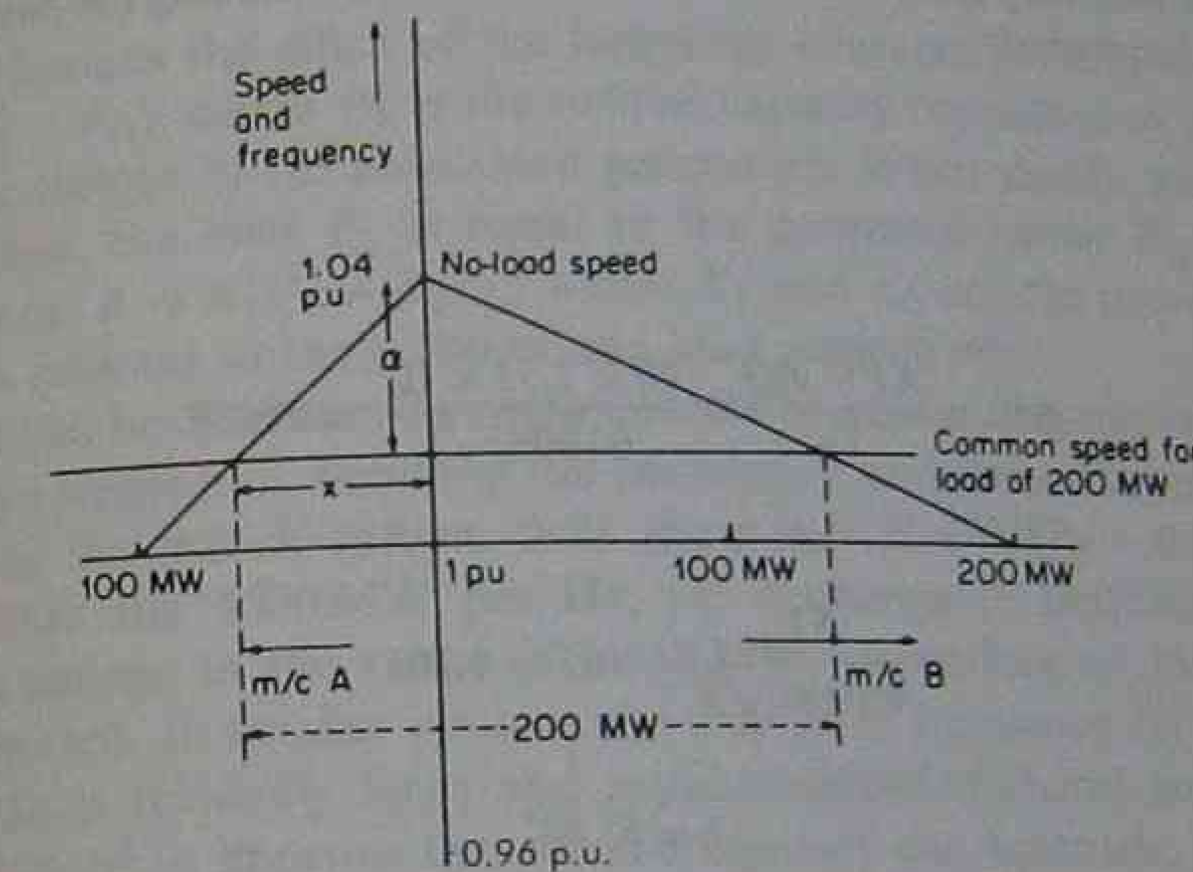

Figure 4.8 Speed–load diagram for Example 4.2

Solution

Let x megawatts be the power supplied from a 100 MW generator. Referring to Figure 4.8,

$$\frac{4}{100} = \frac{\alpha}{x}$$

For the 200 MW machine,

$$\frac{4}{200} = \frac{\alpha}{200 - x}$$

$$\therefore \frac{4x}{100} = \frac{800 - 4x}{200}$$

and $x = 66.6$ MW = load on the 100 MW machine. The load on the 200 MW machine = 133.3 MW.

It will be noticed that when the governor droops are the same the machines share the total load in proportion to their capacities or ratings. Hence it is advantageous for the droops of all turbines to be equal.

Example 4.3

Two units of generation maintain 66 kV and 60 kV (line) at the ends of an interconnector of inductive reactance per phase of 40 Ω and with negligible resistance and shunt capacitance. A load of 10 MW is to be transferred from the 66 kV unit to the other end. Calculate the necessary conditions between the two ends, including the power factor of the current transmitted.

Solution

$$\Delta V_q = \frac{XP - RQ}{V_R} = \frac{40 \times 3.33 \times 10^6}{60\,000/\sqrt{3}} = 3840\text{V}$$

Also,

$$\frac{\Delta V_q}{66\,000/\sqrt{3}} = \sin\delta = 0.101$$

$$\therefore \delta = 5^\circ\ 44'$$

Hence the 66 kV busbars are 5° 44′ in advance of the 60 kV busbars.

$$\Delta V_p = \frac{66\,000 - 60\,000}{\sqrt{3}} = \frac{RP + XQ}{V_R} = \frac{40Q}{60\,000/\sqrt{3}}$$

Hence

$$Q = 3 \text{ MVAr per phase (9 MVAr total)}$$

The p.f. angle $\phi = \tan^{-1} Q/P = 42^\circ$ and hence the p.f. = 0.74.

4.5 The Power–Frequency Characteristic of an Interconnected System

The change in power for a given change in the frequency in an interconnected system is known as the *stiffness* of the system. The smaller the change in frequency for a given load change the stiffer the system. The power–frequency characteristic may be approximated by a straight line and $\Delta P/\Delta f = K$, where

K is a constant (MW per Hz) depending on the governor and load characteristics.

Let ΔP_G be the change in generation with the governors operating 'free acting' resulting from a sudden increase in load ΔP_L. The resultant out-of-balance power in the system

$$\Delta P = \Delta P_L - \Delta P_G \tag{4.4}$$

and therefore

$$K = \frac{\Delta P_L}{\Delta f} - \frac{\Delta P_G}{\Delta f} \tag{4.5}$$

$\Delta P_L/\Delta f$ measures the effect of the frequency characteristics of the load and $\Delta P_G \propto (P_T - P_G)$, where P_T is the turbine capacity connected to the network and P_G the output of the associated generators. When steady conditions are again reached, the load P_L is equal to the generated power P_G (neglecting losses): hence, $K = K_1 P_T - K_2 P_L$, where K_1 and K_2 are the power-frequency coefficients relevant to the turbines and load respectively.

Here, K can be determined experimentally by connecting two large separate systems by a single link, breaking the connection and measuring the frequency change. For the British system, tests show that $K = 0.8P_T - 0.6P_L$ and lies between 2000 and 5500 MW per Hz, i.e. a change in frequency of 0.1 Hz requires a change in the range 200–550 MW, depending on the amount of plant connected. In smaller systems the change in frequency for a reasonable load change is relatively large and rapid-response electrical governors have been introduced to improve the power-frequency characteristic.

In 1977, owing to a series of events triggered off by lightning, New York City was cut off from external supplies and the internal generation available was much less than the city load. The resulting fall in frequency with time is shown in Figure 4.9, illustrating the time–frequency characteristics of an isolated power system.

4.6 System Connected by Lines of Relatively Small Capacity

Let K_A and K_B be the respective power–frequency constants of two separate power systems A and B, as shown in Figure 4.10. Let the change in the power transferred from A to B when a change resulting in an out-of-balance power ΔP occurs in system B, be ΔP_t, where ΔP_t is positive when power is transferred from A to B. The change in frequency in system B, due to an extra load ΔP and an extra input of ΔP_t from A, is $-(\Delta P - \Delta P_t)/K_B$ (the negative sign indicates a fall in frequency). The drop in frequency in A due to the extra load

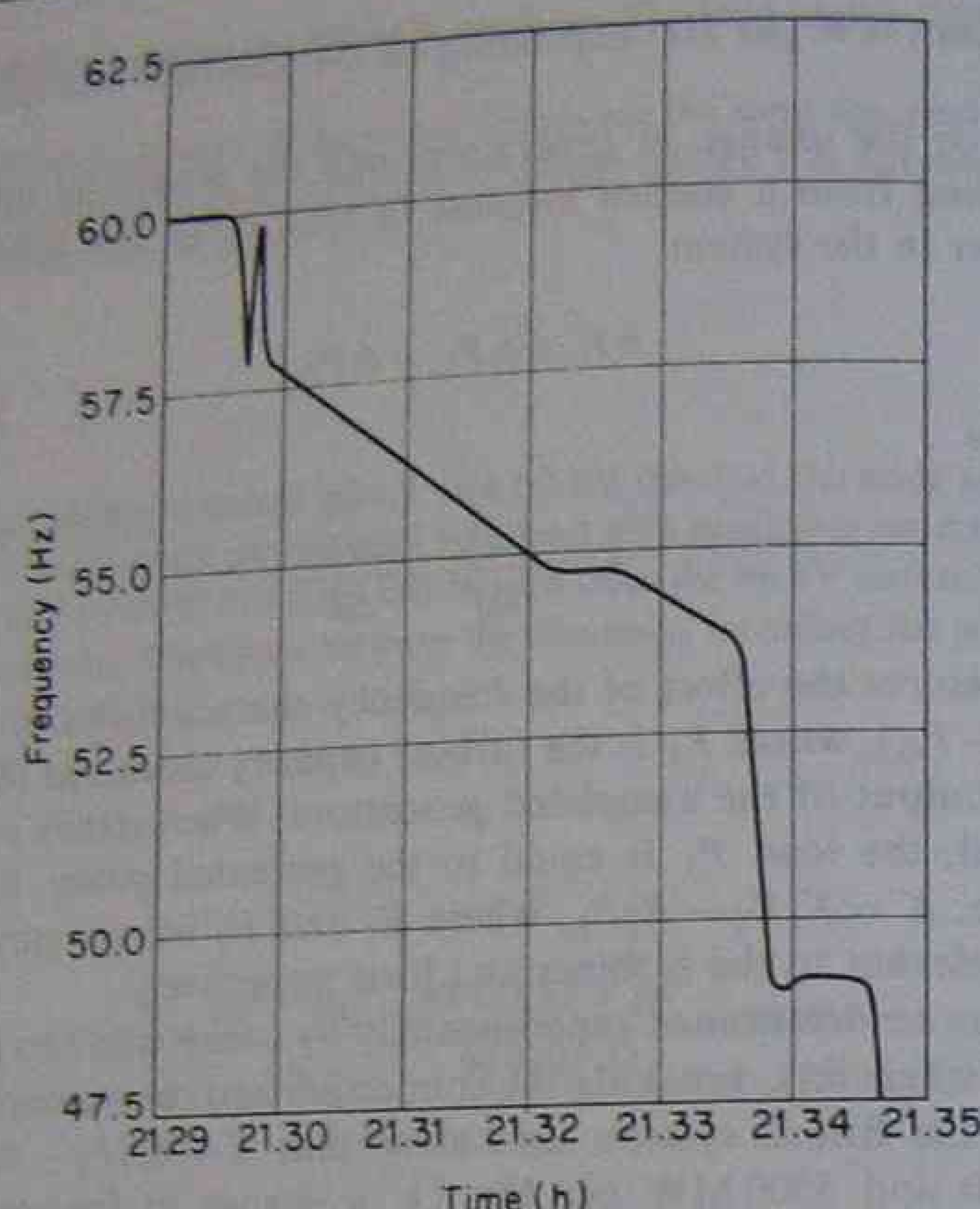

Figure 4.9 Decline of frequency with time of New York City system when isolated from external supplies (*Copyright © 1977 Institute of Electrical and Electronics Engineers, Inc. Reprinted by permission from I.E.E.E. Spectrum, Vol. 15, No. 2 (Feb. 1978) pp. 38–46*)

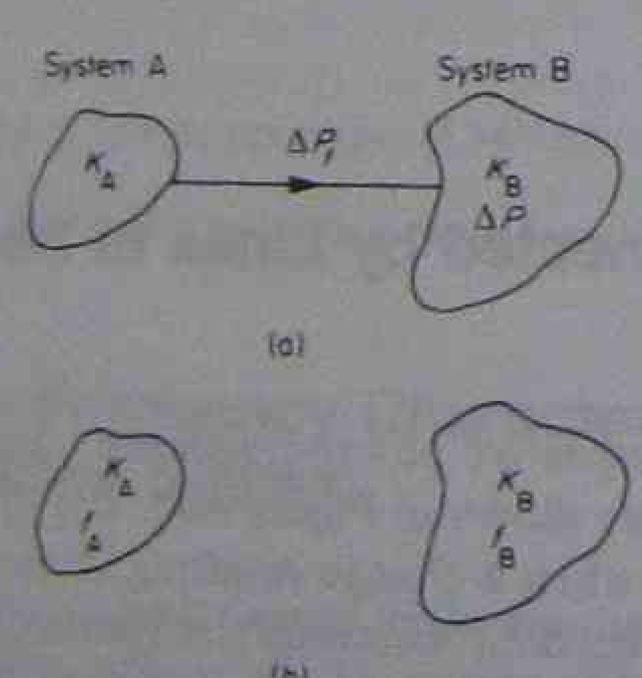

Figure 4.10 (a) Two interconnected power systems connected by a tie-line. (b) The two systems with the tie-line open

ΔP_t is $-\Delta P_t/K_A$, but the changes in frequency in each system must be equal, as they are electrically connected. Hence,

$$\frac{-(\Delta P - \Delta P_t)}{K_B} = \frac{-\Delta P_t}{K_A}$$

$$\therefore \Delta P_t = +\left(\frac{K_A}{K_A + K_B}\right)\Delta P \tag{4.6}$$

Next, consider the two systems operating at a common frequency f with A exporting ΔP_t to B. The connecting link is now opened and A is relieved of ΔP_t and assumes a freqency f_A, and B has ΔP_t more load and assumes f_B. Hence

$$f_A = f + \frac{\Delta P_t}{K_A} \quad \text{and} \quad f_B = f - \frac{\Delta P_t}{K_B}$$

from which

$$\frac{\Delta P_t}{f_A - f_B} = \frac{K_A K_B}{K_A + K_B} \tag{4.7}$$

Hence, by opening the link and measuring the resultant change in f_A and f_B the values of K_A and K_B can be obtained.

When large interconnected systems are linked electrically to others by means of tie-lines the power transfers between them are usually decided by mutual agreement and the power is controlled by regulators. As the capacity of the tie-lines is small compared with the systems, care must be taken to avoid excessive transfers of power and corresponding cascade tripping.

4.6.1 Effect of governor characteristics

A fuller treatment of the performance of two interconnected systems in the steady state requires further consideration of the control aspects of the generation process.

A more complete block diagram for the steam turbine-generator connected to a power system is shown in Figure 4.11. For this system the following equation holds:

$$Ms^2\delta + Ks\delta = \Delta P - \Delta P_L$$

where M is a constant depending on inertia (see Chapter 8); K is the stiffness or damping coefficient (i.e. change of power with speed) in MW/Hz or MW per rad/s

$$= \frac{\partial\,(\text{load power})}{\partial(s\delta)} - \frac{\partial\,(\text{turbine power})}{\partial(s\delta)}$$

where

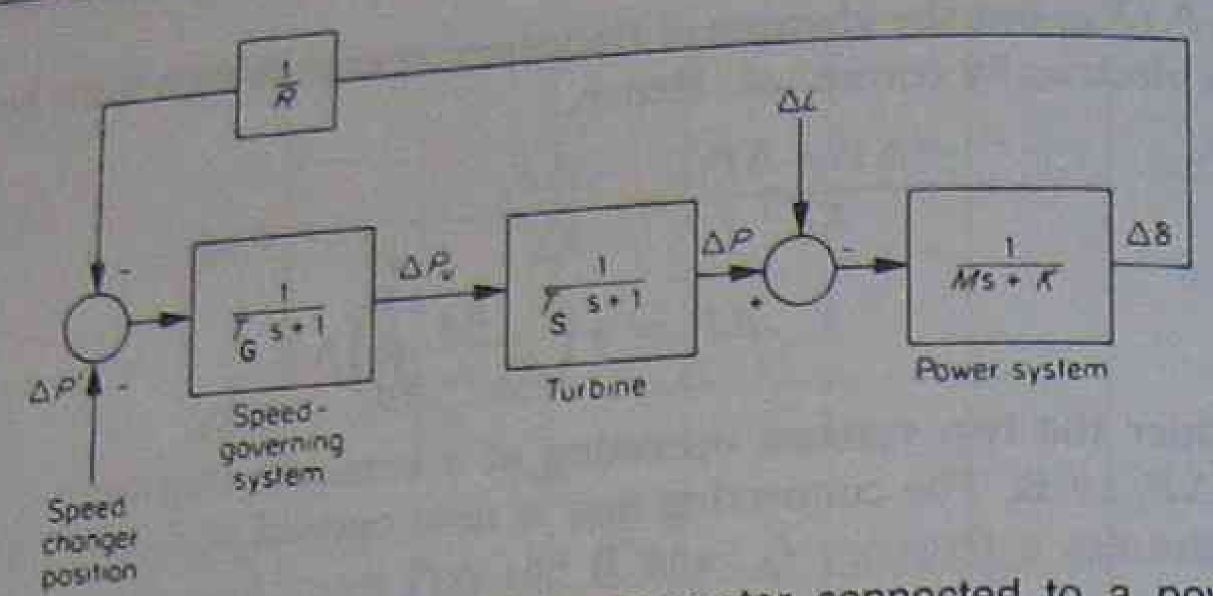

Figure 4.11 Block diagram for turbine-generator connected to a power system (*From Economic Control of Interconnected Systems, by L. K. Kirchmayer. Copyright © 1959 John Wiley & Sons Ltd. By permission of John Wiley & Sons Inc.*)

ΔP and ΔP_L = change in prime mover and load powers;

$\Delta\delta$ = change from initial angular position;

R = present speed regulation (or governor droop), i.e. drop in speed or frequency when combined machines of an area change from no load to full load, expressed as p.u. or Hz or rad/s per MW;

$\Delta P'$ = change in speed-changer setting.

Therefore, change from normal speed or frequency,

$$s\delta = \frac{1}{Ms + K}(\Delta P - \Delta P_L)$$

This analysis holds for steam-turbine generation; for hydro-turbines, the large inertia of the water must be accounted for and the analysis is more complicated.

The representation of two systems connected by a tie-line is shown in Figure 4.12. The general analysis is as before except for the additional power terms due to the tie-line. The machines in the inidividual power systems are considered to be closely coupled and to possess one equivalent rotor.

For system (1),

$$M_1 s^2 \delta_1 + K_1 s\delta_1 + T_{12}(\delta_1 - \delta_2) = \Delta P_1 - \Delta P_{L1} \tag{4.8}$$

where T_{12} is the synchronizing torque coefficient of the tie-line.

For system (2),

$$M_2 s^2 \delta_2 + K_2 s\delta_2 + T_{12}(\delta_2 - \delta_1) = \Delta P_2 - \Delta P_{L2} \tag{4.9}$$

The steady-state analysis of two interconnected systems may be obtained from the transfer functions given in the block diagram.

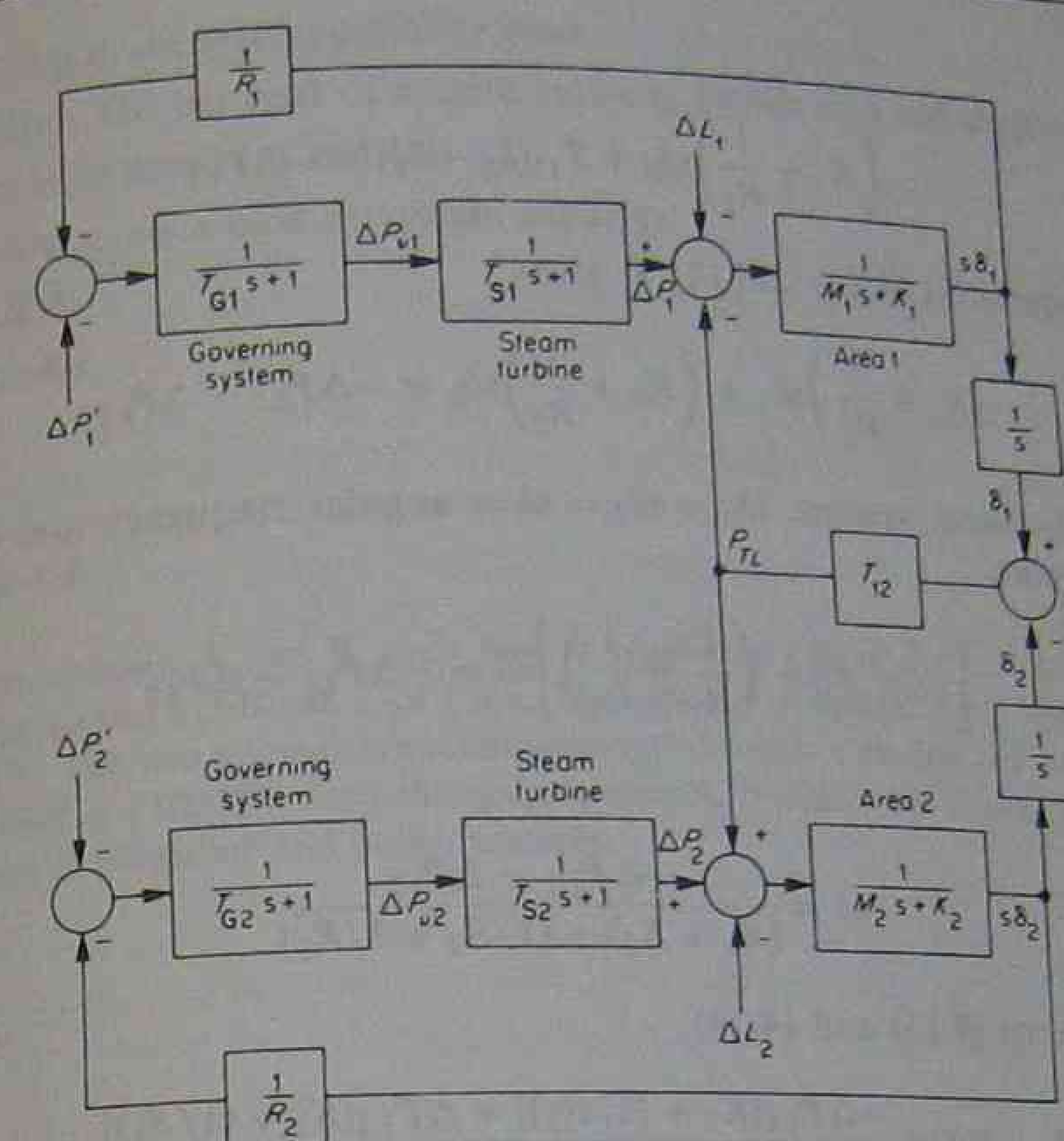

Figure 4.12 Block control diagram of two power systems connected by a tie-line (*From Economic Control of Interconnected Systems by L. K. Kirchmayer. Copyright © 1959 John Wiley & Sons Ltd. By permission of John Wiley & Sons Inc.*)

The speed governor response is given by

$$\Delta P_V = \frac{-1}{(T_G s + 1)}\left(\frac{1}{R} s\delta + \Delta P'\right) \tag{4.10}$$

In the steady state, from equation (4.10),

$$\Delta P_1 = \frac{1}{R_1} s\delta_1 \tag{4.11}$$

and

$$\Delta P_2 = -\frac{1}{R_2} s\delta_2 \tag{4.12}$$

Similarly, from equations (4.8) and (4.9), in the steady state,

$$\left(K_1 + \frac{1}{R_1}\right)s\delta_1 + T_{12}(\delta_1 - \delta_2) = -\Delta P_{L1} \tag{4.13}$$

and

$$\left(K_2+\frac{1}{R_2}\right)s\delta_2+T_{12}(\delta_2-\delta_1)=-\Delta P_{L2} \qquad (4.14)$$

Adding equations (4.13) and (4.14) gives

$$\left(K_1+\frac{1}{R_1}\right)s\delta_1+\left(K_2+\frac{1}{R_2}\right)s\delta_2=-\Delta P_{L1}-\Delta P_{L2} \qquad (4.15)$$

In a synchronous system, $s\delta_1=s\delta_2=s\delta=$ angular frequency and equation (4.15) becomes

$$\left[K_1+K_2+\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\right]s\delta=-\Delta P_{L1}-\Delta P_{L2}$$

and

$$s\delta=\frac{-\Delta P_{L1}-\Delta P_{L2}}{(K_1+K_2)+(1/R_1+1/R_2)} \qquad (4.16)$$

From equations (4.13) and (4.14),

$$T_{12}(\delta_1-\delta_2)=\frac{-\Delta P_{L1}[K_2+(1/R_2)]+\Delta P_{L2}[K_1+(1/R_1)]}{[K_2+(1/R_2)+K_1+(1/R_1)]} \qquad (4.17)$$

It is normally required to keep the system frequency constant and to maintain the interchange through the tie-line at its scheduled value. To achieve this, additional controls are necessary to operate the speed-changer settings, as follows.

For area (1),

$$\begin{aligned} s\Delta P_1^{\prime} &\propto \gamma_{1t}T_{12}(\delta_1-\delta_2)+\gamma_{1f}s\delta_1 \\ &\propto \gamma_{1t}\left[T_{12}(\delta_1-\delta_2)+\frac{\gamma_{1f}}{\gamma_{1t}}s\delta_1\right] \end{aligned} \qquad (4.18)$$

Similarly, for area (2),

$$s\Delta P_2^{\prime}\propto \gamma_{2t}\left[T_{12}(\delta_2-\delta_1)+\frac{\gamma_{2f}}{\gamma_{2t}}s\delta_2\right] \qquad (4.19)$$

where γ_t and γ_f refer to the control constants for power transfer and frequency, respectively. When a load change occurs in a given area the changes in tie-line power and frequency have opposite signs, i.e. the frequency falls for a load increase and the power transfer increases, and vice versa. In the interconnected area, however, the changes have the same sign. Typical system parameters have the following orders of magnitude:

K = 0.75 p.u. on system-capacity base
T_{12} = 0.1p.u. (10 per cent of system capacity results in 1 rad displacement between areas (1) and (2))
R = 0.04p.u. on a base of system capacity
γ_{t} = 0.005
γ_{f} = 0.0009

Example 4.4

Two power systems, A and B, each have a regulation (R) of 0.1 p.u. (on respective capacity bases) and a stiffness K of 1 p.u. The capacity of system A is 1500 MW and of B 1000 MW. The two systems are interconnected through a tie-line and are initially at 60 Hz. If there is a 100 MW load change in system A, calculate the change in the steady-state values of frequency and power transfer.

Solution

$$K_A = 1\times 1500\text{ MW per Hz}$$
$$K_B = 1\times 1000\text{ MW per HZ}$$
$$R_A=\frac{\Delta f(\text{no load to full load})}{\text{full load capacity}}=\frac{0.1\times 60}{1500}$$
$$= 6/1500\text{ Hz per MW}$$
$$R_B=(6/1000)\text{ Hz per MW}$$

From equation (4.16),

$$\Delta f=\frac{-\Delta P_1}{(K_1+1/R_1)+(K_2+1/R_2)}$$
$$=\frac{-100}{1500+\frac{1500}{6}+1000+\frac{1000}{6}}$$
$$=\frac{-600}{17500}=-0.034\text{ Hz}$$
$$P_{12}=T_{12}(\delta_1-\delta_2)=\frac{-\Delta P_1(K_2+1)/R_2)}{(K_1+1/R_1)+(K_2+1/R_2)}$$
$$=\frac{-100\left(\frac{7000}{6}\right)}{10500/6}=\frac{-7000}{105}=-6\text{ MW}$$

Note that without the participation of governor control,

$$P_{12} = \left(\frac{-K_B}{K_A + K_B}\right) \qquad \Delta P = -\frac{1000}{2500} \times 100 = -40\,\text{MW}$$

4.6.2 Frequency-bias–tie-line control

Consider three interconnected power systems A, B and C, as shown in Figure 4.13, the systems being of similar size. Assume that initially A and B export to C, their previously agreed power transfers. If C has an increase in load the overall frequency tends to decrease and hence the generation in A, B, and C increases. This results in increased power transfers from A and B to C. These transfers, however, are limited by the tie-line power controller to the previously agreed values and therefore instructions are given for A and B to reduce generation and hence C is not helped. This is a severe drawback of what is known as straight tie-line control, which can be overcome if the systems are controlled by using consideration of both load transfer and frequency, such that the following equation holds:

$$\sum \Delta P + K\Delta f = 0 \tag{4.20}$$

where $\sum \Delta P$ is the net transfer error and depends on the size of the system and the governor characteristic, and Δf is the frequency error and is positive for high frequency. In the case above, after the load change in C, the frequency error is negative (i.e. low frequency) for A and B and the sum of ΔP for the lines AC and BC is positive. For correct control,

$$\sum \Delta P_A + K_A\Delta f = \sum \Delta P_B + K_B\Delta f = 0$$

Systems A and B take no regulating action despite their fall in frequency. In C, $\sum \Delta P_C$ is negative as it is importing from A and B and therefore the governor speeder motors in C operate to increase ouput and restore frequency. This system is known as frequency-bias–tie-line control and is often implemented automatically in interconnected systems.

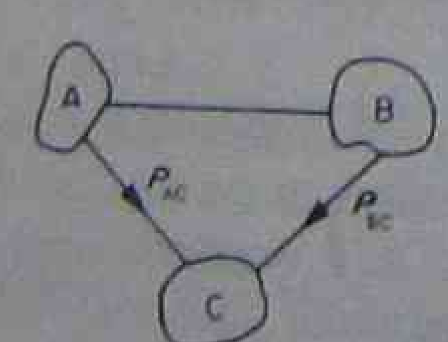

Figure 4.13 Three power systems connected by tie-lines

4.7 Economic Power-System Operation

4.7.1 Introduction

The extensive interconnection of power sources has made the operation of a system in the most economical manner a complex subject (see also Chapter 12). Economy must be balanced against considerations such as security of supply. The use of the *merit order* ensures that as far as possible the most economical generating sets are used. A knowledge of the flows of real and reactive power and other parameters in the network, and effective means of dealing with the analysis of large systems, is required for the operation to attain an economic optimum, although experienced operators certainly approach this aim.

Nowadays, the use of digital computers for load flows and fault calculations and the development of optimization techniques in control theory have resulted in much attention being given to this topic.

Apart from financial considerations, it is becoming difficult for operators to cope with the information produced by large complex systems in times of emergency, such as with major faults. Computers with on-line facilities can more readily digest this information and take correcting action by instructing control gear and settings, or by the suitable display of relevant information to enable human operators to take appropriate action. In the attempt to obtain economic optimization the limitations of the system, such as plant ratings and stability limits, must be observed.

Optimization may be considered in a number of ways according to the time scale involved, namely: daily, yearly (especially with hydro stations), and over much longer periods when planning for future developments, although this latter is not strictly operational optimization. In an existing system the various factors involved are the fixed and variable costs. The former includes labour, administration, interest and depreciation, etc., and the latter, mainly fuel. A major problem is the effective prediction of the future load whether it occurs in 10 minutes, a few hours, or in several year' time.

For operational planning, daily operation, and the setting of economic schedules, the following data is normally required.

- *For each generator:*
  1. maximum and economic output capacities;
  2. fixed and incremental heat rates;
  3. minimum shut-down time;
  4. minimum stable output, maximum run-up and run-down rates.

- *For each station:*
  1. cost and calorific value of fuel (thermal stations);
  2. factors reflecting recent operational performance of the station;
  3. minimum time between loading and unloading successive generators;
  4. any constraints on station output.

- *For the system:*
  1. load demand at given intervals for the specified period;
  2. specified constraints imposed by transmission capability;
  3. running-spare requirements;
  4. transmission circuit parameters, including maximum capacities and reliability factors.

The input–output characteristic of a turbine is of great importance when economical operation is considered. A typical characteristic is shown in Figure 4.14. The *incremental heat rate* is defined as the slope of the input–output curve at any given output. The graph of the incremental heat rate against output is known as the Willans line. For large turbines with a single valve, and for gas turbines, the incremental heat rate is approximately constant over the operating range (most steam turbines in Britain are of this type); with multivalve turbines (as used in the U.S.A.) the Willans line is not horizontal but curves upwards and is often represented by the closest linear law. The value taken for the incremental heat rate of a generating set is sometimes complicated because if only one or two shifts are being operated (there are normally three shifts per day) heat has to be expended in banking boilers when the generator is not required to produce output.

Instead of plotting incremental heat rate or fuel consumption against power output for the turbine-generator, the incremental fuel cost may be used. This is

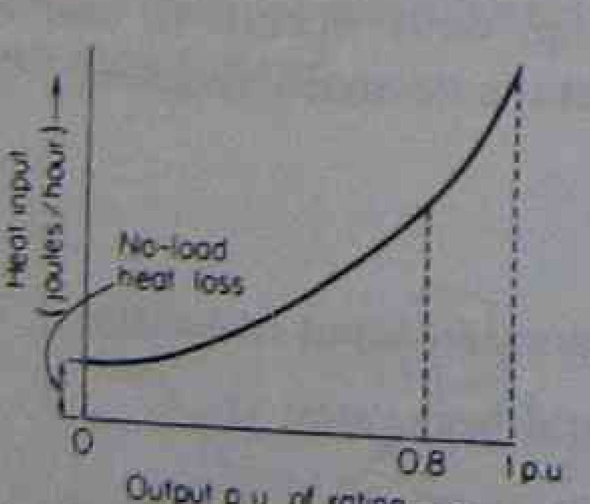

Figure 4.14 Input–output characteristic of a turbogenerator set. Often the curve above an output of 0.8 p.u. is steeper than the remainder and the machine has a maximum rating at 1 p.u. and a maximum economic rating at, say 0.8 p.u.

advantageous when allocating load to generators for optimum economy as it incorporates differences in the fuel costs of the various generating stations. Usually, the graph of incremental fuel cost against power output can be approximated by a straight line (Figure 4.15). Consider two turbine-generator sets having the following different incremental fuel costs, dC_1/dP_1 and dC_2/dP_2, where C_1 is the cost of the fuel input to unit number 1 for a power output of P_1, and, similarly, C_2 and P_2 relate to unit number 2. It is required to load the generators to meet a given requirement in the most economical manner. Obviously the load on the machine with the higher dC/dP will be reduced by increasing the load taken by the machine with the lower dC/dP. This transfer will be beneficial until the values of dC/dP for both sets are equal, after which the machine with the previously higher dC/dP now becomes the one with the lower value, and vice versa. There is no economic advantage in further transfer of load, and the condition when $dC_1/dP_1 = dC_2/dP_2$ therefore gives optimum economy; this can be seen by considering Figure 4.15. The above argument can be extended to several machines supplying a load. Generally, for optimum economy the *incremental fuel cost should be identical for all contributing turbine-generator sets* on free-governor action. In practice, most generators will be loaded to their maximum output.

The above reasoning must be modified when the distances of generating stations from the common loads are different; here the cost of transmission losses will affect the argument.

As important as the transmission losses is the optimum method of transporting fuel from the production centres to the generating stations. The transport of both electrical energy and fuel in the optimum manner forms *transportation problems*, which may be dealt with by special techniques or by the general method of linear programming. In a competitive situation the transportation costs will be included in the generator bid price.

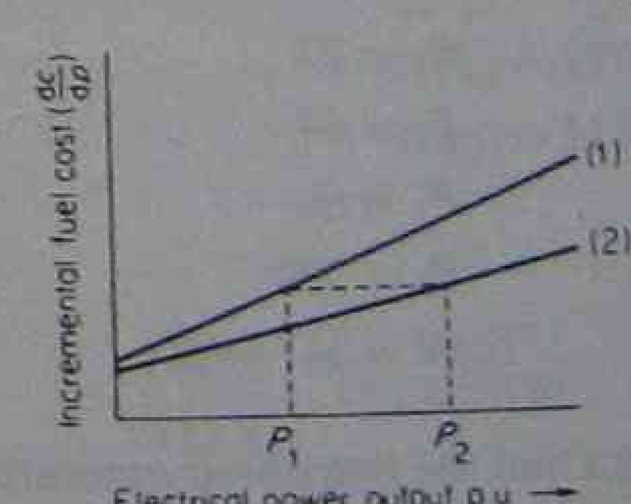

Figure 4.15 Idealized graphs of incremental fuel cost against output for two machines sharing a load equal to P_1 and P_2

Example 4.5

Four generators are available to supply a power system peak load of 472.5 MW. The cost of power $C(P_i)$ from each generator, and maximum output, is given in (\$ U.S.) by:

$$C(P_1) = 200 + 15P_1 + 0.20P_1^2 \qquad \text{Max. output 100 MW}$$
$$C(P_2) = 300 + 17P_2 + 0.10P_2^2 \qquad \text{Max. output 120 MW}$$
$$C(P_3) = 150 + 12P_3 + 0.15P_3^2 \qquad \text{Max.output 160 MW}$$
$$C(P_4) = 500 + 2P_4 + 0.07P_4^2 \qquad \text{Max. output 200 MW}$$

The spinning reserve is to be 10 per cent of peak load and the transmission losses can be neglected.

Calculate the optimal loading of each generator and the cost of operating the system for 1 h at peak.

Solution

The generators' combined output is to be 472.5 MW.

Marginal costs are given (in U.S.\$/MW) by:

$$\frac{dC(P_1)}{dP_1} = 15 + 0.40P_1 \qquad P_1 \leqslant 100\,\text{MW}$$
$$\frac{dC(P_2)}{dP_2} = 17 + 0.20P_2 \qquad P_2 \leqslant 120\,\text{MW}$$
$$\frac{dC(P_3)}{dP_3} = 12 + 0.30P_3 \qquad P_3 \leqslant 160\,\text{MW}$$
$$\frac{dC(P_4)}{dP_4} = 2 + 0.14P_4 \qquad P_4 \leqslant 200\,\text{MW}$$

The marginal cost curves are plotted in Figure 4.16 for eachof the generators up to their maximum output.

From the curves, at 30 \$/MW the outputs are

$$P_1 = 23$$
$$P_2 = 64$$
$$P_3 = 60$$
$$P_4 = 200$$
$$\text{Total} = 347\,\text{MW}$$

Hence, P_4 runs at full output and the remaining generators must sum to 272.5 MW.

At 40 \$/MW the outputs sum to 446 MW. At 41 \$/MW P_2 reaches its maximum of 120 MW, and $P_1 = 40$ MW and $P_3 = 98$ MW (total 458 MW).

This leaves 14.5 MW to be found from P_1 and P_3.

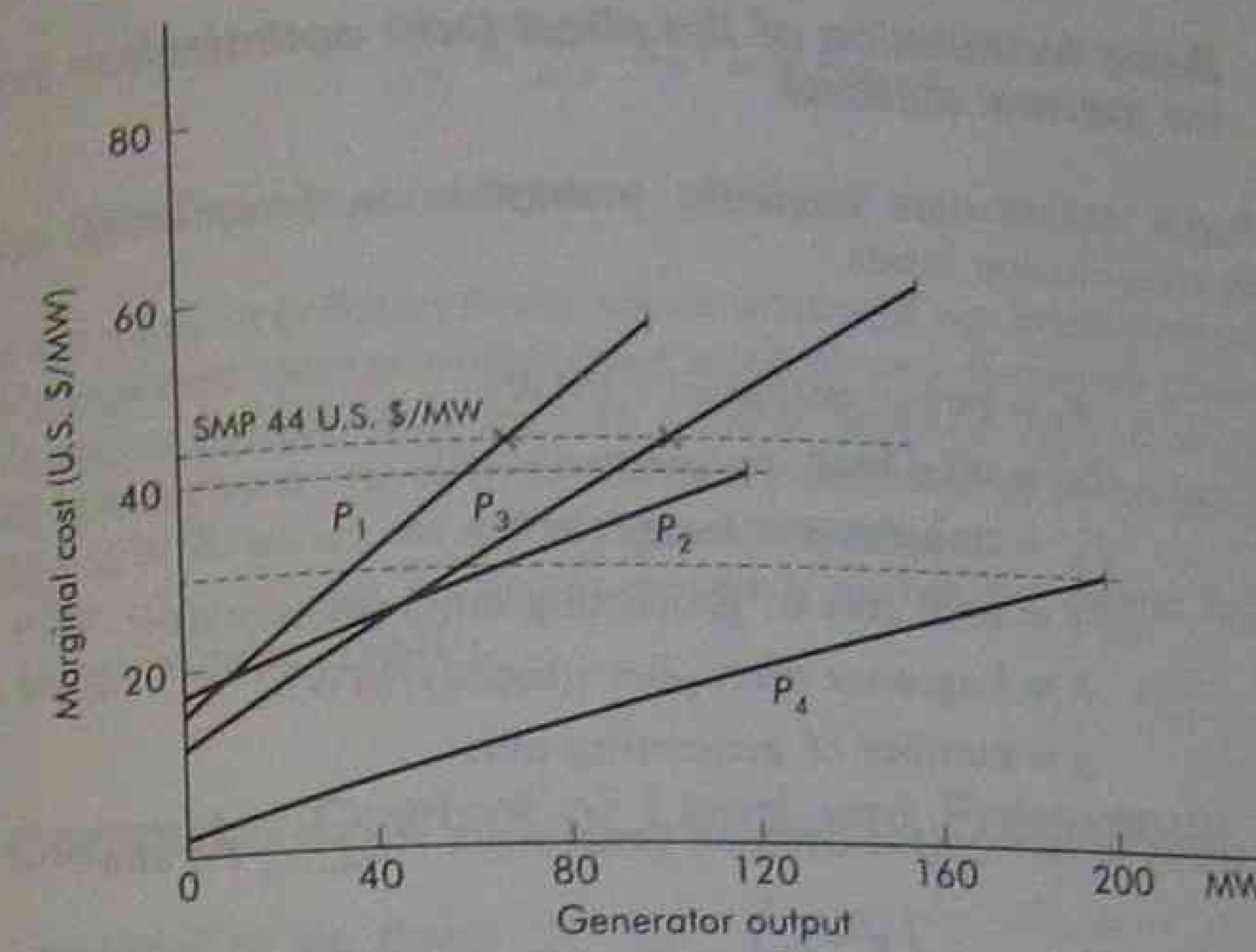

Figure 4.16 Marginal cost curves for four generators

Adjusting the marginal cost line now to 44 \$/MW provides 44 MW from P_1 and 108 MW from P_3, giving a total of 472 MW, which is near enough when using this graphical method. The spinning reserve on these generators is 108 MW, more than enough to cover 10 per cent of 472.5 MW.

The cost of operating the system for 1 h at peak is

$$C(P_1) = 200 + 15 \times 44 + 0.2 \times 44^2 = \$1247$$

Similarly

$$C(P_2) = \$3780$$
$$C(P_3) = \$3195$$
$$C(P_4) = \$3700$$
$$\text{Total} = \$11922$$

i.e.

$$\frac{11922}{472.5} = 25.2\,\$/\text{MW}$$

(Note: This is the *average* cost per hour whereas the *marginal cost* was calculated as 44 \$/MW. It can be seen that the use of large units on base load (e.g. the 200 MW generator) reduces system marginal price (SMP) considerably (see Chapter 12).)

4.7.2 Basic formulation of the short-term optimization problem for thermal stations

Kirchamayer (1958) uses Lagrange multipliers in formulating equations, including transmission losses.

Let

P_i = power output of i (MW)
P_R = total load on system (MW)
P_L = transmission losses (MW)
F_T = total cost of generating units (money/h)
λ = Lagrange multiplier ((money/MWh)
n = number of generating units

The total input to the system from all generators $P_T \sum_{i=1}^{n} P_i$, and

$$\left(\sum_{1}^{n} P_i\right) - P_L - P_R = 0$$

Using Lagrangian multipliers, the expression

$$\gamma = F_T - \lambda\left(\sum_{1}^{n} P_i - P_L - P_R\right)$$

is formulated, where for minimum cost (F_T), $\partial\gamma/\partial P_i = 0$ for all values of i. (Note the use of partial differentiation here.) This is given by

$$\frac{\partial \gamma}{\partial P_i} = \frac{dFi}{dP_i} - \lambda + \lambda\frac{\partial P_L}{\partial P_i} = 0$$

since P_R is assumed fixed. Hence

$$\frac{dF_i}{dP_i} + \lambda\frac{\partial P_L}{\partial P_i} = \lambda \tag{4.21}$$

In equation (4.21), $\partial P_L/\partial P_i$ is the incremental transmission loss. One way of solving the equations described by (4.21) is known as the penalty-factor method in which equation (4.21) is rewritten as

$$\frac{dF_i}{dP_i}L_i = \lambda \tag{4.22}$$

where

$$L_i = \left(\frac{1}{1 - \partial P_L/\partial P_i}\right) = \text{penalty factor of plant } i$$

where $i = 1 \ldots n$ (number of plants). In practice the determination of $\partial P_L/\partial P_i$ is difficult and the use of the so-called loss or 'B' coefficients is made, i.e.

$$\frac{\partial P_L}{\partial P_i} = \sum_{1} 2B_{mi}P_m + B_{i0}$$

where the B coefficients are determined from the network (see Kirchmayer, 1958).

There are many drawbacks to the above treatment e.g. limitations on power flows by equipment ratings transformer settings, and maximum phase angles allowable across transmission lines on stability grounds. Also, it is concerned only with active power, reactive power being neglected or taken into account by limiting MW flows across defined group boundaries.

The value of λ at any particular period is known as 'system lambda' or 'system marginal price' (SMP).

4.8 Computer Control of Load and Frequency

4.8.1 Control of tie lines

Automatic control of area power systems connected by tie-lines has already been discussed. The methods used will now be extended to include optimum economy as well as power transfer and frequency control. The basic systems described are typical of U.S. and European practice and have been comprehensively discussed by many authors. In the previous section, methods for economic analysis and optimization as developed by Kirchmeyer (1958) have been summarized. The choice of generating units to be operated is largely decided by spinning reserve, voltage control, stability, and protection requirments. The methods discussed decide the allocation of load to particular machines.

If transmission losses are to be neglected, it has been shown that optimum economy results when $dF_i/dP_i = \lambda$. Control equipment to adjust the governor speed-change settings such that all units comply with the appropriate value of dF_i/dP_i is incorporated in the control loops for frequency and power-transfer adjustment, as shown in Figure 4.17. The frequency and load-transfer control acts quickly, and once these quantities have been decided the slower acting economic controls then act. For example, if an increase in load occurs in the controlled area, a signal requiring increased generation transmits through the control system. These changes alter the value of λ and cause the economic control apparatus to call for generation to be operated at the same incremental cost. Eventually the system is again in the steady state, the load change having been absorbed, and all units operate at an identical value of incremental loss.

If transmission losses are included, the basic economic requirement calls for $dF_i/dP_i = \lambda/L_i$ where L_i is the penalty factor; $(1/L_i)$ signals are generated by a computer from a knowledge of system parameters in the form of the so-called B coefficients. However, in practice, most generators are being run at their

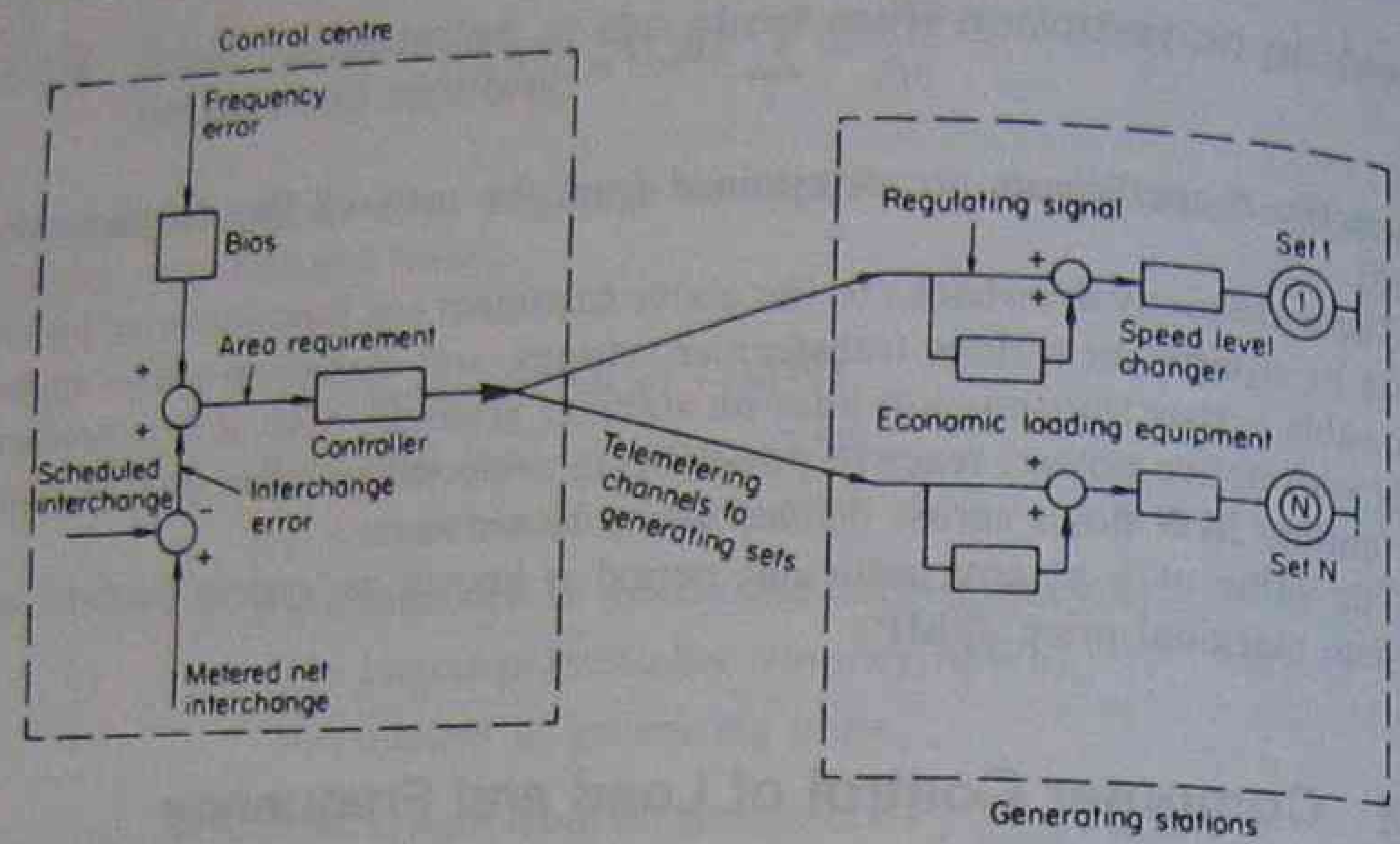

Figure 4.17 Schematic diagram of automatic control arrangements covering frequency, tie-line power flows, and economic loading of generating sets (*From Economic Control of Interconnected Systems by L. K. Kirchmayer. Copyright © 1959 John Wiley & Sons Ltd. By permission of John Wiley & Sons Inc.*)

maximum rated output because their cost is below the system λ. Only a few units on the cost margin are setting the value of λ.

In the U.S.A. the many separate power companies are connected into power pools. The aim is optimal control of the participating machines in each pool, within security constraints. Figure 4.18 shows a block diagram of the New York Power Pool. Among the advantages of such pools are; the more economic use of large generators, emergency assistance to neighbouring utilities, reduced spinning reserves, and lower overall generation costs. Each pool is connected to others by tie-lines, e.g. the New York Pool to Ontario Hydro, New England Power Exchange and the Pennsylvania–New Jersey–Maryland interconnection. Control of the generators in the pool is achieved either centrally or from each of the constituent areas (i.e. area control centres) as indicated in Figure 4.19

The central control mechanisms for pools are basically the same as for areas and the systems of Figure 4.17 may be used with 'pool' substituted for 'area'. Allocation between generators in an area, or between areas in a pool, may be accomplished by the use of base points or loadings and participation factors. The former gives the economic allocation for a specified total generation and they are normally established every few minutes or when loadings change. When the generation allocations are established they are compared with the actual values being generated and a control error formed. The unit participation factor (K_n) for any unit (n) in the pool is given by

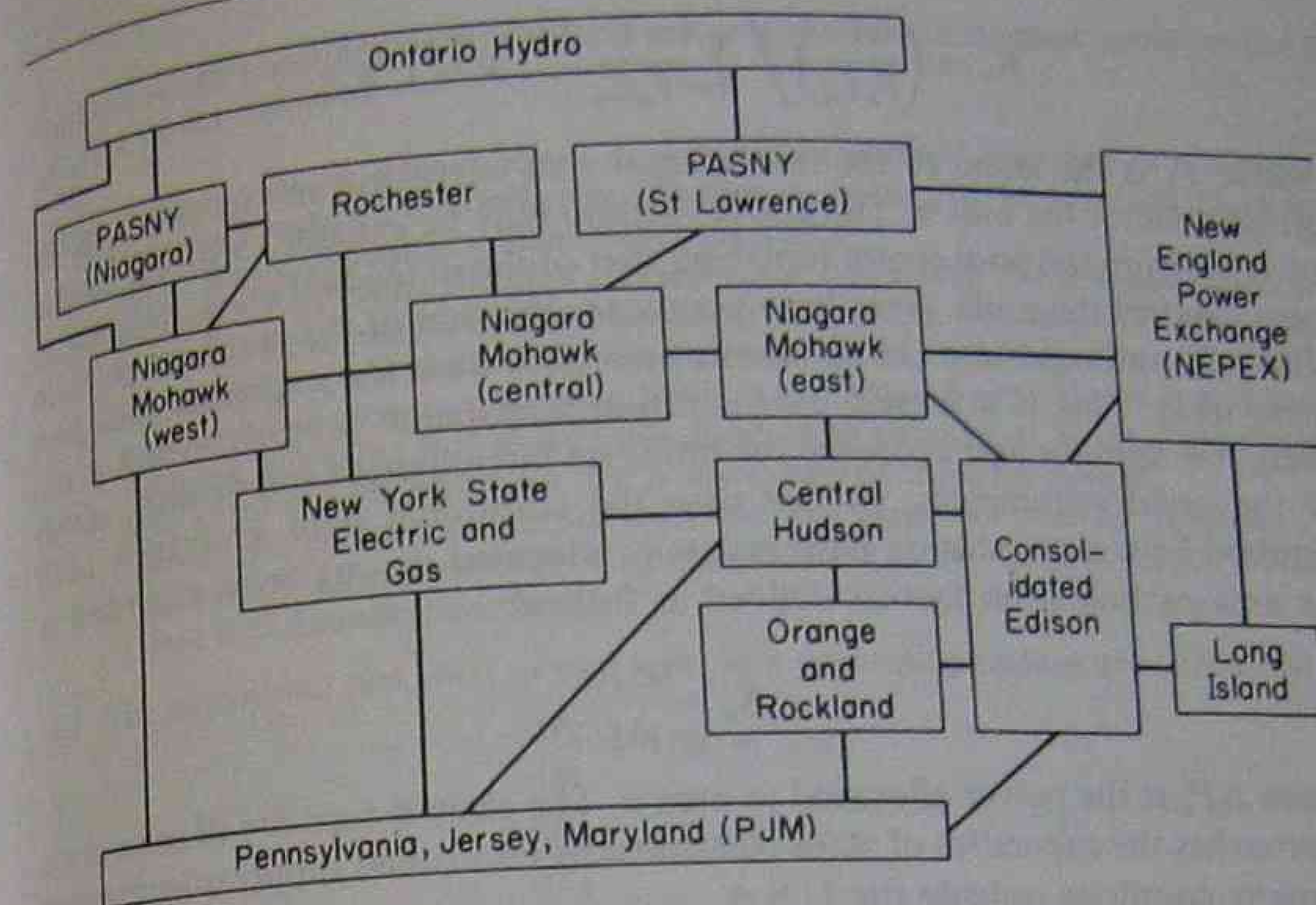

Figure 4.18 Block diagram showing the eight power companies within the New York Power Pool, and three other pools that are interconnected with it (*Copyright © 1973 Institute of Electrical and Electronics Engineers, Inc. Reprinted by permission from I.E.E.E. Spectrum, Vol. 10, No. 3, March 1973, pp. 54–61*)

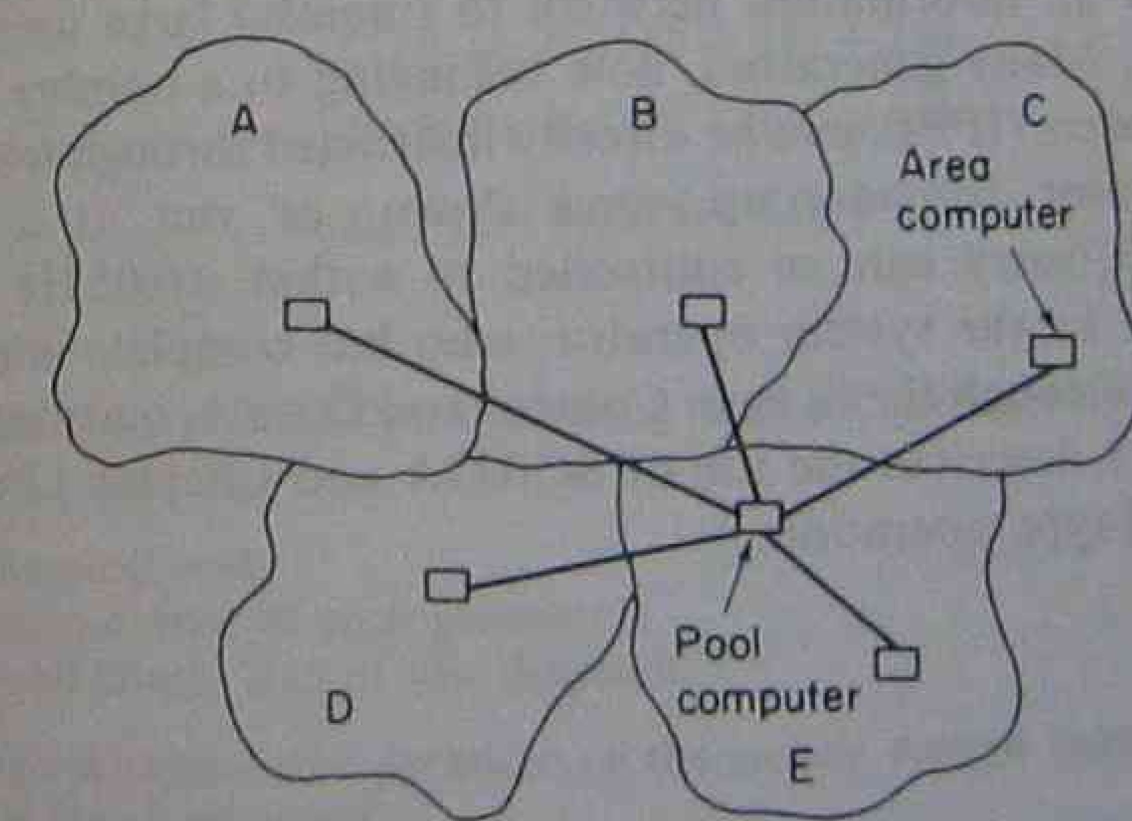

Figure 4.19 Multicomputer configuration formed by a five-area pool (*Copyright © 1973 Institute of Electrical and Electronics Engineers, Inc. Reprinted by permission from I.E.E.E. Spectrum, Vol. 10, No. 3, March 1973, pp. 54–61*)

$$K_n = \left(\frac{1}{F_n L_n}\right) \Big/ \sum \frac{1}{F_n L_n} \qquad n = 1, 2, 3, \ldots$$

where F_n is the slope of the incremental cost curve for unit n, and L_n is the penalty factor for unit n. The above method may be extended with the control of areas from the pool centre replacing that of the individual units; the areas, in turn, control the units. Area base points are the sum of the unit base points in that area and represent the economic area allocation for a specified total pool generation. This is achieved by a multi-area dispatch program which is run every few minutes and accounts for limits on interchanges and tie-lines, as well as the usual parameters. In this case the additional pool generation (ΔG) required between updating base points is allocated among areas according to the area participation factors defined as follows:

$$K_a = \frac{\Delta P_a}{\Delta G}$$

where ΔP_a is the power allocated to area a. The electric capacity of some pools approaches the capacities of state-run centrally controlled supply organizations in many countries outside the U.S.A.

In the U.K., following privatization in 1989/90, the England and Wales transmission network is run on a pool basis into which generators (including contributions from Scotland and France) bid a price for their electrical energy for the following day. The pool then computes an economic schedule of generation from a load forecast for the following day, predicting the system λ price for each half-hour if the schedule is adhered to as planned. Although some producers may be required to provide a variable output to cover reserves and contingencies, consumers can vary their requirements depending on the SMP as displayed over an information network to (mainly) large users and other interested parties. Many generators, now belonging to a number of independent power producers (IPPs), can be directly instructed through tele-control by the system operator to start up, shut down, or run at a part load. Consequently, frequency can be controlled to within ± 0.05 Hz throughout the day and night by the system operator, who has complete information at the control centre through the System Control and Data Acquisition (SCADA) system. Chapter 11 describes the use of SCADA and Chapter 12 outlines the economics of pool-type operation.

Problems

4.1 Two identical 60 MW synchronous generators operate in parallel. The governor settings on the machines are such that they have 4 per cent and 3 per cent droops (no-load to full-load percentage speed drop). Determine (a) the load taken by each machine for a total of 100 MW; (b) the percentage adjustment in the no-load speed to be made by the speeder motor if the machines are to share the load equally.

(Answer: (a) 42.8 and 57.2 MW; (b) 0.83 per cent increase in no-load speed on the 4 per cent droop machine)

4.2 (a) Explain how the output power of a turbine-generator operating in a constant frequency system is controlled by adjusting the setting of the governor. Show the effect on the generator power–frequency curve.
(b) Generator A of rating 200 MW and generator B of rating 350 MW have governor droops of 5 per cent and 8 per cent, respectively, from no-load to full-load. They are the only supply to an isolated system whose nominal frequency is 50 Hz. The corresponding generator speed is 3000 r.p.m. Initially, generator A is at 0.5 p.u. load and generator B is at 0.65 p.u. load, both running at 50 Hz. Find the no-load speed of each generator if it is disconnected from the system.
(c) Also determine the total output when the first generator reaches its rating.
(Answer: (b) Generator B 3156 r.p.m; generator A 3075 r.p.m; (c) 337 MW)
(From E C Examination, 1996)

4.3 The incremental fuel costs of two units in a generating station are as follows:

$$\frac{dF_1}{dP_1} = 0.003P_1 + 0.7$$

$$\frac{dF_2}{dP_2} = 0.004P_2 + 0.5$$

where F is in £/h and P is in MW.
Assuming continuous running with a total load of 150 MW calculate the saving per hour obtained by using the most economical division of load between the units as compared with loading each equally. The maximum and minimum operational loadings are the same for each unit and are 125 MW and 20 MW.
(Answer: $P_1 = 57$ MW, $P_2 = 93$ MW; saving £1.14 per hour)

4.4 What is the merit order used for when applied to generator scheduling?
A power system is supplied by three generators. The functions relating the cost (in £/h) to active power output (in MW) when operating each of these units are:

$$C_1(P_1) = 0.04P_1^2 + 2P_1 + 250$$

$$C_2(P_2) = 0.02P_2^2 + 3P_2 + 450$$

$$C_3(P_3) = 0.01P_3^2 + 5P_3 + 250$$

The system load is 525 MW. Assuming that all generators operate at the same marginal cost, calculate:

(a) the marginal cost;
(b) optimum output of each generator;
(c) the total hourly cost of this dispatch.

How is the economy of operation of the power system balanced against security requirements for all demands?
(Answer: (a) £10/MWh; (b) P_1 100 MW, P_2, 175 MW, P_3, 250 MW; (c) £4562.5/h)
(From E.C. Examination, 1997)

4.5 Two power systems A and B are interconnected by a tie-line and have power-frequency constants K_A and K_B MW/Hz. An increase in load of 500 MW on system A

causes a power transfer of 300 MW from B to A. When the tie-line is open the frequency of system A is 49 Hz and of system B 50 Hz. Determine the values of K_A and K_B, deriving any formulae used.
(Answer: K_A 500 MW/Hz; K_B 750 MW/Hz)

4.6 Two power systems, A and B, having capacities of 3000 and 2000 MW, respectively, are interconnected through a tie-line and both operate with frequency-bias–tie-line control. The frequency bias for each area is 1 per cent of the system capacity per 0.1 Hz frequency deviation. If the tie-line interchange for A is set at 100 MW and for B is set (incorrectly) at 200 MW, calculate the steady-state change in frequency.
(Answer: 0.6 Hz; use $\Delta P_A + \sigma_A \Delta f = \Delta P_B + \sigma_B \Delta f$)

4.7 (a) (i) Why do power systems operate in an interconnected arrangement?
(ii) How is the frequency controlled in a power system?
(iii) What is meant by the *stiffness* of a power system?

(b) Two 50 Hz power systems are interconnected by a tie-line, which carries 1000 MW from system A to system B, as shown in Figure 4.20. After the outage of the line shown in the figure, the frequency in system A increases to 50.5 Hz, while the frequency in system B decreases to 49 Hz.

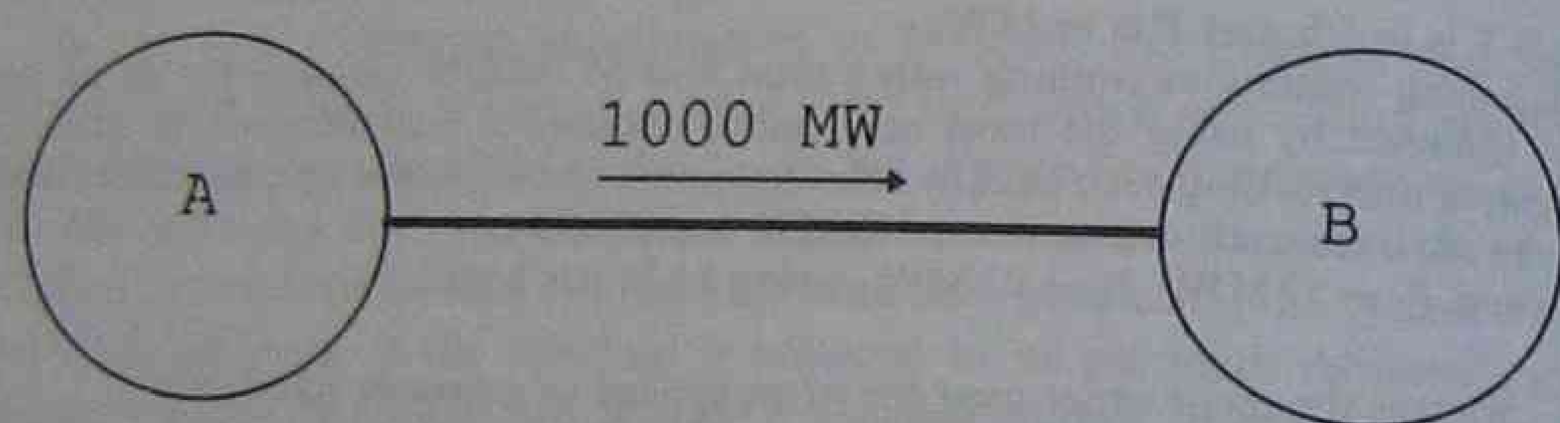

Figure 4.20 Interconnected systems of Problem 4.7 (b)

(i) Calculate the stiffness of each system.
(ii) If the systems operate interconnected with 1000 MW being transferred from A to B, calculate the flow in the line after outage of a 600 MW generator in system B.
(Answer: (b) (i) K_A 2000 MW/Hz, K_B 1000 MW/Hz; (ii) 1400 MW)
(*From*, *E.C. Examination, 1997*)

5 Control of Voltage and Reactive Power

5.1 Introduction

The approximate relationship between the scalar voltage difference between two nodes in a network and the flow of reactive power was shown in Chapter 2 to be

$$\Delta V = \frac{RP + XQ}{V} \tag{2.9}$$

Also it was shown that the transmission angle δ is proportional to

$$\frac{XP - RQ}{V} \tag{2.10}$$

For networks where $X \gg R$, i.e. most power circuits, ΔV, the voltage difference, determines Q. Consider the simple interconnector linking two generating stations A and B, as shown in Figure 5.1(a). The machine at A is in phase advance of that at B and V_1 is greater than V_2; hence there is a flow of power and reactive power from A to B. This can be seen from the phasor diagram shown in Figure 5.1(b). It is seen that I_d and hence P is determined by $\angle\delta$ and the value of I_q, and hence Q mainly, by $V_1 - V_2$. In this case $V_1 > V_2$ and reactive power is transferred from A to B; by varying the generator excitations such that $V_2 > V_1$, the direction of the reactive power is reversed, as shown in Figure 5.1(c). Hence, power can be sent from A to B or B to A by suitably adjusting the amount of steam (or water) admitted to the turbine, and reactive power can be sent in either direction by adjusting the voltage magnitudes. These two operations are approximately independent of each other if $X \gg R$, and the flow of reactive power can be studied almost independently of the power flow. The phasor diagrams show that if a scalar voltage difference exists across a largely reactive link, the reactive power flows towards the node of lower voltage. From another point of view, if, in a network, there is

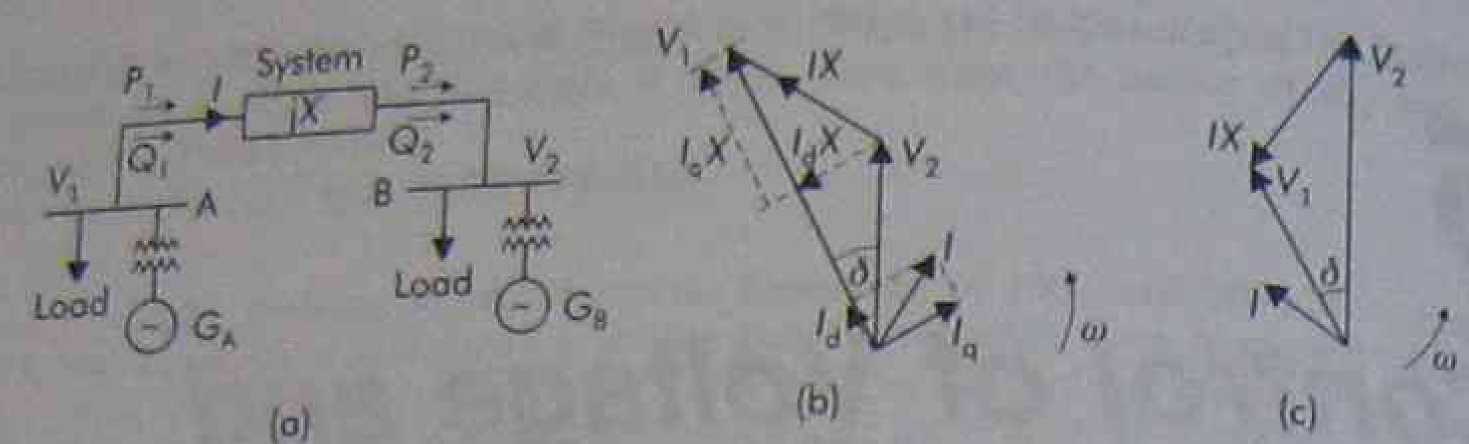

Figure 5.1 (a) System of two generators interconnected. (b) Phasor diagram when $V_1 > V_2$. I_d and I_q are components of I. (c) Phasor diagram when $V_2 > V_1$

a deficiency of reactive power at a point, this has to be supplied from the connecting lines and hence the voltage at that point falls. Conversely if there is a surplus of reactive power generated (e.g. lightly loaded cables generate positive vars), then the voltage will rise. This is a convenient way of expressing the effect of the power factor of the transferred current, and although it may seem unfamiliar initially, the ability to think in terms of var flows, instead of exclusively with power factors and phasor diagrams, will make the study of power networks much easier.

If it can be arranged that Q_2 in the system in Figure 5.1(a) is zero, then there will be no voltage drop between A and B, a very satisfactory state of affairs. Assuming that V_1 is constant, consider the effect of keeping V_2, and hence the voltage drop ΔV, constant. From equation (2.9)

$$Q_2 = \frac{V_2 \cdot \Delta V - R \cdot P_2}{X} = K - \frac{R}{X} P_2 \tag{5.1}$$

where K is a constant and R is the resistance of the system.

If this value of Q_2 does not exist naturally in the circuit then it will have to be obtained by aritificial means, such as the connection at B of capacitors or inductors. If the value of the power changes from P_2 to P_2' and if V_2 remains constant, then the reactive power at B must change to Q_2' such that

$$Q_2' - Q_2 = \frac{R}{X}(P_2' - P_2)$$

i.e. an increase in power causes an increase in reactive power. The change, however, is proportional to (R/X), which is normally small. It is seen that voltage can be controlled by the injection into the network of reactive power of the correct sign. Other methods of a more obvious kind for controlling voltage are the use of tap-changing transformers and voltage boosters.

5.2 The Generation and Absorption of Reactive Power

In view of the findings in the previous section, a review of the characteristics of a power system from the viewpoint of reactive power is now appropriate.

5.2.1 Synchronous generators

These can be used to generate or absorb reactive power. The limits on the capability for this can be seen in Figure 3.14. The ability to supply reactive power is determined by the short-circuit ratio (1/synchronous reactance) as the distance between the power axis and the theoretical stability-limit line in Figure 3.14 is proportional to the short-circuit ratio. In modern machines the value of this ratio is made low for economic reasons, and hence the inherent ability to operate at leading power factors is not large. For example, a 200 MW 0.85 p.f. machine with a 10 per cent stability allowance has a capability of 45 MVAr at full power output. The var capacity can, however, be increased by the use of continuously acting voltage regulators, as explained in Chapter 3. An over-excited machine, i.e. one with greater than normal excitation, generates reactive power whilst an underexcited machine absorbs it. The generator is the main source of supply to the system of both positive and negative vars.

5.2.2 Overhead lines and transformers

When fully loaded, lines absorb reactive power. With a current I amperes for a line of reactance per phase $X(\Omega)$ the vars absorbed are I^2X per phase. On light loads the shunt capacitances of longer lines may become predominant and the lines then become var generators.

Transformers always absorb reactive power. A useful expression for the quantity may be obtained for a transformer of reactance X_T p.u. and a full-load rating of $3V \cdot I_{rated}$.

The ohmic reactance

$$= \frac{V \cdot X_T}{I_{rated}}$$

Therefore the vars absorbed

$$= 3 \cdot I^2 \cdot \frac{V \cdot X_T}{I_{rated}}$$

$$= 3 \cdot \frac{I^2 V^2}{(IV)_{rated}} \cdot X_T = \frac{(VA \text{ of load})^2}{\text{Rated } VA} \cdot X_T$$

5.2.3 Cables

Cables are generators of reactive power owing to their high shunt capacitance. A 275 kV, 240 MVA cable produces 6.25–7.5 MVAr per km; a 132 kV cable roughly 1.9 MVAr per km; and a 33kV cable, 0.125 MVAr per km.

5.2.4 Loads

A load at 0.95 power factor implies a reactive power demand of 0.33 kVAr per kW of power, which is more appreciable than the mere quoting of the power factor would suggest. In planning a network it is desirable to assess the reactive power requirements to ascertain whether the generators are able to operate at the required power factors for the extremes of load to be expected. An example of this is shown in Figure 5.2, where the reactive losses are added for each item until the generator power factor is obtained.

Example 5.1

In the radial transmission system shown in Figure 5.2, all p.u. values are referred to the voltage bases shown and 100 MVA. Determine the power factor at which the generator must operate.

Solution

Voltage drops in the circuits will be neglected and the nominal voltages assumed.

Busbar A,

$$P = 0.5\,\text{p.u.} \qquad Q = 0$$

I^2X loss in 132 kV lines and transformers

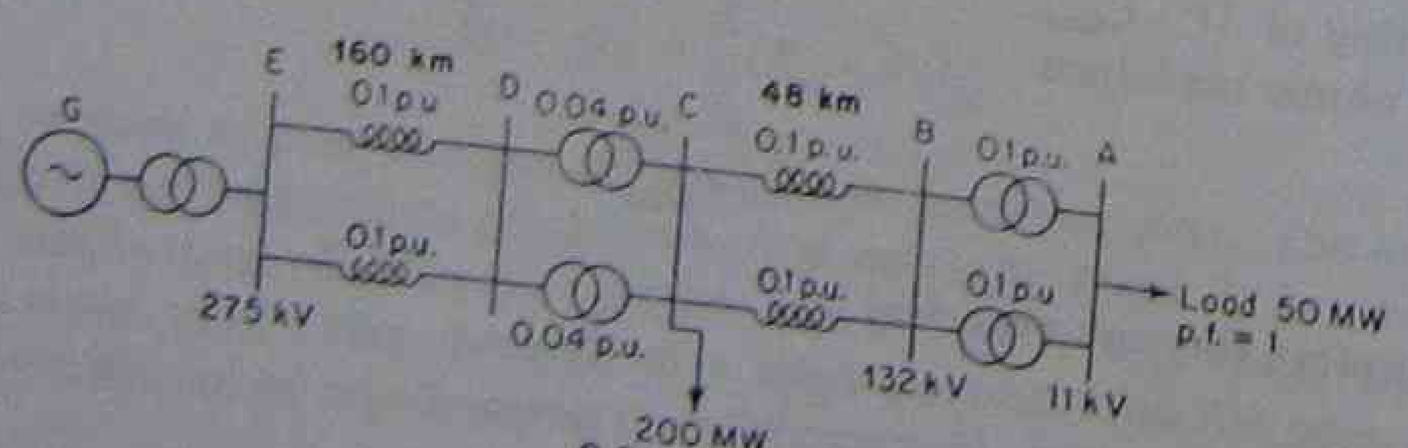

Figure 5.2 Radial transmission system with intermediate loads. Calculation of reactive-power requirement

$$= \frac{P^2 + Q^2}{V^2} X_{CA} = \frac{0.5^2}{1^2} \cdot 0.1$$
$$= 0.025\,\text{p.u.}$$

Busbar C,

$$P = 2 + 0.5 = 2.5\,\text{p.u.}$$
$$Q = 1.5 + 0.025\,\text{p.u.}$$
$$= 1.525\,\text{p.u.}$$

I^2X loss in 275 kV lines and transformers

$$= \frac{2.5^2 + 1.525^2}{1^2} 0.07$$
$$= 0.6\,\text{p.u.}$$

The I^2X loss in the large generator-transformer will be negligible, so that the generator must deliver $P = 2.5$ and $Q = 2.125$ p.u. and operate at a power factor of 0.76 lagging. It is seen in this example that, starting with the consumer load, the vars for each circuit, in turn, are added to obtain the total.

5.3 Relation Between Voltage, Power, and Reactive Power at a Node

The phase voltage V at a node is a function of P and Q at that node, i.e.

$$V = \phi(P, Q)$$

The voltage is also dependent on adjacent nodes and the present treatment assumes that these are infinite buses.

The total differential of V,

$$\mathrm{d}V = \frac{\partial V}{\partial P} \cdot \mathrm{d}P + \frac{\partial V}{\partial Q} \cdot \mathrm{d}Q$$

and using

$$\frac{\partial P}{\partial V} \cdot \frac{\partial V}{\partial P} = 1 \quad \text{and} \quad \frac{\partial Q}{\partial V} \cdot \frac{\partial V}{\partial Q} = 1$$

$$\mathrm{d}V = \frac{\mathrm{d}P}{(\partial P/\partial V)} + \frac{\mathrm{d}Q}{(\partial Q/\partial V)} \tag{5.2}$$

It can be seen from equation (5.2) that the change in voltage at a node is defined by the two quantities

$$\left(\frac{\partial P}{\partial V}\right) \quad \text{and} \quad \left(\frac{\partial Q}{\partial V}\right).$$

If, at a particular system load, the line voltage of M falls below its nominal value by 5 kV, calculate the magnitude of the reactive volt-ampere injection required at M to re-establish the original voltage.

The p.u. values are expressed on a 500 MVA base and resistance may be neglected throughout.

Solution

The line diagram and equivalent single-phase circuit are shown in Figures 5.4 and 5.5.

It is necessary to determine the value of $\partial Q/\partial V$ at the node or busbar M; hence the current flowing into a three-phase short-circuit at M is required.

The base value of reactance in the 132 kV circuit is

$$\frac{132^2 \times 1000}{500\,000} = 35\,\Omega$$

Therefore the line reactances

$$= \frac{j50}{35} = j1.43\text{ p.u.}$$

The equivalent reactance from M to N = $j0.5$ p.u.

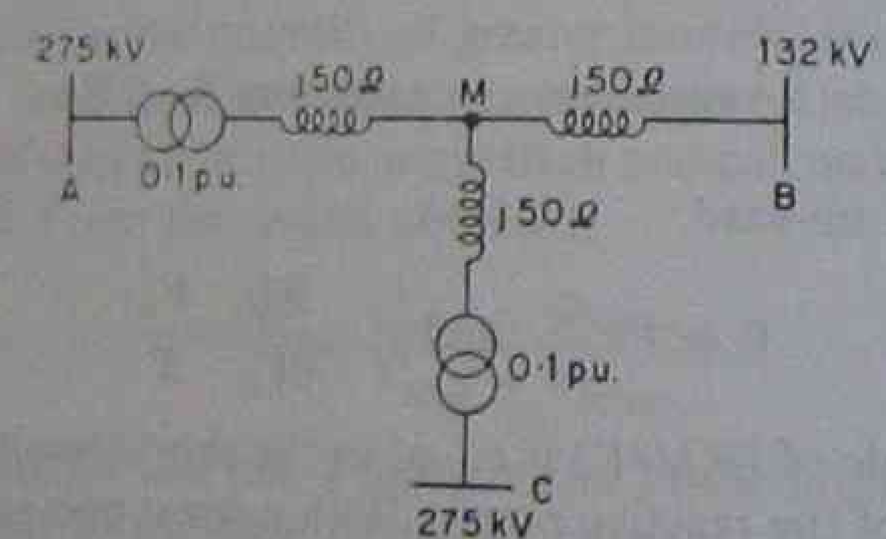

Figure 5.4 Schematic diagram of the system for Example 5.2

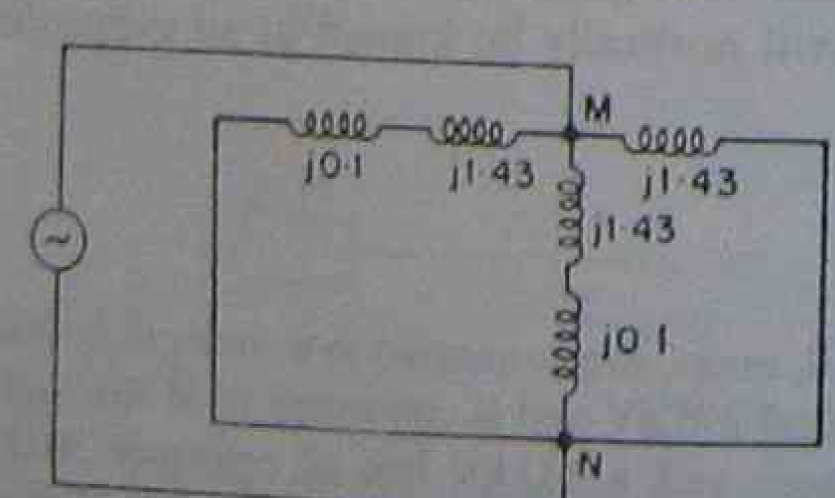

Figure 5.5 Equivalent single-phase network with the node M short-circuited to neutral (refer to Chapter 7 for full explanation of the derivation of this circuit)

Hence the fault MVA at M

$$= \frac{500}{0.5} = 1000\text{ MVA}$$

and the fault current

$$= \frac{1000 \times 10^6}{\sqrt{(3)} \times 132\,000}$$

$$= 4380\text{ A} \quad \text{at zero power factor lagging}$$

It has been shown that $\partial Q_M/\sqrt{(3)}\partial V_M$ = three-phase short-circuit current when Q_M and V_M are three-phase and line values

$$\therefore \frac{\partial Q_M}{\partial V_M} = 7.6\text{ MVAr/kV}$$

The natural voltage drop at M = 5 kV. Therefore the value of the injected vars required, ΔQ_M, to offset this drop

$$= 7.6 \times 5 = 38\text{ MVAr}$$

5.4 Methods of Voltage Control: (i) Injection of Reactive Power

The background to this method has been given in the previous sections. This is the most fundamental method, but in transmission systems it lacks the flexibility and economy of transformer tap-changing. Hence it is only used in schemes when transformers alone will not suffice. The provision of static capacitors to improve the power factors of factory loads has been long established. The capacitance required for the power-factor improvement of loads for optimum economy is determined as follows.

Let the tariff of a consumer be

$$£A \times \text{kVA} + B \times \text{kWh}$$

A load of P_1 kilowatts at power factor ϕ_1 lagging has a kVA of $P_1/\cos\phi_1$. If this power factor is improved to $\cos\phi_2$, the new kVA is $P_1/\cos\phi_2$. The saving is therefore

$$£P_1 A\left(\frac{1}{\cos\phi_1} - \frac{1}{\cos\phi_2}\right)$$

The reactive power required from the correcting capacitors

$$= (P_1 \tan\phi_1 - P_1 \tan\phi_2)\text{ kVAr}$$

Let the cost per annum in interest and depreciation on the capacitor installation be £C per kVAr or

$$£C(P_1 \tan\phi_1 - P_1 \tan\phi_2)$$

The net saving

$$= £\left[AP_1\left(\frac{1}{\cos\phi_1} - \frac{1}{\cos\phi_2}\right) - CP_1(\tan\phi_1 - \tan\phi_2)\right]$$

This saving is a maximum when

$$\frac{d(\text{saving})}{d\phi_2} = 0$$

i.e. when $\sin\phi_2 = C/A$.

It is interesting to note that the optimum power factor is independent of the original one. The improvement of load power factors in such a manner will help to alleviate the whole problem of var flow in the transmission system.

The main effect of transmitting power at non-unity power factors is as follows. It is evident from equation (2.9) that the voltage drop is largely determined by the reactive power (Q). The line currents are larger, giving increased I^2R losses and hence reduced thermal capability. One of the obvious places for the artificial injection of reactive power is at the loads themselves. In general, four methods of injection are available, involving the use of:

1. static shunt capacitors;
2. static series capacitors;
3. synchronous compensators;
4. current compensation by series injection.

5.4.1 Shunt capacitors and reactors

Shunt capacitors are used for lagging power-factor circuits, whereas reactors are used on those with leading power factors such as created by lightly loaded cables. In both cases the effect is to supply the requisite reactive power to maintain the values of the voltage. Capacitors are connected either directly to a busbar or to the tertiary winding of a main transformer and are disposed along the route to minimize the losses and voltage drops. Unfortunately, as the voltage falls, the vars produced by a shunt capacitor or reactor fall; thus, when needed most, their effectiveness falls. Also, on light loads when the voltage is high, the capacitor output is large and the voltage tends to rise to excessive levels, requiring some capacitors or cable circuits to be switched out by local overvoltage relays. Switched capacitors in parallel with semiconductor-controlled reactors provides modern variable var compensators for fast control of voltages in power systems.

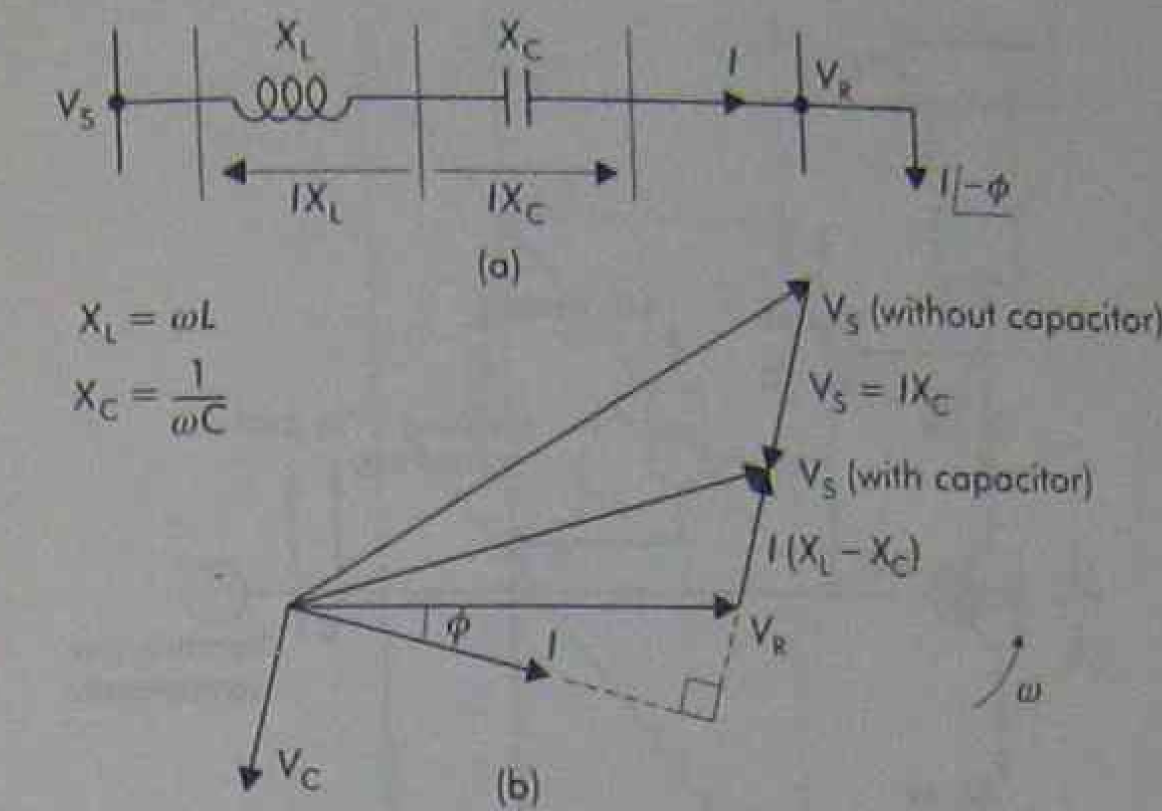

Figure 5.6 (a) Line with series capacitor, C load. (b) Phasor diagram for fixed V_R

5.4.2 Series capacitors

These are connected in series with the line conductors and are used to reduce the inductive reactance between the supply point and the load. One major drawback is the high overvoltage produced when a short-circuit current flows through the capacitor, and special protective devices need to be incorporated (e.g. spark gaps) and non-linear resistors. The phasor diagram for a line with a series capacitor is shown in Figure 5.6(b).

The relative merits between shunt and series capacitors may be summarized as follows:

1. If the load var requirement is small, series capacitors are of little use.
2. With series capacitors the reduction on line current is small; hence if thermal considerations limit the current, little advantage is obtained and shunt compensation should be used
3. If voltage drop is the limiting factor, series capacitors are effective; also, voltage fluctuations due to arc furnaces, etc., are evened out.
4. If the total line reactance is high, series capacitors are very effective and stability is improved.

5.4.3 Synchronous compensators

A synchronous compensator is a synchronous motor running without a mechanical load and, depending on the value of excitation, it can absorb or

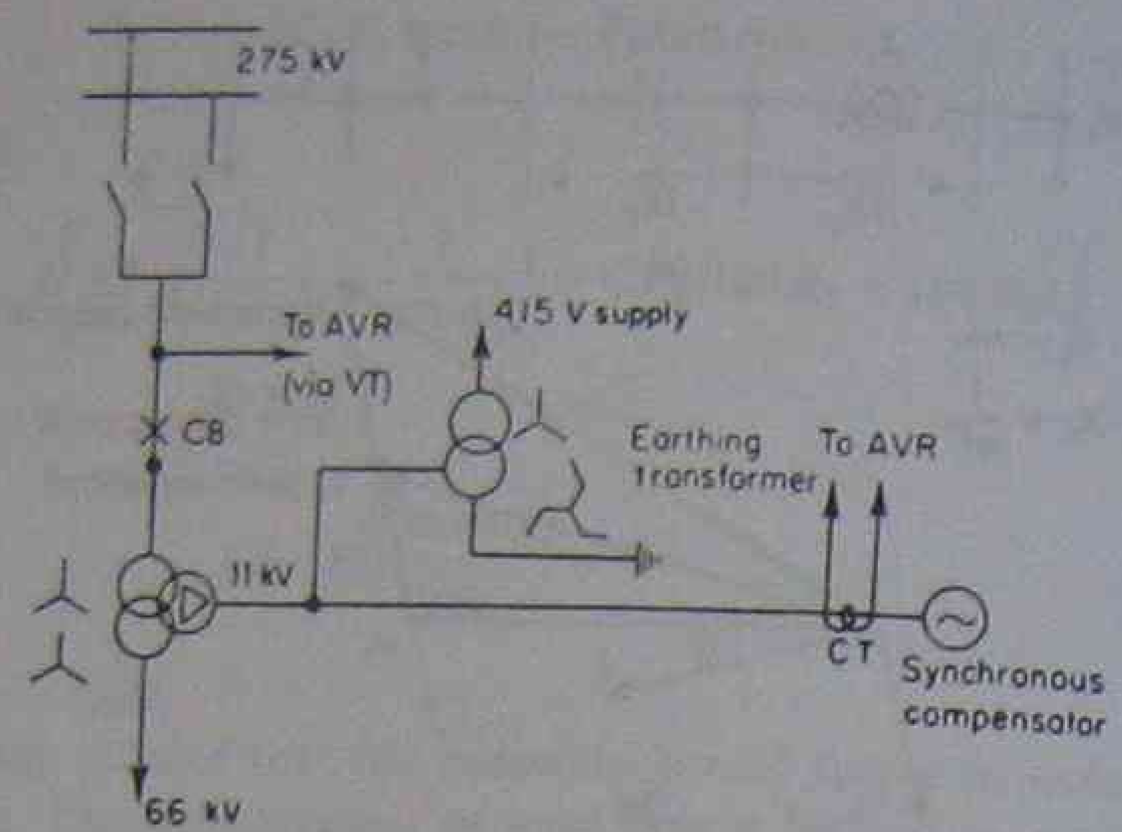

Figure 5.7 Typical installation with synchronous compensator connected to tertiary (delta) winding of main transformer. A neutral point is provided by the earthing transformer shown. The automatic voltage regulator on the compensator is controlled by a combination of the voltage on the 275 kV system and the current output; this gives a droop to the voltage-var output curve which may be varied as required

generate reactive power. As the losses are considerable compared with static capacitors, the power factor is not zero. When used with a voltage regulator the compensator can automatically run overexcited at times of high load and underexcited at light load. A typical connection of a synchronous compensator is shown in Figure 5.7 and the associated voltage–var output characteristic in Figure 5.8. The compensator is run up as an induction motor in 2.5 min and then sychronized.

A great advantage is the flexibility of operation for all load conditions. Although the cost of such installations is high, in some circumstances it is justified, e.g. at the receiving-end busbar of a long high-voltage line where transmission at power factors less than unity cannot be tolerated. Being a rotating machine, its stored energy is useful for riding through transient disturbances, including voltage sags.

5.4.4 Series injection

With the development of high-power, high-voltage semiconductor-controlled devices, including pulse turn-on and turn-off (e.g. IGBT, as discussed in Chapter 9), inverters are now being designed and constructed that can inject a voltage in series with a line whose angle can have any desired relation with the phase voltage. This is equivalent to the series capacitor case of Figure 5.6, except that V_C is not confined to being only 90° out of phase with the current

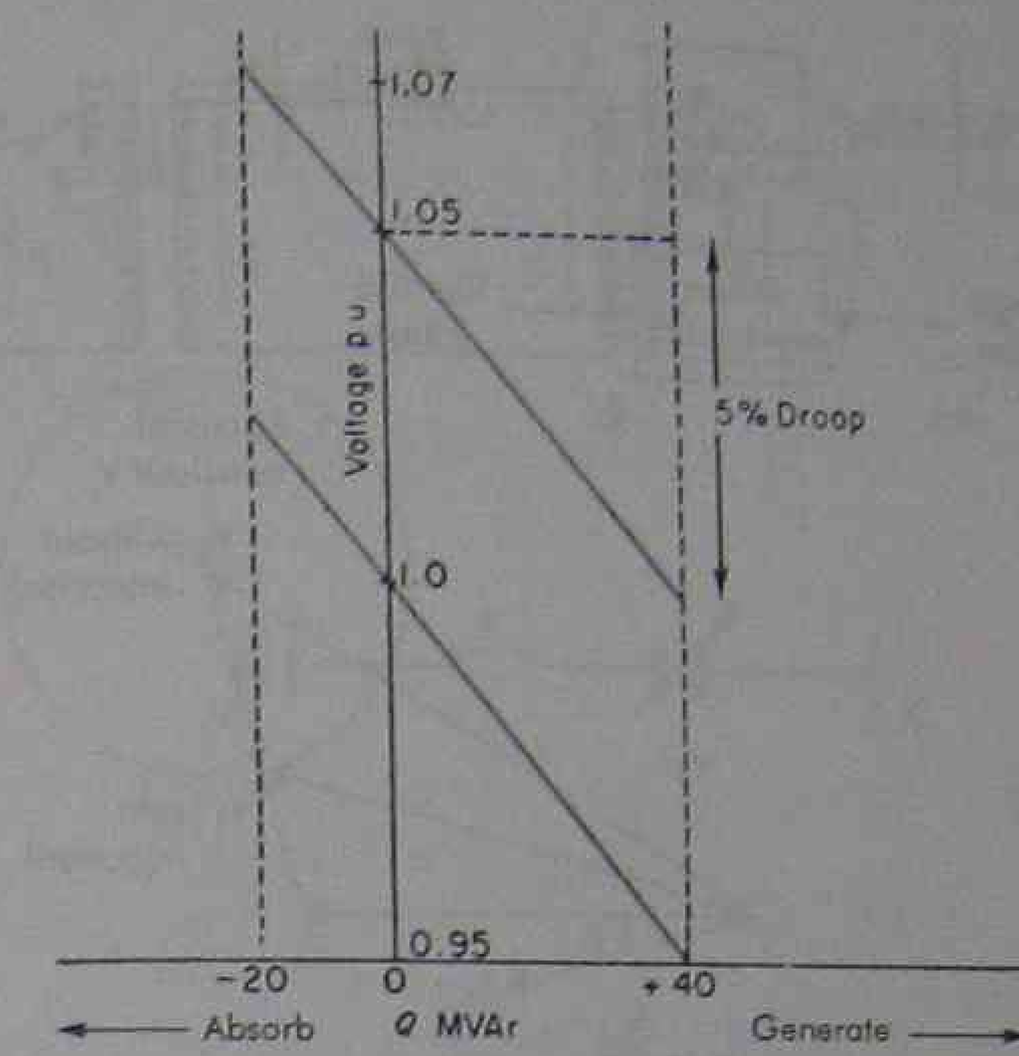

Figure 5.8 Voltage-reactive power output of a typical 40 MVAr synchronous compensator

and dependent upon the IX_C voltage rise. Such a regulator, known as a Universal Power Controller (UPC) is shown, in principle, in Figure 5.9.

It can be realised that if V_i is at 90° to the current I, then no energy is required from the source. At any other angle, energy is either drawn from the system or required from the source. Most conveniently, the source is a transformer connected to the system busbars feeding a rectifier, from which a sinusoidal injected voltage at the desired magnitude and angle is synthesized. Alternatively, the source could be a storage device (battery, capacitor, superconducting energy store, etc.), in which case some auxiliary charging may be necessary, but peak lopping of even uninterruptible power may also be provided.

5.5 Methods of Voltage Control: (ii) Tap-Changing Transformers

The basic operation of the tap-changing transformer has been discussed in Chapter 3; by changing the transformation ratio, the voltage in the secondary circuit is varied and voltage control is obtained. This constitutes the most popular and widespread form of voltage control at all voltage levels.

Consider the operation of a radial transmission system with two tap-changing transformers, as shown in the equivalent single-phase circuit of

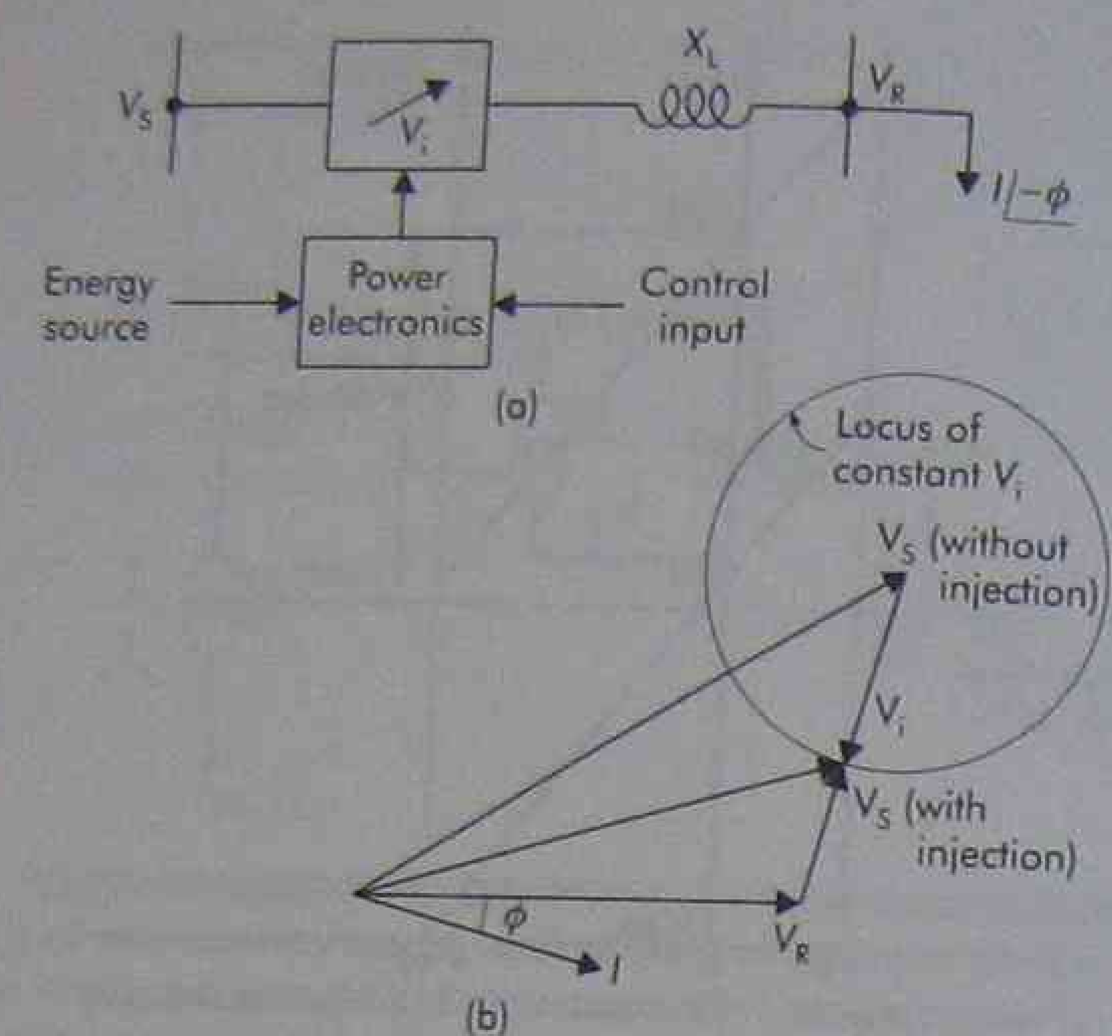

Figure 5.9 Principle of Universal Power Controller for series injection. (a) System diagram. (b) Phasor diagram

Figure 5.10. Here, t_s and t_r are fractions of the nominal transformation ratios, i.e. the tap ratio/nominal ratio. For example, a transformer of nominal ratio 6.6 to 33 kV when tapped to give 6.6 to 36 kV has a $t_s = 36/33 = 1.09$. V_1 and V_2 are the nominal voltages; at the ends of the line the actual voltages are t_sV_1 and t_rV_2. It is required to determine the tap-changing ratios required to completely compensate for the voltage drop in the line. The product t_st_r will be made unity; this ensures that the overall voltage level remains in the same order and that the minimum range of taps on both transformers is used.

(Note that all values are in per unit; t is the off-nominal tap ratio.)

Transfer all quantitites to the load circuit. The line impedance becomes $(R + jX)/t_r^2$; $V_s = V_1t_s$ and, as the impedance has been transferred, $V_r = V_1t_s$. The input voltage to the load circuit becomes V_1t_s/t_r and the equivalent circuit is as shown in Figure 5.10(c). The arithmetic voltage drop

$$= (V_1t_s/t_r) - V_2 \approx \frac{RP + XQ}{t_r^2V_2}$$

When $t_r = 1/t_s$,

$$t_s^2V_1V_2 - V_2^2 = (RP + XQ)t_s^2$$

and

$$V_2 = \tfrac{1}{2}[t_s^2V_1 \pm t_s(t_s^2V_1^2 - 4(RP + XQ))^{\frac{1}{2}}] \qquad (5.7)$$

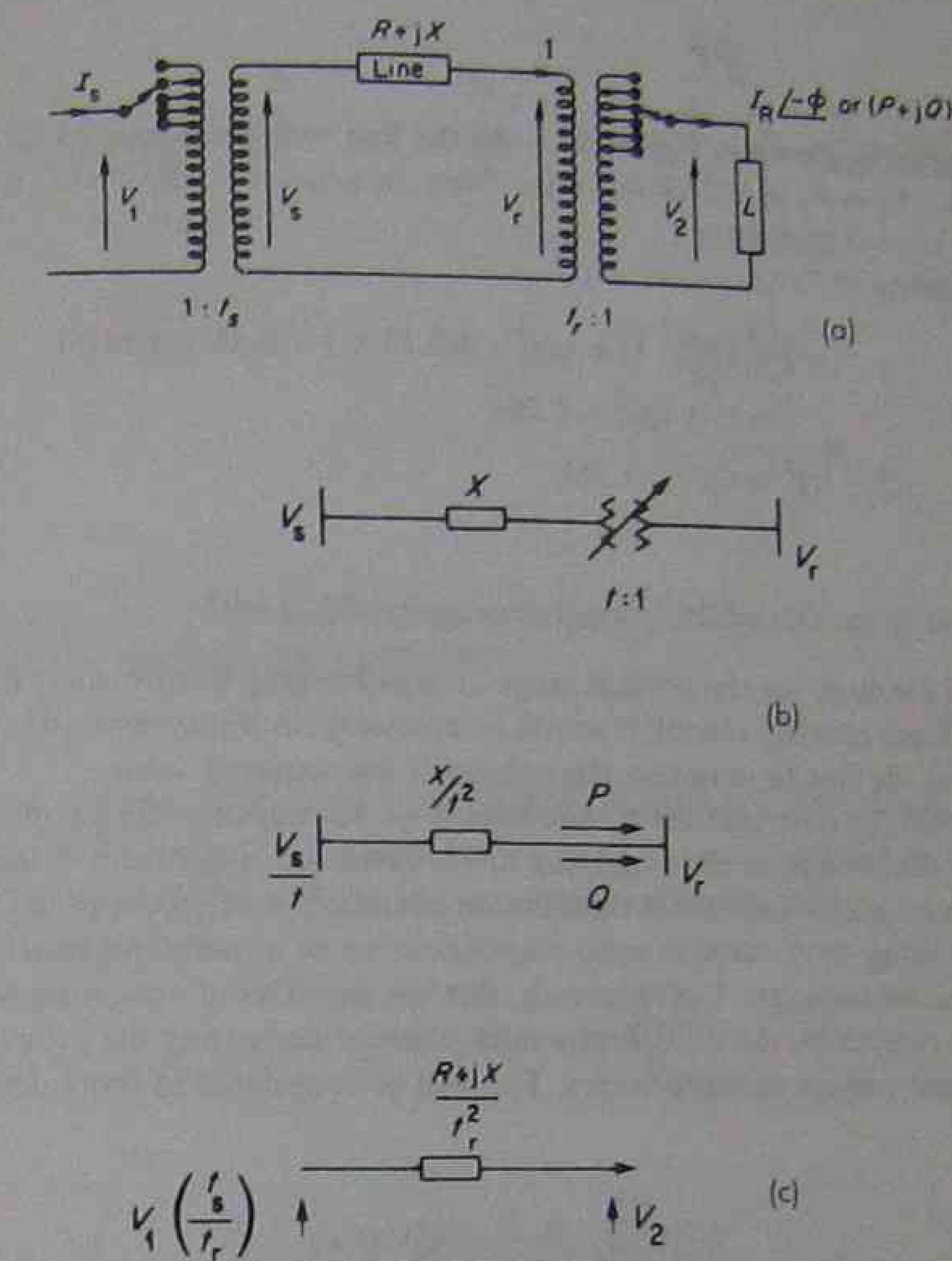

Figure 5.10 (a) Coordination of two tap-changing transformers in a radial transmission link (b) and (c) Equivalent circuits for dealing with off-nominal tap ratio. (b) Single transformer. (c) Two transformers

Hence if t_s is specified, there are two values of V_2 for a given V_1.

Example 5.3

A 13 kV line is fed through an 11/132 kV transformer from a constant 11 kV supply. At the load end of the line the voltage is reduced by another transformer of nominal ratio 132/11 kV. The total impedance of the line and transformers at 132 kV is (25 + j66) Ω. Both transformers are equipped with tap-changing facilities which are arranged so that the product of the two off-nominal settings is unity. If the load on the system is 100 MW at 0.9 p.f. lagging, calculate the settings of the tap-changers required to maintain the voltage of the load busbar at 11 kV. Use a base of 100 MVA.

Solution

The line diagram is shown in Figure 5.11. As the line voltage drop is to be completely compensated, $V_1 = V_2 = 132\,\text{kV} = 1$ p.u. Also, $t_s \times t_r = 1$. The load is 100 MW, 48.3 MVAr., i.e. $1 + j0.483$ p.u.

Using equation (5.7)

$$1 = \tfrac{1}{2}[(t_s^2 \cdot 1) \pm t_s(t_s^2 - 4(0.14 \times 1 + 0.38 \times 0.48))^{\frac{1}{2}}]$$
$$\therefore 2 = t_s^2 \pm t_s(t_s^2 - 1.28)^{\frac{1}{2}}$$
$$\therefore (2 - t_s^2)^2 = t_s^2(t_s^2 - 1.28)$$

Hence,

$$t_s = 1.21 \quad \text{and} \quad t_r = 1/1.21 = 0.83$$

These settings are large for the normal range of tap-changing transformers (usually not more than ±20 per cent tap range). It would be necessary, in this system, to inject vars at the load end of the line to maintain the voltage at the required value.

It is important to note that the transformer does not improve the var-flow position and also that the current in the supplying line is increased if the ratio is increased. In countries with long and inadequate distribution circuits, it is often the practice to boost the received voltage by a variable ratio transformer so as to maintain rated voltage as the power required increases. Unfortunately, this has the effect of increasing the primary supply circuit current by the transformer ratio, thereby decreasing the primary voltage still further until voltage collapse occurs. This will be considered in more detail later in this chapter.

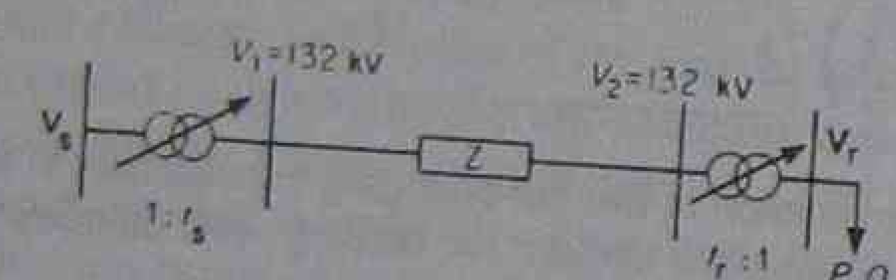

Figure 5.11 Schematic diagram of system for Example 5.3

5.6 Combined Use of Tap-Changing Transformers and Reactive-Power Injection

The usual practical arrangement is shown in Figure 5.12, where the tertiary winding of a three-winding transformer is connected to a compensator. For given load conditions it is proposed to determine the necessary transformation ratios with certain outputs of the compensator.

The transformer is represented by the equivalent star connection and any line impedance from V_1 or V_2 to the transformer can be lumped together with the transformer branch impedances. Here, V_n is the phase voltage at the star point of the equivalent circuit in which the secondary impedance (X_s) is usually

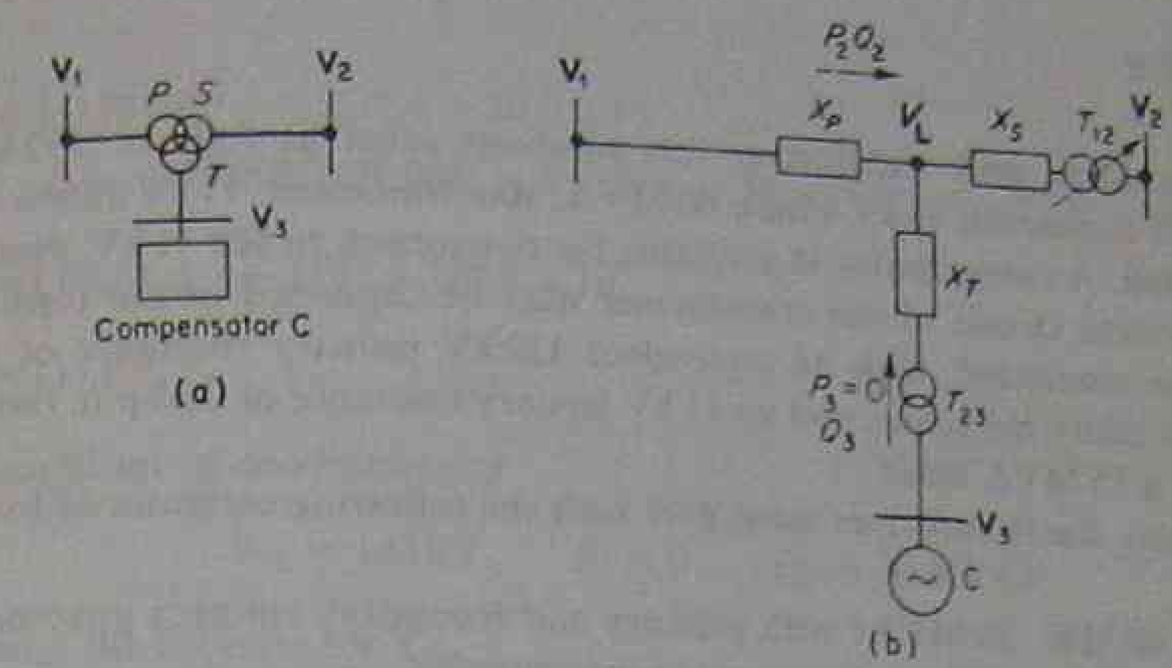

Figure 5.12 (a) Schematic diagram with combined tap-changing and synchronous compensation. (b) Equivalent network

approaching zero and hence is neglected. Resistance and losses are neglected. The allowable ranges of voltage for V_1 and V_2 are specified and the values of P_2, Q_2, P_3, and Q_3 are given; P_3 is usually taken as zero.

The volt drop V_1 to V_L

$$= \Delta V_p \approx X_p \frac{Q_2/3}{V_n}$$

or

$$\Delta V_p = X_p \frac{Q_2}{V_L\sqrt{3}},$$

where V_L is the line voltage $= \sqrt{(3)}V_n$, and Q_2 is the total vars. Also,

$$\Delta V_q = X_p \frac{P_2}{V_L\sqrt{3}}$$
$$\therefore (V_n + \Delta V_p)^2 + (\Delta V_q)^2 = V_1^2$$

(see phasor diagram of Figure 2.22; phase values used) and

$$\left(V_n + X_p \frac{Q_2}{V_L\sqrt{3}}\right)^2 + X_p^2\left(\frac{P_2^2}{3V_L^2}\right) = V_1^2$$
$$\therefore (V_L^2 + X_pQ_2)^2 + X_p^2P_2^2 = V_{1L}^2V_L^2$$

where V_{1L} is the line voltage $= \sqrt{(3)}V_1$

$$\therefore V_L^2 = \frac{V_{1L}^2 - 2X_pQ_2}{2} \pm \tfrac{1}{2}\sqrt{[V_{1L}^2(V_{1L}^2 - 4X_pQ_2) - 4 \cdot X_p^2P_2^2]}$$

Once V_L is obtained, the transformation ratio is easily found. The procedure is best illustrated by an example.

Example 5.4

A three-winding grid transformer has windings rated as follows: 132 kV (line), 75 MVA, star connected; 33 kV (line), 60 MVA, star connected; 11 kV (line), 45 MVA, delta connected. A compensator is available for connection to the 11 kV winding.

The equivalent circuit of the transformer may be expressed in the form of three windings, star connected, with an equivalent 132 kV primary reactance of 0.12 p.u., negligible secondary reactance, and an 11 kV tertiary reactance of 0.08 p.u. (both values expressed on a 75 MVA base).

In operation, the transformer must deal with the following extremes of loading:

1. Load of 60 MW, 30 MVAr with primary and secondary voltages governed by the limits 120 kV and 34 kV; compensator disconnected.
2. No load, Primary and secondary voltage limits 143 kV and 30 kV; compensator in operation and absorbing 20 MVAr.

Calculate the range of tap-changing required. Ignore all losses.

Solution

The value of X_p, the primary reactance (in ohms)

$$= 0.12 \times 132^2 \times 1000/75 \times 1000 = 27.8\ \Omega$$

Similarly, the effective reactance o the tertiary winding is 18.5 Ω. The equivalent star circuit is shown in Figure 5.13.

The first operating conditions are as follows:

$$P_1 = 60\,\text{MW} \qquad Q_1 = 30\,\text{MVAr} \qquad V_{1L} = 120\,\text{kV}$$

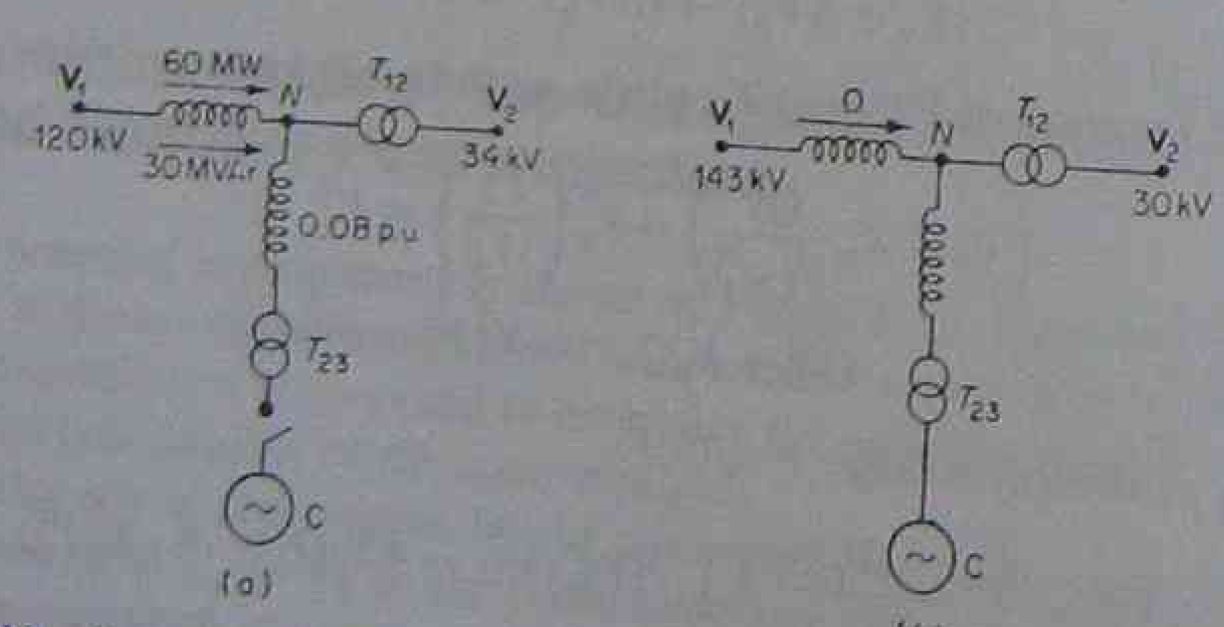

Figure 5.13 Systems for Example 5.4. (a) System with loading condition 1. (b) System with loading condition 2

Hence,

$$
\begin{aligned}
V_L^2 &= \tfrac{1}{2}(120\,000^2 - 2 \times 27.8 \times 30 \times 10^6)\\
&\quad \pm \tfrac{1}{2}\sqrt{[120\,000^2(120\,000^2 - 4 \times 27.8 \times 30 \times 10^6) - 4 \times 27.8^2 \times 60^2 \times 10^{12}]}\\
&= \left(63.61 \pm \frac{122}{2}\right)10^8 = 124.4 \times 10^8\\
\therefore V_L &= 111\,\text{kV}
\end{aligned}
$$

The second set of conditions are:

$$V_{1L} = 143\,\text{kV} \qquad P_2 = 0 \qquad Q_2 = 20\,\text{MVAr}$$

Again, using the formula for V_L,

$$V_L = 138.5\,\text{kV}$$

The transformation ratio under the first condition

$$= 111/34 = 3.27$$

and, for the second condition

$$\frac{138.5}{30} = 4.61$$

The actual raio will be taken as the mean of these extremes, i.e. 3.94, varying by ±0.67 or 3.94 ± 17 per cent. Hence the range of tap-changing required is ±17 per cent.

A further method of var production is the use of adjustment of tap settings on transformers connecting large interconnected systems. Consider the situation in Figure 5.14(a), in which V_s and V_r are constant voltages representing the two connected systems. The circuit may be rearranged as shown in Figure 5.14(b), where t is the off-nominal (per unit) tap setting; resistance is zero. The voltage drop between busbars

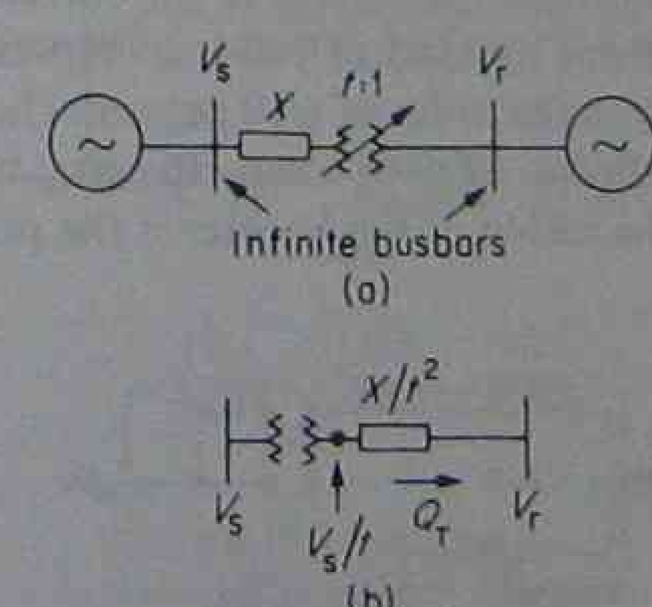

Figure 5.14 (a) Two power systems connected via a tap-change transformer. (b) Equivalent circuit with impedance transferred to receiver side

$$= \left(\frac{V_s}{t}\right) - V_r = \frac{X}{t^2} \cdot \frac{Q_T}{V_r}$$

Hence,

$$(V_s V_r t - V_r^2 t^2) 1/X = Q_T$$

and

$$t(1-t)V^2/X = Q_T \qquad \text{when} \qquad V_s = V_r = V$$

Also, $Q_T = t(1-t)S$, where S = short-circuit level, i.e. V^2/X. When

$t < 1$, Q_T is positive, i.e. a flow of vars into V_r

$t > 1$, Q_T is negative, a flow of vars out of V_r

Thus, by suitable adjustment of the tap setting, an appropriate injection of reactive power is obtained.

The idea can be extended to two transformers in parallel between networks. If one transformer is set to an off-nominal ratio of, say, 1:1.1 and the other to 1:0.8 (i.e. in opposite directions), then a circulation of reactive power occurs round the loop, resulting in a net absorption of vars. This is known as 'tap-stagger' and is a comparatively inexpensive method of var absorption.

5.7 Booster Transformers

It may be desirable, on technical or economic grounds, to increase the voltage at an intermediate point in a line rather than at the ends as with tap-changing transformers, or the system may not warrant the expense of tap-changing. Here, booster transformers are used as shown in Figure 5.15. The booster can be brought into the circuit by the closure of relay B and the opening of A, and vice versa. The mechanism by which the relays are operated can be controlled from a change in either the voltage or the current. The latter method is the more sensitive, as from no-load to full-load represents a 100 per cent change in current, but only in the order of a 10 per cent change in voltage. This booster gives an in-phase boost, as does a tap-changing transformer. An economic advantage is that the rating of the booster is the product of the current

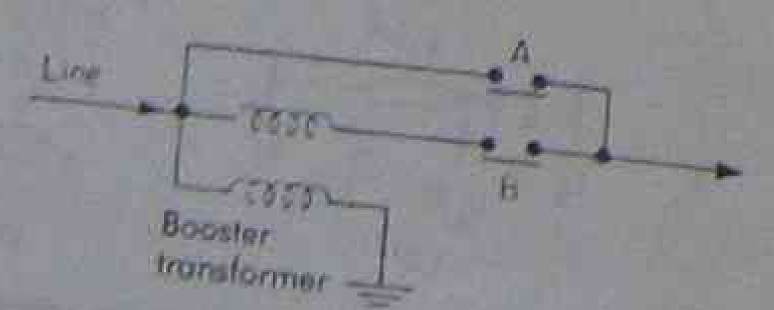

Figure 5.15 Connection of in-phase booster transformer. One phase only shown

and the injected voltage, and is hence only about 10 per cent of that of a main transformer. Boosters are often used in distribution feeders where the cost of tap-changing transformers is not warranted.

Example 5.5

In the system shown by the line diagram in Figure 5.16, each of transformers T_A and T_B have tap ranges of ± 10 per cent in 10 steps of 1.0 per cent. It is required to find the voltage boost needed on transformer T_A to share the power flow equally between the two lines.

The system data is as follows:

$$\text{All transformers: } X_T = 0.1 \text{ p.u.}$$

$$\left.\begin{aligned} \text{Transmission lines: } R = R' &= 0 \\ X &= 0.20 \text{ p.u.} \\ X' &= 0.15 \text{ p.u.} \end{aligned}\right\} \text{All to same base}$$

$$V_A = 1.1\underline{/5^\circ} \qquad V_B = 1.0\underline{/0^\circ}$$

Solution

We must first calculate the current sharing in the two parallel lines:

$$I_1 = \frac{1.1\underline{/5^\circ} - 1.0\underline{/0}}{j0.40} = 0.2397 - j0.2397$$

$$I_2 = \frac{1.1\underline{/5^\circ} - 1.0\underline{/0}}{j0.35} = 0.2740 - j0.2740$$

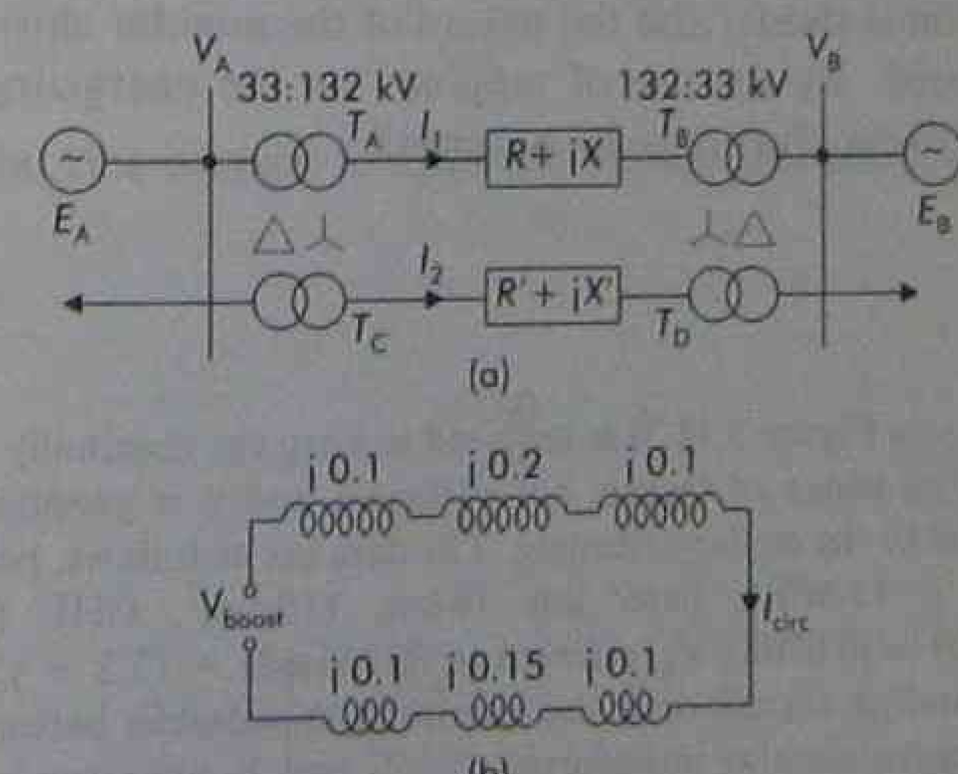

Figure 5.16 (a) Line diagram of system for Example 5.5. (b) Equivalent network with voltage boost V_{boost} acting

Any boost by transformer T_A will cause a current to circulate between the two busbars because the voltages V_A and V_B are assumed to be held constant by the voltage regulators on the generators.

To equalize the currents, a circulating current

$$I_{\text{circ}} = \frac{I_2 - I_1}{2}$$

is required, as in Figure 5.16(b), giving

$$I_{\text{circ}} = \frac{0.0343 - \mathrm{j}0.0343}{2} = 0.0241\underline{/-45^\circ}$$

$$\therefore V_{\text{Boost}} = 0.0241\underline{/-45} \times \mathrm{j}0.75$$

$$= 0.0180\underline{/45^\circ}\ V$$

To achieve this boost, ideally T_A should be equipped with a phase changer of 45° and taps to give 1.8 per cent boost. In practice, a tap of 2 per cent would be used in either an in-phase boost (such as obtainable from a normal tapped transformer) or a quadrature boost (obtainable from a phase-shift transformer—(see next section). In transmission networks it should be noted that because of the generally high X/R ratio, an in-phase boost gives rise to a quadrative current whereas a quadrature boost produces an in-phase circulating current, thereby adding to or subtracting from the real power flow. This technique has also been used to de-ice lines in winter by producing extra I^2R losses for heating. Alternatively, two transformers in parallel can be tap-staggered to produce I^2X absorption under light-load, high-voltage conditions.

5.7.1 Phase-shift transformer

A quadrature phase shift can be achieved by the connections shown in Figure 5.17(a). The booster arrangement shows the injection of voltage into one phase only; it is repeated for the other two phases. In Figure 5.17(b), the corresponding phasor diagram is shown and the nature of the angular shift of the voltage boost V'_{YB} indicated. By the use of tappings on the energizing transformer, several values of phase shift may be obtained.

Example 5.6

In the system shown in Figure 5.18, it is required to keep the nominally 11 kV busbar at constant voltage. The range of taps is not sufficient and it is proposed to use shunt capacitors connected to the tertiary winding. The data are as follows, per unit quantities being referred to a 15 MVA base: line 16 km, 115 mm², OHL (overhead line), 33 kV, $Z_L = (0.0304 = \mathrm{j}0.0702)$; Z_L referred to 33 kV side = $(2.2 + \mathrm{j}5.22)\,\Omega$.

For the three-winding transformer the measured impedances between the windings and the resulting equivalent star impedances Z_1, Z_2 and Z_3 are given in Table 5.1. The equivalent circuit referred to 33 kV is shown in Figure 5.18(b).

The voltage across the receiving-end load

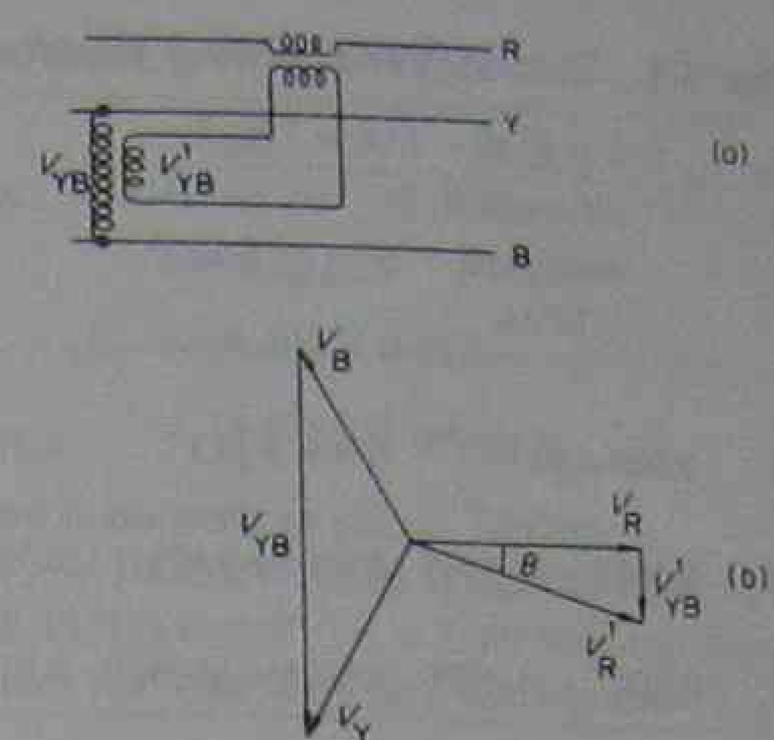

Figure 5.17 (a) Connections for one phase of a phase-shifting transformer. Similar connections for the other two phases. (b) Corresponding phasor diagram

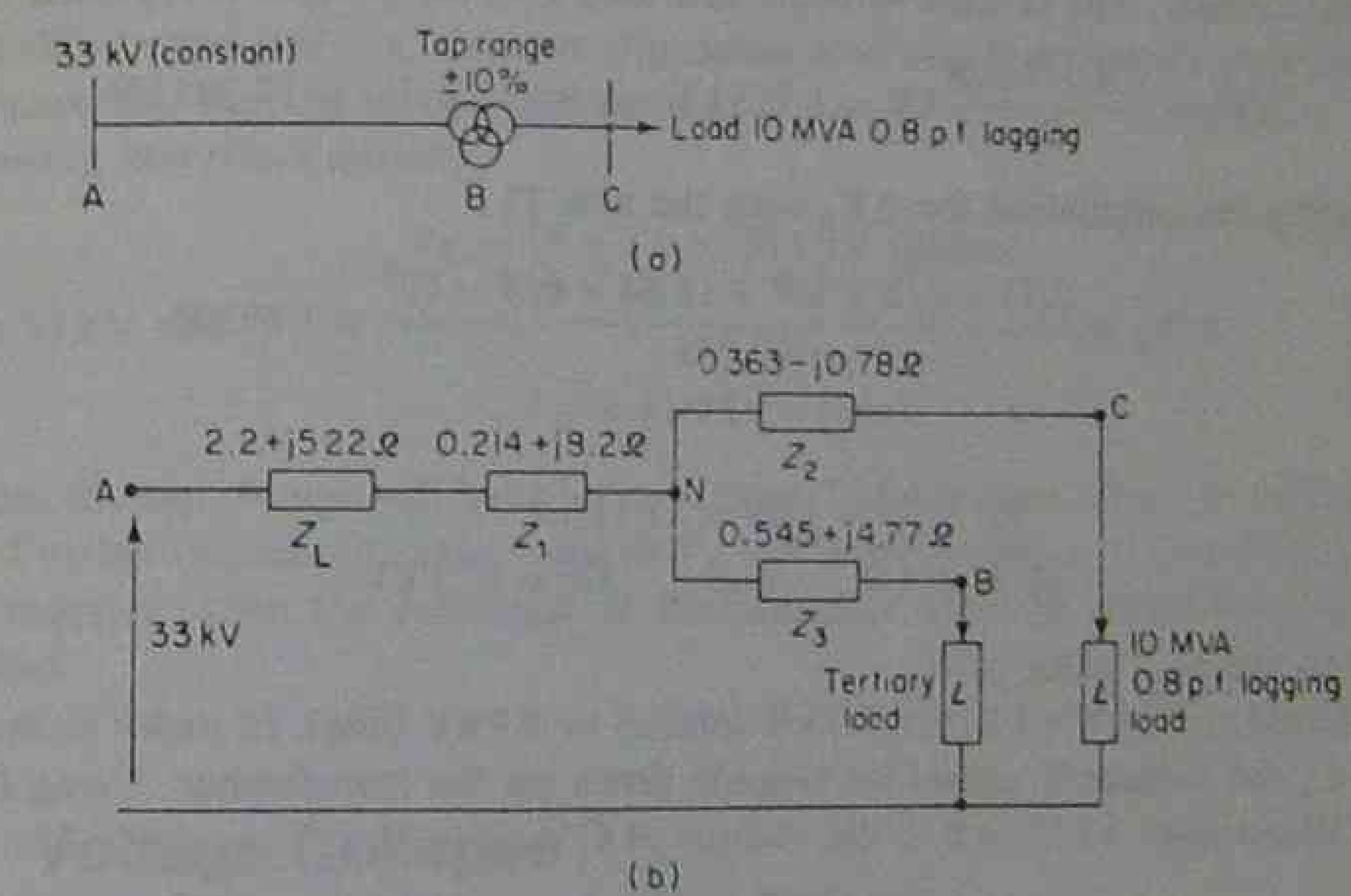

Figure 5.18 (a) Line diagram for Example 5.6. (b) Equivalent network—referred to 33kV

$$= \frac{33\,000}{\sqrt{3}} - \Delta V_p$$

where

$$\Delta V_p \approx \frac{RP + XQ}{V_C}$$

$$\therefore \Delta V_p \approx \frac{2.77 \times 8/3 \times 10^6 + 12.64 \times 6/3 \times 10^6}{33\,000/\sqrt{3}}$$

Table 5.1 Data for three-winding transformer

Winding	MVA	Voltage (kV)	p.u. Z referred to nameplate MVA	p.u. Z on 15 MVA base	$(Z(\Omega))$ referred to 33 kV	Equivalent $(Z(\Omega))$ referred to 33 kV
P-S	15	33/11	0.008 + j0.1	0.008 + j0.1	0.57 + j7.3	Z_1 = 0.214 + j8.2
P-T	5	33/1.5	0.0035 + j0.0595	0.0105 + j.179	0.76 + j4.32	Z_2 = 0.363 − j0.78
S-T	5	11/1.5	0.0042 + j0.0175	0.0126 + j0.0525	0.915 + j1.27	Z_3 = 0.545 + j4.77

As V_C referred to the 33 kV base is not known because of the system volt drop, 33 kV is assumed initially. The revised value is then used and the process is repeated.

$$\Delta V_p = \frac{7.4 + 25.28}{19}\,\text{kV} = 1.715\,\text{kV} \qquad \text{and} \qquad V_C = 17.285\,\text{kV}$$

Repeating the calculation for ΔV_p with the new V_C,

$$\Delta' V_p = \frac{2.77 \times 8/3 \times 10^6 + 12.64 \times 6/3 \times 10^6}{17.285} = 1.89\,\text{kV}$$

Hence

$$V_C' = 19 - 1.89 = 17.11\,\text{kV}$$
$$\Delta' V = 1.9 \qquad \text{and} \qquad V_C' = 17.1\,\text{kV}$$

This will be the final value of V_C.

V_C referred to 11 kV = 17.1/3 = 5.7 kV (phase) or 9.9 kV (line). In order to maintain 11 kV at C, the voltage is raised by tapping down on the transformer. Using the full range of 10 per cent, i.e. $t_r = 0.9$, the voltage at C is

$$\frac{29.7}{(33 \times 0.9)/11} = 11\,\text{kV}$$

The true voltage will be less than this as the primary current will have increased by (1/0.9) because of the change in transformer ratio. It is evident that the tap-changing transformer is not able to maintain 11 kV at C and the use of a static capacitor connected to the tertiary will be investigated.

Consider a shunt capacitor of capacity 5 MVAr (the capacity of the tertiary). Assume the transformer to be at its nominal ratio 33/11 kV. The voltage drop

$$= \frac{2.414 \times 8/3 \times 10^6 + 13.42 \times 1/3 \times 10^6}{V_N(\approx 19\,\text{kV})}$$
$$= 0.587\,\text{kV}$$
$$V_N' = 19 - 0.587 = 18.413\,\text{kV (phase)}$$
$$\therefore \Delta' V_N = 0.606 \qquad \text{and} \qquad V_N' = 18.394\,\text{kV}$$

Therefore the volt drop N to C

$$\Delta V_C = \frac{0.363 \times 8/3 - 0.78 \times 6/3}{18.394}\,\text{kV}$$
$$= -0.032\,\text{kV}$$
$$\therefore V_C = 18.394 + 0.032$$
$$= 18.426\,\text{kV (phase)}$$

As ΔV_C is so small there is no need to iterate further.

Referred to 11 kV, V_C = 10.55 kV (line). Hence, to have 11 kV the transformer will tap such that $t_r = (1 - 0.35/11) = 0.97$, i.e. a 3 per cent tap change, which is well within the range and leaves room for load increases. On *no-load*

$$\Delta V_p = \frac{2.959 \times 0 + 18.19 \times (-5.3)}{19}\,\text{kV}$$
$$= \frac{30.3}{19} = -1.595\,\text{kV (phase)}$$

The shunt capacitor is a constant impedance load and hence as V_N rises the current taken increases, causing further volt increase.

Ignoring this effect initially,

$$V_C = 19 + 1.6 = 20.6\,\text{kV (phase)}$$

On the 11 kV side

$$V_C = 11.85\,\text{kV (line)}$$

therefore the tap change will have to be at least 7.15 per cent, which is well within the range. Further refinement in the value of V_C will be unnecessary. If an accurate value of V_C is required, then the reactance of the capacitor must be found and the current evaluated.

5.8 Voltage Collapse

Voltage collapse is essentially an aspect of system stability to be discussed in Chapter 8. As the voltages to be maintained in a system are influenced by system stability it is appropriate to discuss the subject here.

Consider the circuit shown in Figure 5.19(a). If V_s is fixed (i.e. an infinite busbar supply), the graph of V_R against P for given power factors is as shown in Figure 5.19(b). In Figure 5.19(b), Z represents the series impedance of a 160 km long, double-circuit, 400 kV, 260 mm^2 conductor overhead line. The fact that two values of voltage exist for each value of power is easily demonstrated by considering the analytical solution of this circuit. At the lower voltage a very high current is taken to produce the power. The seasonal thermal ratings of the line are also shown, and it is apparent that for loads of power factor less than unity the possibility exists that, before the thermal rating is reached, the operating power may be on that part of the characteristic where

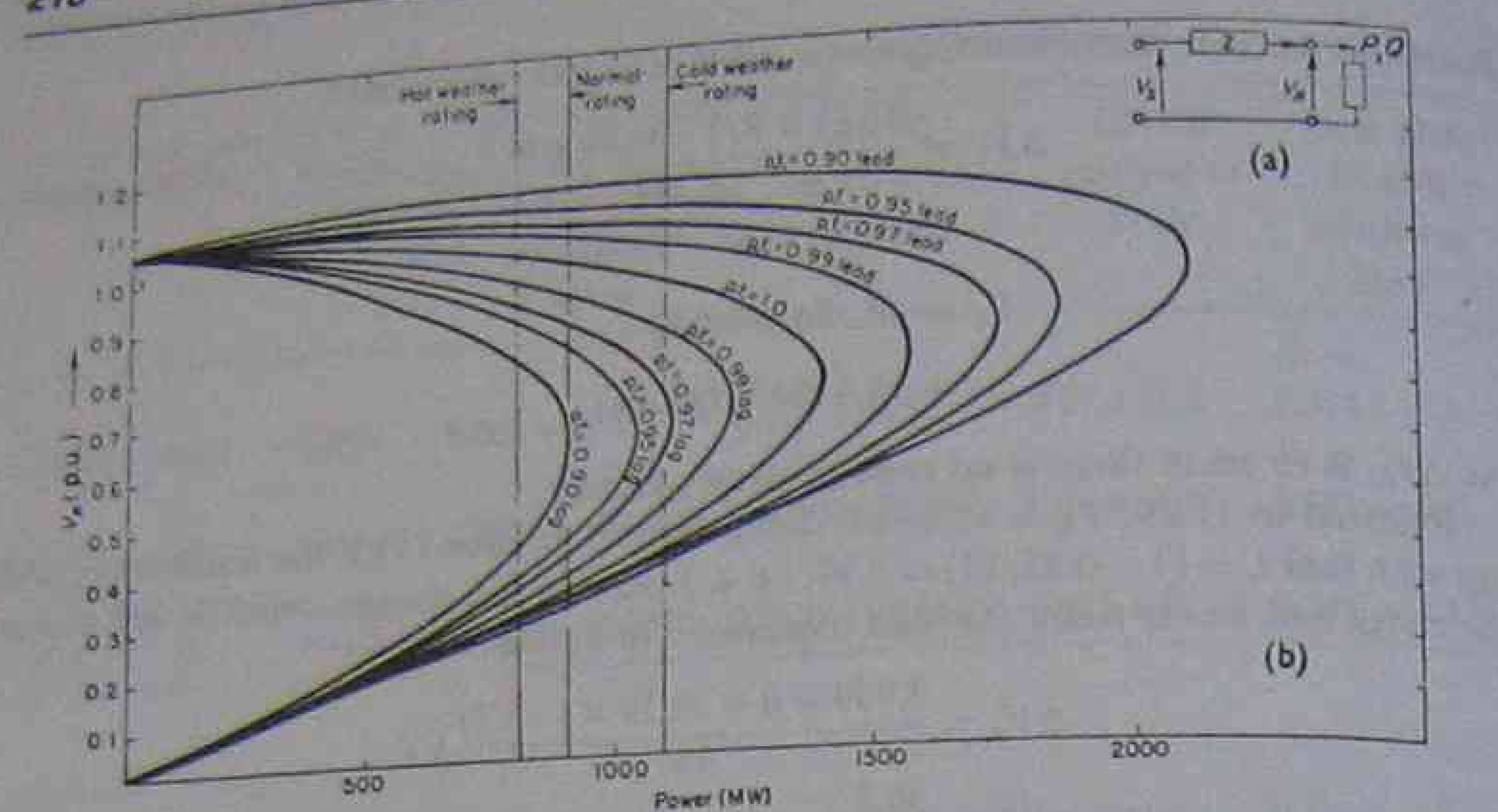

Figure 5.19 (a) Equivalent circuit of a line supplying a load $P + jQ$. (b) Relation between load voltages and received power at constant power factor for a 400 kV, $2 \times 260\,mm^2$ conductor line, 160 km in length. Thermal ratings of the line are indicated

small changes in load cause large voltage changes and *voltage instability* will have occurred. In this condition the action of tap-changing transformers is interesting. If the receiving-end transformers 'tap up' to maintain the load voltage, the line current increases, thereby causing further increase in the voltage drop. It would, in fact, be more profitable to 'tap down', thereby reducing the current and voltage drop. It is feasible therefore for a 'tapping-down' operation to result in increased secondary voltage, and vice versa.

The possibility of an actual voltage collapse depends upon the nature of the load. If this is stiff (constant power), e.g. induction motors, the collapse is aggravated. If the load is soft, e.g. heating, the power falls off rapidly with voltage and the situation is alleviated. Referring to Figure 5.19 it is evident that a critical quantity is the power factor; at full load a change in lagging power factor from 0.99 to 0.90 will precipitate voltage collapse. On long lines, therefore, for reasonable power transfers it is necessary to keep the power factor of transmission approaching unity, certainly above 0.97 lagging, and it is economically justifiable to employ var injection by static capacitors, synchronous compensators or UPC close by the load.

A problem arises with the operation of two or more lines in parallel, e.g. the system shown in Figure 5.20, in which the shunt capacitance has been represented as in a π section. If one of the three lines is removed from the circuit because of a fault, the total system reactance will increase from $X/3$ to $X/2$, and the capacitance, which normally improves the power factor, decreases to $2C$ from $3C$. Thus the overall voltage drop is greatly increased and, owing to the increased I^2X loss of the lines and the decreased generation of vars by the

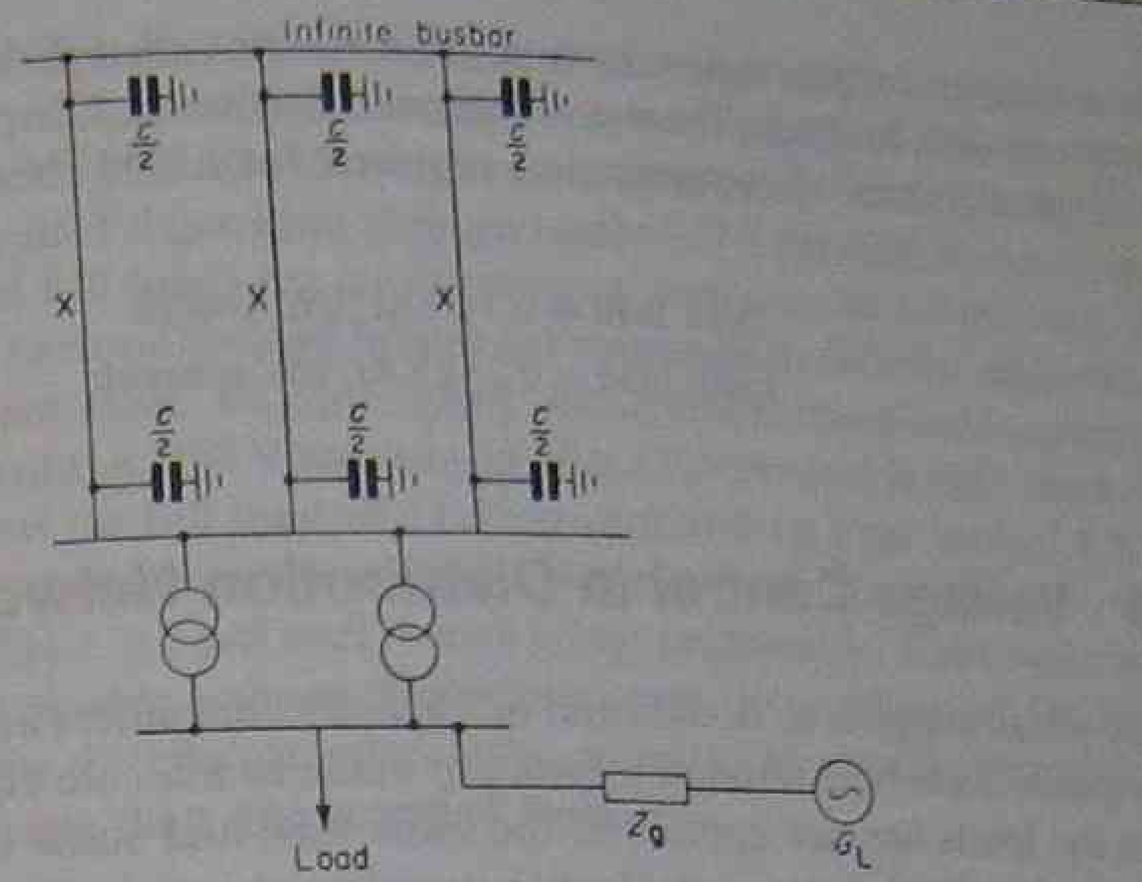

Figure 5.20 Line diagram of three long lines in parallel—effect of the loss of one line. G_L = local generators

shunt capacitances, the power factor decreases; hence the possibility of voltage instability. The same argument will, of course, apply to two lines in parallel.

Usually, there will be local generation or compensation feeding the receiving-end busbars at the end of long lines. If this generation is electrically close to the load busbars, i.e. low connecting impedance Z_g, a fall in voltage will automatically increase the local var generation, and this may be sufficient to keep the reactive power transmitted low enough to avoid large voltage drops in the long lines. Often, however, the local generators supply lower voltage networks and are electrically remote from the high-voltage busbar of Figure 5.20, and Z_g is high. The fall in voltage now causes little change in the local generator var output and the use of static-controlled capacitors at the load may be required. As Z_g is inversely proportional to the three-phase short-circuit level at the load busbar because of the local generation, the reactive-power contribution of the local machines is proportional to this fault level. When a static or synchronous compensator reaches its rated limit, voltage can no longer be controlled and rapid collapse of voltage can follow because any vars demanded by the load must now be supplied from sources further away electrically over the high-voltage system.

In the U.K. and some other countries where 'unbundled' systems are now extant, many generators are some distance from the load-centres. Consequently, the transmission system operator is required to install local flexible var controllers or compensators to maintain a satisfactory voltage at the delivery substations supplying the local distribution systems. Such flexible controllers, based on semiconductor devices which can vary the var absorption in a reactor or generation in a capacitor, are called FACTS (Flexible a.c. Transmission System). These are discussed further in Chapter 9.

Typical values of compensation required for a 400 kV or 500 kV network are:

Peak load = 0.3 kVAr/kW absorb

Light load = 0.25 kVAr/kW generate

5.9 Voltage Control in Distribution Networks

Single-phase supplies to houses and other small consumers are tapped off from three-phase feeders. Although efforts are made to allocate equal loads to each phase the loads are not applied at the same time and some unbalance occurs.

In the distribution network (British practice) shown in Figure 5.21 an 11 kV distributor supplies a number of lateral feeders in which the voltage is 420 V, and then each phase, loaded separately.

The object of design is to keep the consumers' nominal 415 V supply within the statutory ±6 per cent of the declared voltage. The main 33/11 kV transformer gives a 5 per cent rise in voltage between no-load and full-load. The distribution transformers have taps that are only adjustable off-load, and a

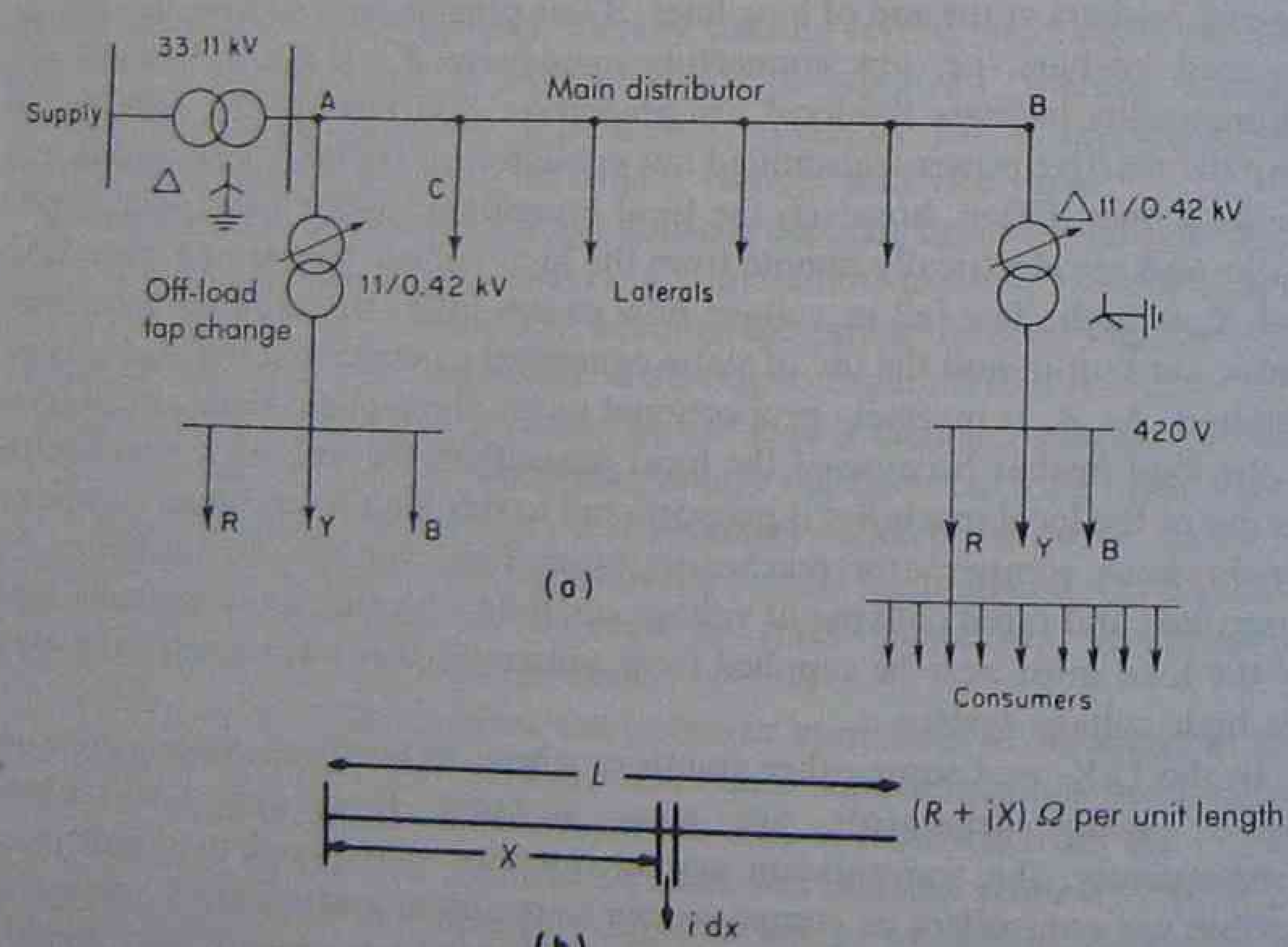

Figure 5.21 Line diagrams of typical radial distribution schemes

secondary phase voltage of 250 V which is 4 per cent high on the nominal value of 240 V. A typical distribution of voltage drops would be as follows: main distributor, 6 per cent; 11/0.42 kV transformer, 3 per cent; 420 V circuit, 7 per cent; consumer circuit, 1.5 per cent; giving a total of 17.5 per cent. On very light load (10 per cent of full load) the corresponding drop may be 1.5 per cent. To offset these drops, various voltage boosts are employed as follows: main transformer, +5 per cent (zero on light load); distribution transformer, inherent boost of +4 per cent (i.e. 250 V secondary) plus a 2.5 per cent boost. These add to give a total boost on full load of 11.5 per cent and on light load of 6.5 per cent. Hence the consumers' voltage varies between (−17.5 + 11.5), i.e. −6 per cent and (6.5 − 1.5) i.e. +5 per cent, which is just permissible. There will also be a difference in consumer voltage depending upon the position of the lateral feeder on the main distributor; obviously, a consumer supplied from C will have a higher voltage than one supplied from B.

5.9.1 *Uniformly loaded feeder from one end*

In areas with high load densities a large number of tappings are made from feeders and a uniform load along the length of a feeder may be considered to exist. Consider the voltage drop over a length dx of the feeder distant x metres from the supply end. Let iA be the current tapped per unit length and R and X be the resistance and reactance per phase per metre, respectively. The length of the feeder is L (m) (see Figure 5.21(b)).

The voltage across $dx = Rix\,dx\cos\phi + Xix\,dx\sin\phi$, where $\cos\phi$ is the power factor (assumed constant) of the uniformly distributed load.

The total voltage drop

$$= R\int_0^L ix\,dx\cos\phi + X\int_0^L ix\,dx\sin\phi$$

$$= R_i\frac{L^2}{2}\cos\phi + Xi\frac{L^2}{2}\sin\phi$$

$$= \frac{LR}{2}I\cos\phi + \frac{LX}{2}I\sin\phi$$

where $I = Li$, the total current load. Hence the uniformly distributed load may be represented by the total load tapped at the centre of the feeder length.

5.10 Long Lines

On light loads the charging volt-amperes of a line exceed the inductive vars consumed and the voltage rises, causing problems for generators. With very long lines the voltage drop can be massive. A length of 1500 km at 50 Hz corresponds to a quarter-wavelength line. Series capacitors would normally be installed to improve the power capacity and these effectively shorten the line electrically. In addition, shunt reactors are switched in circuit at times of light load to absorb the generated vars.

A 500 km line can operate within ± 10 per cent voltage variation without shunt reactors. However, with, say, a 800 km line, shunt reactors are essential and the effects of these are shown in Figure 5.22. For long lines in general, it is usual to divide the system into sections with compensation at the ends of each section. This controls the voltage profile, helps switching, and reduces short-circuit currents. Shunt compensation can be varied by switching discrete amounts of inductance. A typical 500 kV, 1000 km scheme uses compensation totalling 1200 MVAr.

Improvement in voltage profile may be obtained by compensation, using FACTS devices, at intermediate points, as well as at the ends of the line, as shown in Figure 5.22. If the natural load is transmitted there is, of course, constant voltage along the line with no compensation. If the various busbars of a sectioned line can be maintained at *constant voltage* regardless of load, each section has a theoretical maximum transmission angle of 90°. Thus, for a three-section line a total angle of much greater than 90° would be possible. This is illustrated in Figure 5.23 for a three-section, 1500 km line with a unity power-factor load.

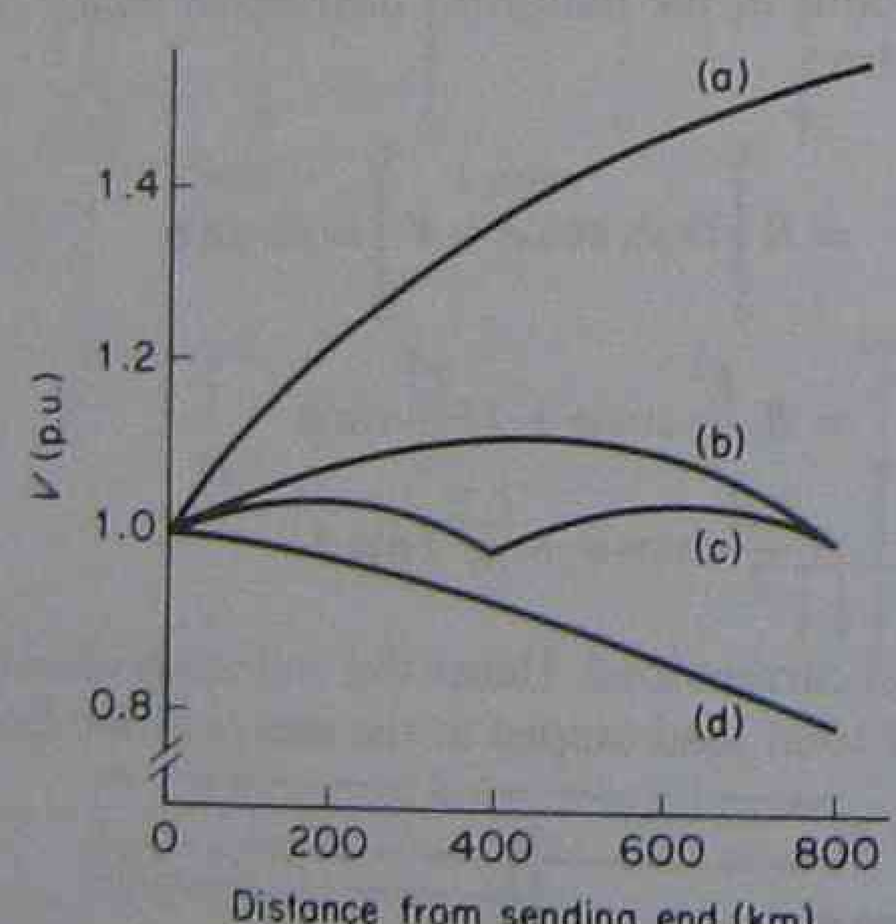

Figure 5.22 Voltage variation along a long line; (a) on no load with no compensation; (b) on no load with compensation at ends; (c) on no load with compensation at ends and at centre. (d) Transmitting natural load, compensation at ends and centre

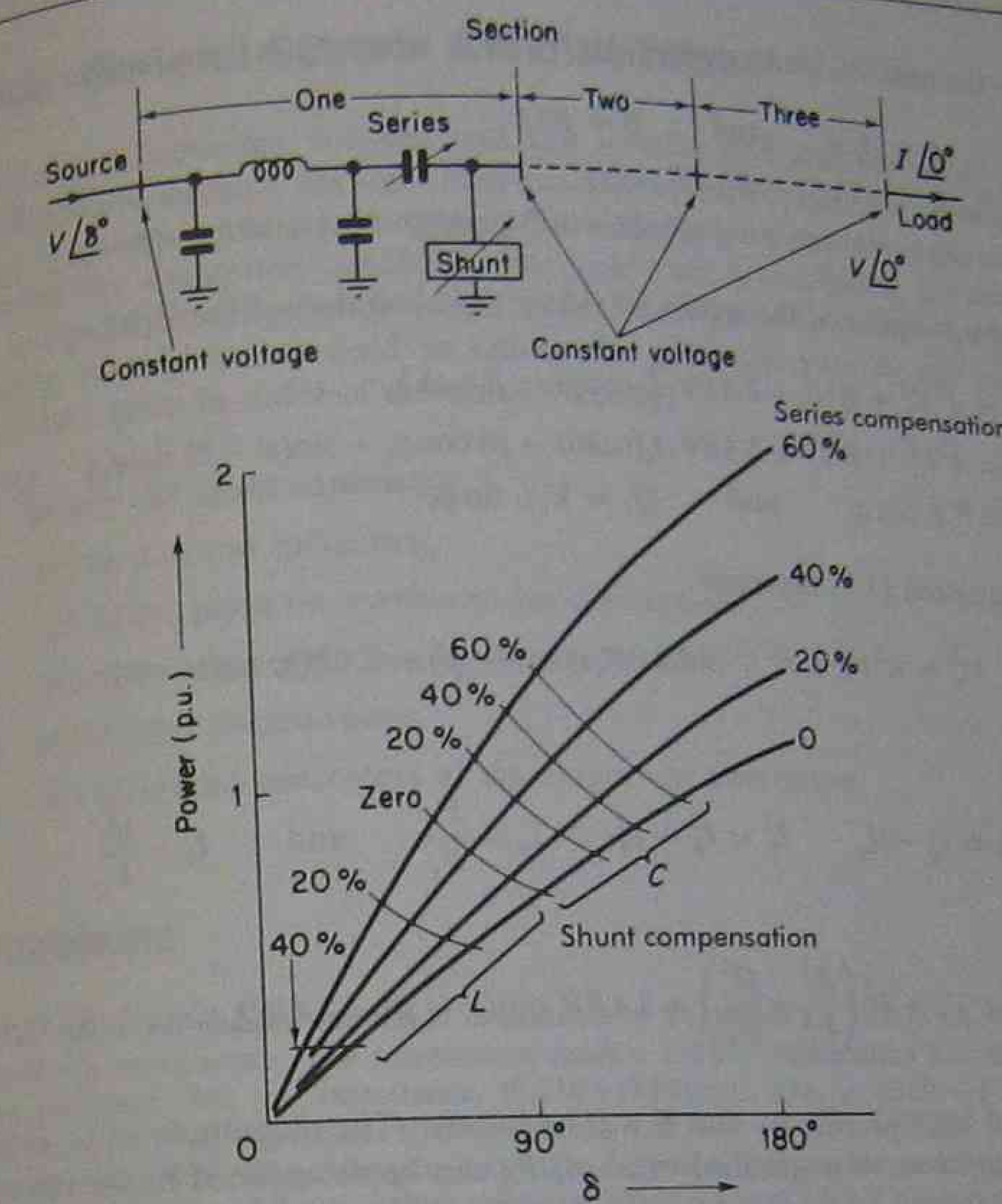

Figure 5.23 Power-angle curves for 1500 km line in three sections. Voltages at section-busbars maintained constant by variable compensation. Percentage of series and shunt compensation indicated (Reproduced by permission of the *Institution of Electrical Engineers*)

5.10.1 *Reactive-power requirements for the voltage control of long lines*

It is advantageous to use the generalized line equation,

$$\mathbf{V}_S = \mathbf{A}\mathbf{V}_r + \mathbf{B}\mathbf{I}_r$$

which takes into account the presence of transformers.

Let the received load current $\mathbf{I}_r$ lag $\mathbf{V}_r$ by ϕ_r where $\mathbf{V}_r$ is the reference phasor

$$\mathbf{A} = A\angle\alpha \quad \text{and} \quad \mathbf{B} = B\angle\beta$$

$$V_s\angle\delta_s = AV_r\angle\alpha + BI_r\angle(\beta - \phi_r)$$

$$\therefore V_s = AV_r\cos\alpha + \mathrm{j}AV_r\sin\alpha + BI_r\cos(\beta - \phi_r) + \mathrm{j}BI_r\sin(\beta - \phi_r)$$

As the modulus of the left-hand side is equal to that of the right

$$\begin{aligned} V_s^2 &= A^2V_r^2 + B^2I_r^2 + 2ABV_rI_r\cos(\alpha = \beta + \phi_r) \\ &= A^2V_r^2 + B^2I_r^2 + 2ABV_rI_r[\cos(\alpha - \beta)\cos\phi_r - \sin(\alpha - \beta)\sin\phi_r] \end{aligned} \tag{5.8}$$

$$P_r = V_rI_r\cos\phi_r \quad \text{and} \quad Q_r = V_rI_r\sin\phi_r$$

Hence equation (5.8) becomes

$$V_s^2 = A^2V_r^2 + B^2I_r^2 + 2ABP_r\cos(\alpha - \beta) - 2ABQ_r\sin(\alpha - \beta)$$

Also

$$\mathbf{I}_r = I_p - \mathrm{j}I_q \qquad I_r^2 = I_p^2 + I_q^2 \qquad I_p = \frac{P_r}{V_r} \quad \text{and} \quad I_q = \frac{Q_r}{V_r}$$

$$\therefore V_s^2 = A^2V_r^2 + B^2\left(\frac{P_r^2}{V_r^2} + \frac{Q_r^2}{V_r^2}\right) + 2ABP_r\cos(\alpha - \beta) - 2ABQ_r\sin(\alpha - \beta) \tag{5.9}$$

For a given network, P_r, **A**, and **B** will be known. The magnitude of Q_r such that V_r is equal to, or a specified ratio of, V_S can be determined by the use of equation (5.9).

5.10.2 Subsynchronous oscillation

The combination of series capacitors and the natural inductance of the line (plus that of the connected systems) creates a resonant circuit of subsynchronous resonant frequency. This resonance can interact with the generator-shaft critical-torsional frequency, and a mechanical oscillation is superimposed on the rotating generator shaft that may have sufficient magnitude to cause mechanical failure.

Subsynchronous resonance has been reported, caused by line-switching in a situation where trouble-free switching was normally carried out with all capacitors in service, but trouble occurred when one capacitor bank was out of service. Although this phenomena may be a rare occurrence, the damage resulting is such that, at the design stage, an analysis of possible resonance effects is required.

5.11 General System Considerations

Because of increasing voltages and line lengths, and also the wider use of underground circuits, the light-load reactive-power problem for an interconnected system becomes substantial, particularly with modern generators of limited var absorption capability. At peak load, transmission systems need to increase their var generation, and as the load reduces to a minimum (usually during the night) they need to reduce the generated vars by the following methods, given in order of economic viability:

1. switch out shunt capacitors;
2. switch in shunt inductors;
3. run hydro plant on maximum var absorption;
4. switch out one cable in a double-circuit link;
5. tap-stagger transformers;
6. run base-load generators at maximum var absorption.

Problems

5.1. An 11 kV supply busbar is connected to an 11/132 kV, 100 MVA, 10 per cent reactance transformer. The transformer feeds a 132 kV transmission link consisting of an overhead line of impedance (0.014 + j0.04) p.u. and a cable of impedance (0.03 + j0.01) p.u. in parallel. If the receiving end is to be maintained at 132 kV when delivering 80 MW, 0.9 p.f. lagging, calculate the power and reactive power carried by the cable and the line. All p.u. values relate to 100 MVA and 132 kV bases.
(Answer: Line (23 + j38) MVA; cable (57 + j3.8) MVA)

5.2 A three-phase induction motor delivers 500 hp at an efficiency of 0.91, the operating power factor being 0.76 lagging. A loaded synchronous motor with a power consumption of 100 kW is connected in parallel with the induction motor. Calculate the necessary kVA and the operating power factor of the synchronous motor if the overall power factor is to be unity.
(Answer: 365 kVA, 0.274)

5.3 The load at the receiving end of a three-phase overhead line is 25 MW, power factor 0.8 lagging, at a line voltage of 33 kV. A synchronous compensator is situated at the receiving end and the voltage at both ends of the line is maintained at 33 kV. Calculate the MVAr of the compensator. The line has resistance of 5 Ω per phase and inductive reactance of 20 Ω per phase.
(Answer: 25 MVAr)

5.4 A transformer connects two infinite busbars of equal voltage. The transformer is rated at 500 MVA and has a reactance of 0.15 p.u. Calculate the var flow for a tap setting of (a) 0.85:1; (b) 1.1:1.

(Answer: (a) 427 MVAr; (b) −367 MVAr)

5.5 A three-phase transmission line has resistance and inductive reactance of 25 Ω and 90 Ω, respectively. With no load at the receiving end, a synchronous compensator there takes a current lagging by 90°; the voltage is 145 kV at the sending end and 132 kV at the receiving end. Calculate the value of the current taken by the compensator.

When the load at the receiving end is 50 MW, it is found that the line can operate with unchanged voltages at the sending and receiving ends, provided that the compensator takes the same current as before, but now leading by 90°.

Calculate the reactive power of the load.
(Answer: 83.5 A; Q_L 24.2 MVAr)

5.6 Repeat Problem 5.3 making use of $\partial Q/\partial V$ at the receiving end.

5.7 In Example 5.3, determine the tap ratios if the receiving-end voltage is to be maintained at 0.9 p.u. of the sending-end voltage.
(Answer: $1.19t_s$, $0.84t_r$)

5.8 In the system shown in Figure 5.24, determine the supply voltage necessary at D to maintain a phase voltage of 240 V at the consumer's terminals at C. The data in Table 5.2 apply.
(Answer: 33 kV)

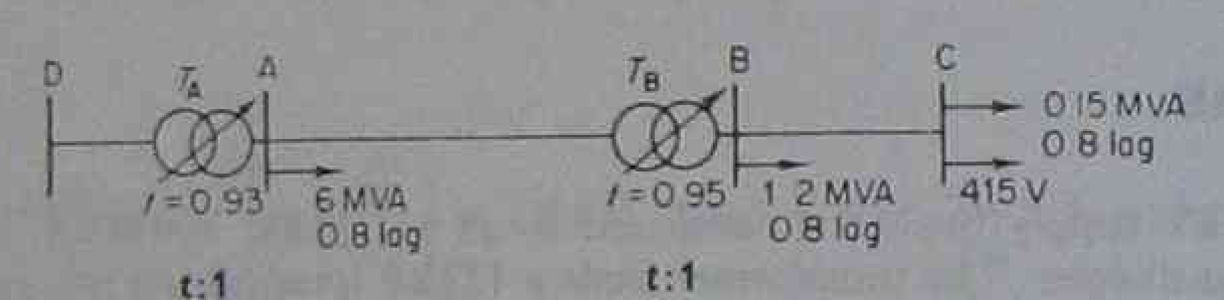

Figure 5.24 Line diagram for system in Problem 5.8

Table 5.2 Data for Problem 5.8

Line or/ transformers	Rated voltage (kV)	Transmission rating	Nominal tap ratio	Impedance (Ω)
BC	0.415			0.0127 + j0.00909
AB	11			1.475 + j2.75
DA	33			1.475 + j2.75
T_A	33/11	10 MVA	30.69/11	1.09 + j9.8 referred to 33 kV
T_B	11/0.415	2.5 MVA	10.450/0.415	0.24 + j1.95 referred to 33 kV

5.9 A load is supplied through a 275 kV link of total reactance 50 Ω from an infinite busbar at 275 kV. Plot the receiving-end voltage against power graph for a constant load power factor of 0.95 lagging. The system resistance may be neglected.

5.10 (a) Describe two methods of controlling voltage in a power system.
(b) Show how the scalar voltage difference between two nodes in a network is given approximately by:

$$\Delta V = \frac{RP + XQ}{V}$$

(c) Each phase of a 50 km, 132 km overhead line has a series resistance of 0.156 Ω/km and an inductive reactance of 0.4125 Ω/km. At the receiving end the voltage is 132 kV with a load of 100 MVA at a power factor of 0.9 lagging. Calculate the magnitude of the sending-end voltage.
(d) Calculate also the approximate angular difference between the sending-end and receiving-end voltages.
(Answer: (c) 144.5 kV; (d) 4.55°)
(From E.C. Examination 1997)

5.11 Explain the limitations of tap-changing transformers. A transmission link (Figure 5.25(a) connects an infinite busbar supply of 400 kV to a load busbar supplying 1000 MW, 400 MVAr. The link consists of lines of effective impedance (7 + j70) Ω feeding the load busbar via a transformer with a maximum tap ratio of 0.9:1. Connected to the load busbar is a compensator. If the maximum overall voltage drop is to be 10 per cent with the transformer taps fully utilized, calculate the reactive power requirement from the compensator.
(Answer: 148 MVAr)
Note: Refer voltage and line Z to load side of transformer in Figure 5.25(b).

$$V_R = \frac{V_s}{t} - \left(\frac{\frac{RP}{t^2} + \frac{X}{t^2}Q}{V_R}\right)$$

5.12 A generating station consists of four 500 MW, 20 kV, 0.95 p.f. (generate) generators, each feeding through a 525 MVA, 0.1 p.u. reactance transformer onto a common busbar. It is required to transmit 2000 MW at 0.95 p.f. lagging to a substation

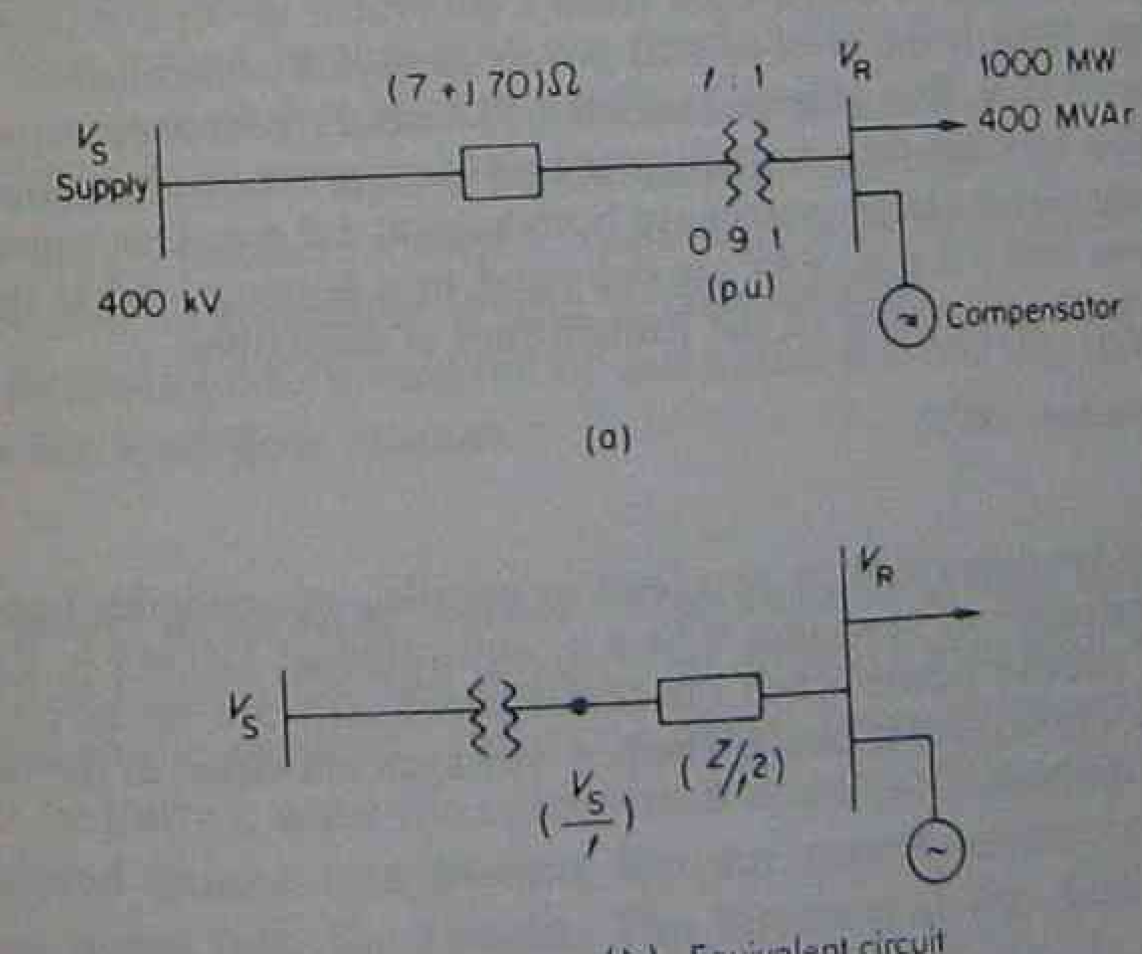

Figure 5.25 Circuits for Problem 5.11

maintained at 500 kV in a power system at a distance of 500 km from the generating station. Design a suitable transmission link of nominal voltage 500 kV to achieve this, allowing for a reasonable margin of stability and a maximum voltage drop of 10 per cent. Each generator has synchronous and transient reactances of 2 p.u. and 0.3 p.u. respectively, and incorporates a fast-acting automatic voltage regulator. The 500 kV transmission lines have an inductive reactance per phase of 0.4 Ω/km and a shunt capacitive reactance per phase of 0.3×10^6 Ω/km. Both series and shunt capacitors may be used if desired and the number of three-phase lines used should be not more than three—fewer if feasible. Use approximate methods of calculation, ignore resistance, and state clearly any assumption made. Assume shunt capacitance to be lumped at the receiving end only. (Use two 500 kV lines with series capacitors compensating to 70 per cent of series inductance.)

5.13 It is required to transmit power from a hydroelectric station to a load centre 480 km away using two lines in parallel for security reasons.

Assume sufficient bundle conductors are used such that there are no thermal limitations, and the effective reactance per phase per km is 0.44 Ω and that the resistance is negligible. The shunt capactive reactance of each line is 0.44 MΩ per phase per km, and each line may be represented by the nominal π-circuit with half the capacitance at each end. The load is 2000 MW at 0.95 lagging and is independent of voltage over the permissible range.

Investigate, from the point of view of stability and voltage drop, the feasibility and performance of the link if the sending-end voltage is 345, 500, and 765 kV assuming the transmission angle is not to exceed 30°.

The lines may be compensated up to 70 per cent by series capacitors and the load-end compensators of 120 MVAr capacity are available. The maximum permissible voltage drop is 10 per cent. As two lines are provided for security reasons, your studies should include the worst-operating case of only one line in use.

5.14 Explain the action of a variable-tap transformer, showing, with a phasor diagram, how reactive power may be despatched from a generator down a mainly reactive line by use of the taps. How is the level of real power despatch controlled?

Power flows down an H.V. line of impedance $0 + j0.15$ p.u. from a generator whose output passes through a variable-ratio transformer to a large power system. The voltage of the generator and the distant large system are both kept at 1.0 p.u. Determine the tap setting if 0.8 p.u. power and 0.3 p.u. VAr are delivered to a lagging load at the power system busbar. Assume the reactance of the transformer is negligible.

(Answer: $t = 1.052$)

(*From E.C. Examination, 1995*)

5.15 Two substations are connected by two lines in parallel, of negligible impedance, each containing a transformer of reactance 0.18 p.u. and rated at 120 MVA. Calculate the net absorption of reactive power when the transformer taps are set to 1:1.15 and 1:0.85, respectively (i.e. tap-stagger is used). The p.u. voltages are equal at the two ends and are constant in magnitude.

(Answer: 30 MVAr)

6 Load Flows

6.1 Introduction

A *load flow* is power-system jargon for the steady-state solution of a network. This does not essentially differ from the solution of any other type of network except that certain constraints are peculiar to power supply. In previous chapters the manner in which the various components of a power system may be represented by equivalent circuits has been demonstrated. It should be stressed that the simplest representation should always be used, consistent with the accuracy of the information available. There is no merit in using very complicated machine and line models when the load and other data are known only to a limited accuracy, e.g. the long-line representation should only be used where absolutely necessary. Similarly, synchronous-machine models of more sophistication than given in this text are needed only for very specialized purposes, e.g. in some stability studies. Usually, the size and complexity of the network itself provides more than sufficient intellectual stimulus without undue refinement of the components. Often, resistance may be neglected with little loss of accuracy and an immense saving in computation.

The following combinations of quantities are usually specified at the system busbars for load-flow studies.

Slack, swing, or floating busbar One node is always specified by a voltage, constant in magnitude and phase. The effective generator at this node supplies the losses to the network; this is necessary because the magnitude of the losses will not be known until the calculation of currents is complete, and this cannot be achieved unless one busbar has no power constraint and can feed the required losses into the system. The location of the slack node can influence the complexity of the calculations; the node most closely approaching an infinite busbar should be used.

Load nodes The complex power $S = P \pm jQ$ is specified. Designated P, Q node.

Generation nodes The voltage magnitude and power are usually specified. Often, limits to the value of the reactive power are given depending upon the characteristics of individual machines. Designated P, V node. Load flow studies are performed to investigate the following features of a power system network:

1. Flow of MW and MVAr in the branches of the network;
2. Busbar (node) voltages;
3. Effect of rearranging circuits and incorporating new circuits on system loading;
4. Effect of temporary loss of generation and transmission circuits on system loading (mainly for security studies);
5. Effect of injecting in-phase and quadrature boost voltages on system loading;
6. Optimum system running conditions and load distribution;
7. Minimizing system losses;
8. Optimum rating and tap-range of transformers;
9. Improvements from change of conductor size and system voltage.

Studies will normally be performed for minimum-load conditions (possibility of instability due to high voltage levels and self-excitation of induction machines) and maximum-load conditions (possibility of instability). Having ascertained that a network behaves reasonably under these conditions, further load flows will be performed to attempt to optimize various quantities. The design and operation of a power network to obtain optimum economy is of paramount importance and the furtherance of this ideal is achieved by the use of centralized automatic control of generating stations through system control centres.

Although the same approach can be used to solve all problems, e.g. the nodal voltage method, the object should be to use the quickest and most efficient method for the particular type of problem. Radial networks will require less sophisticated methods than closed loops. In very large networks the problem of organizing the data is almost as important as the method of solution, and the calculation must be carried out on a systematic basis where the nodal-voltage method is the most advantageous. Such methods as network reduction combined with the Thevenin or superposition theorems are at their best with smaller networks. In the nodal method, great numerical accuracy is

required in the computation as the currents in the branches are derived from the voltage differences between the ends. These differences are small in well-designed networks so the method is ideally suited for computation using digital computers.

6.2 Radial and Simple Loop Networks

In radial networks the phase shifts due to transformer connections along the circuit are not usually important. The following examples illustrate the solution of this type of network.

Example 6.1

Distribution feeders with several tapped loads. A distribution feeder with several tapped inductive loads (or laterals), and fed at one end only, is shown in Figure 6.1(a). Determine the total voltage drop.

Solution

The current in AB

$$=(I_1 \cos\theta_1 + I_2 \cos\theta_2 + I_3 \cos\theta_3 + I_4 \cos\theta_4) - j(I_1 \sin\theta_1 + I_2 \sin\theta_2 + I_3 \sin\theta_3 + I_4 \sin\theta_4)$$

Similarly, the currents in the other sections of the feeder are obtained. The approximate voltage drop is obtained from $\Delta V = RI\cos\theta + XI\sin\theta$ for each section. That is

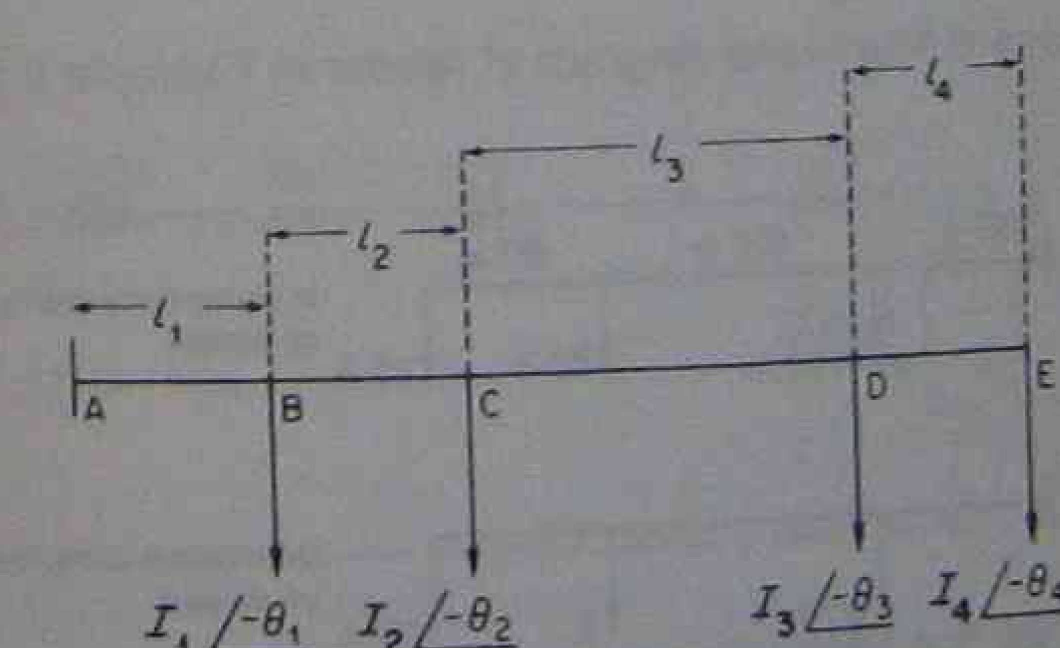

Figure 6.1(a) Feeder with several load tappings along its length

$$\begin{aligned}\Delta V_{AE} = {} & R_1(I_1 \cos\theta_1 + I_2\cos\theta_2 + I_3\cos\theta_3 + I_4\cos\theta_4) \\ & + R_2(I_2\cos\theta_2 + I_3\cos\theta_3 + I_4\cos\theta_4) \\ & + R_3(I_3\cos\theta_3 + I_4\cos\theta_4) + R_4(I_4\cos\theta_4) \\ & + X_1(I_1\sin\theta_1 + I_2\sin\theta_2 + I_3\sin\theta_3 + I_4\sin\theta_4), \text{ and so on}\end{aligned}$$

Rearranging and letting the resistance per loop-metre be $r\,\Omega$ and the reactance per loop-metre be $x\,\Omega$ (the term loop-metre refers to single-phase circuits and includes the go and return conductors),

$$\begin{aligned}\Delta V_{AE} = {} & r[I_1 \cdot l_1\cos\theta_1 + I_2\cos\theta_2 \cdot (l_1 + l_2) + I_3\cos\theta_3(l_1 + l_2 + l_3) \\ & + I_4\cos\theta_4(l_1 + l_2 + l_3 + l_4)] + x[I_1 l_1 \sin\theta_1 + I_2\sin\theta_2(l_1 + l_2) \\ & + I_3\sin\theta_3(l_1 + l_2 + l_3) + I_4\sin\theta_4(l_1 + l_2 + l_3 + l_4)]\end{aligned}$$

In the system shown in Figure 6.1(b), calculate the size of cable required if the voltage drop at the end load on the feeder must not exceed 12 V (line value). Let the resistance and reactance per metre per phase be $r(\Omega)$ and $x(\Omega)$, respectively. Then, referring to Figure 6.1(c), voltage drop is given by

$$r[100 \times 40 + 250 \times 20 + 330 \times 25] + x[100 \times 30 + 250 \times 0 + 330 \times 25] = 12/\sqrt{3}$$

i.e.

$$r(4000 + 5000 + 8250) + x(3000 + 0 + 8250) = 12/\sqrt{3}$$

$$17250r + 11250x = 12/\sqrt{3}$$

The procedure now is to consult the appropriate overhead line or cable specifications to select a cross-section that gives values of r and x which best fit the above equation. Usually, if underground cables are used the size of cross-section is selected on thermal considerations and then the voltage drop is calculated. With overhead lines the volt drop is the prime consideration and the conductor size will be determined accordingly.

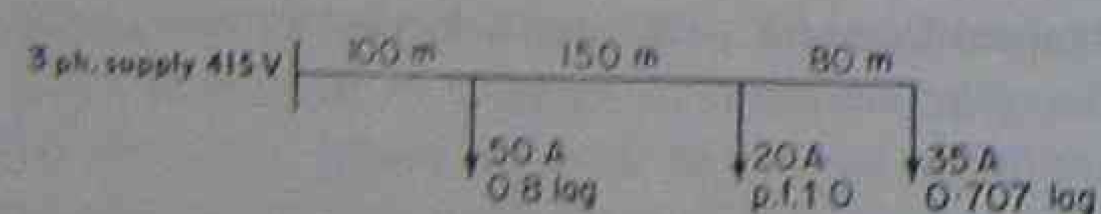

Figure 6.1(b) Line diagram of feeder in Example 6.1

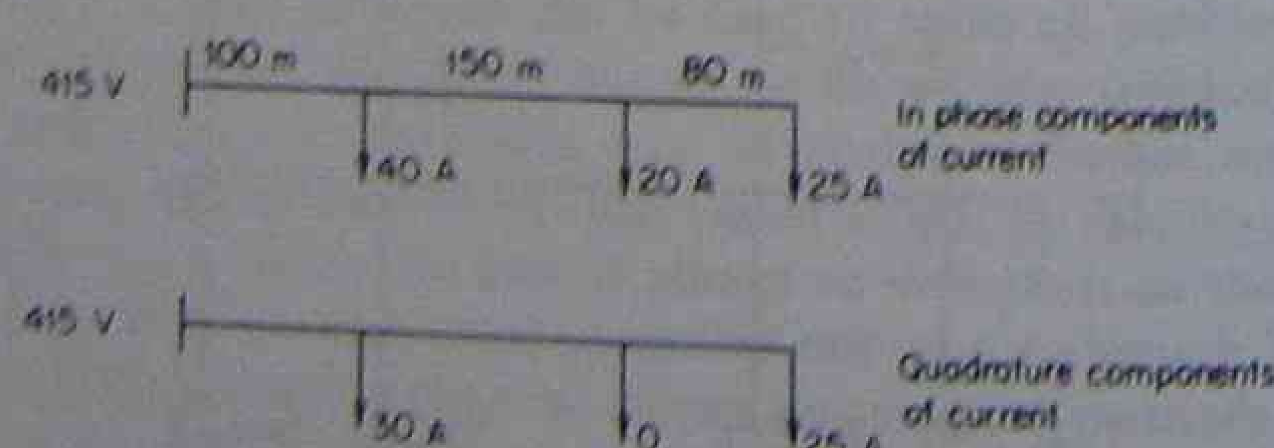

Figure 6.1(c) In-phase and quadrature quantities

Example 6.2

The system shown in figure 6.2(a) feeds two distinct loads, a group of domestic consumers, and a group of induction motors which, on starting, take 5× full-load current at zero power-factor lagging. This is a 'worst-case' calculation. The induction motor voltage is nominally 6 kV. Calculate the dip in voltage on the domestic-load busbar when the induction motor group is started.

Solution

The problem of the drop in the voltage to other consumers when an abnormally large current is taken for a brief period from an interconnected busbar is often serious. The present problem poses a typical situation. The 132 kV system is not an infinite busbar and is represented by a voltage source in series with a 0.04 p.u. reactance on a 50 MVA base. Using a 50 MVA base the equivalent single-phase circuit is as shown in Figure 6.2(b).

Starting current of the induction motors

$$\begin{aligned} &= -\mathrm{j}\frac{15000 \times 10^3}{\sqrt{3} \times 6000} \times 5\text{ A} \\ &= -\mathrm{j}7217\text{ A}\end{aligned}$$

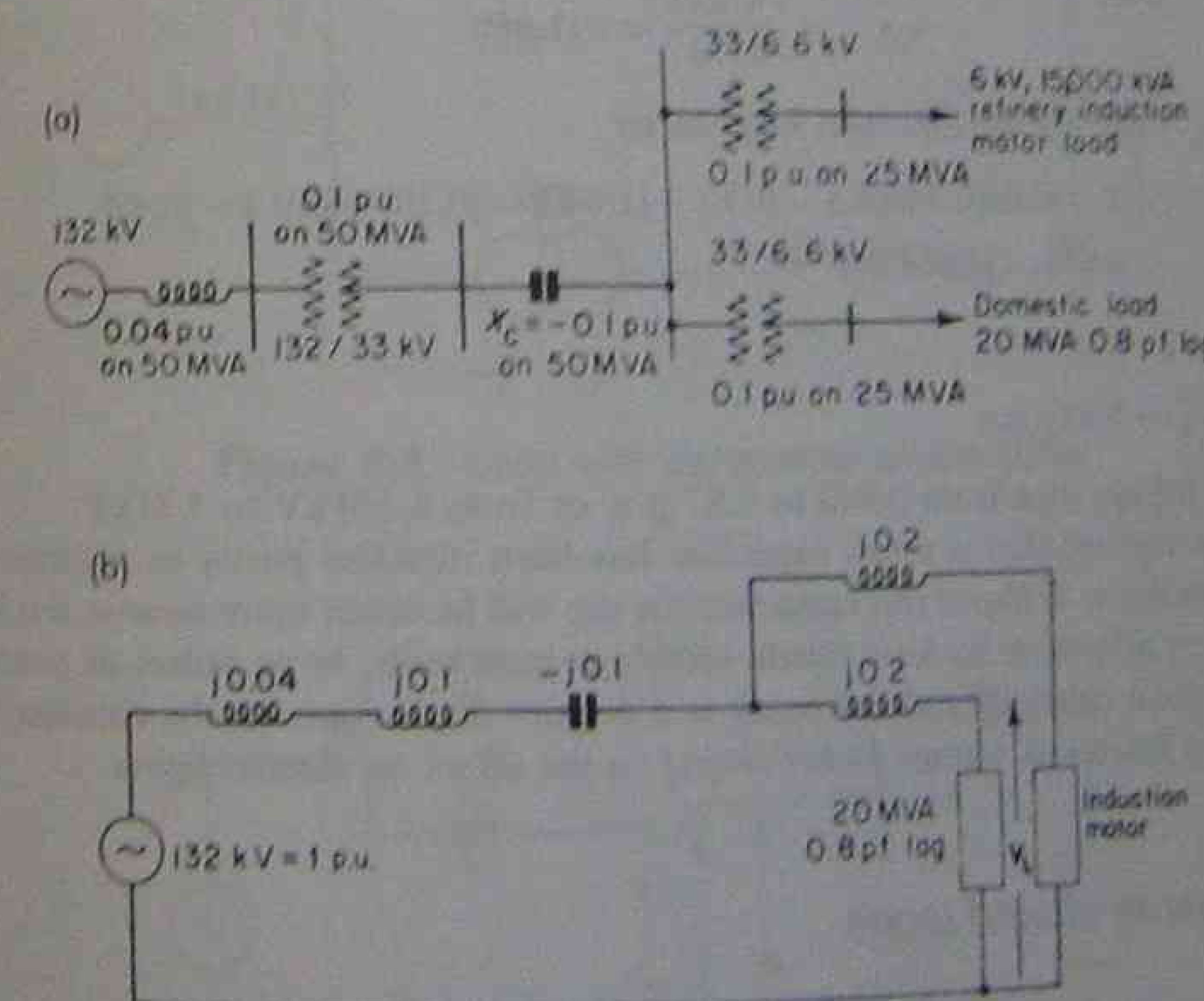

Figure 6.2 (a) Line diagram of the system in Example 6.2; (b) equivalent circuit of the system in Example 6.2

Domestic load voltage before the induction-motor load is switched on

$$\mathbf{V}_L = 1.0 - \mathbf{IZ}$$

where

$$\mathbf{Z} = j(0.04 + 0.1 - 0.1 + 0.2) = j0.24 \text{ p.u.}$$

and

$$\mathbf{I} = \frac{20 \times 10^6}{\sqrt{3} \times 6000} = 1925 \text{ A}, 0.8 \text{ p.f. lagging}$$

But base current at 6.6 kV

$$= \frac{50 \times 10^6}{\sqrt{3} \times 6.6 \times 10^3}$$
$$= 4374 \text{ A}$$
$$\therefore \mathbf{I}_{p.u.} = \frac{1925}{4374} = 0.440 \text{ p.u. at } 0.8 \text{ lag}$$

Hence

$$\mathbf{V}_L = 1.0 - 0.440(0.8 - j0.6)(j0.24) \text{ p.u.}$$
$$= 0.9366 - j0.0845 \text{ p.u.}$$
$$\therefore V_L = 0.9404 \text{ p.u.}$$

Starting current of the induction motors

$$= \frac{-j7217}{4374} = -j1.640$$

Voltage $\mathbf{V}'_L$ when the motors start is given by

$$= 1 - j0.04[0.440(0.8 - j0.6) - j1.640] - j0.2[0.440(0.8 - j0.6)]$$
$$= 0.871 - j0.084 \text{ p.u.}$$

and

$$V'_L = 0.875 \text{ p.u.}$$

Hence the voltage dips from 0.940 to 0.87 p.u. or from 6.204 kV to 5.74 kV.

It will be noticed that a series capacitor has been installed partly to neutralize the network reactance. Without this capacitor the dip will be much more serious; it is left to the reader to determine by how much. Often, in steel mills, large pulses of power are taken at regular intervals and the result is a regular fluctuation on the consumer busbars. This is known as *voltage flicker* owing to the effect on electric lights.

Load flows in closed loops

In radial networks any phase shifts due to transformer connections are not usually of importance as the currents and voltages are shifted by the same amount. In a closed loop, to avoid circulating currents, the product of the

transformer transformation ratios (magnitudes) round the loop should be unity and the sum of the phase shifts in a common direction round the loop should be zero. This will be illustrated by the system shown in Figure 6.3.

Neglecting phase shifts due to the impedance of components, for the closed loop formed by the two lines in parallel, the total shift and transformation ratio is

$$\left(\frac{33}{13.8}\angle 30^\circ\right)\left(\frac{132}{33}\angle -30^\circ\right)\left(\frac{13.8}{132}\angle 0^\circ\right) = 1\angle 0^\circ$$

In practice, the transformation ratios of transformers are frequently changed by the provision of tap-changing equipment. This results in the product of the ratios round a loop being no longer unity, although the phase shifts are still equal to zero. To represent this condition a fictitious autotransformer is connected as shown in Figure 6.4. The increase in voltage on the 132 kV line by tap-changing has the effect, in a largely reactive path, of changing the flow of reactive power and thus the power factor of the current. An undesirable effect is the circulating current set up around the loop, unless load sharing is required (see Section 5.7).

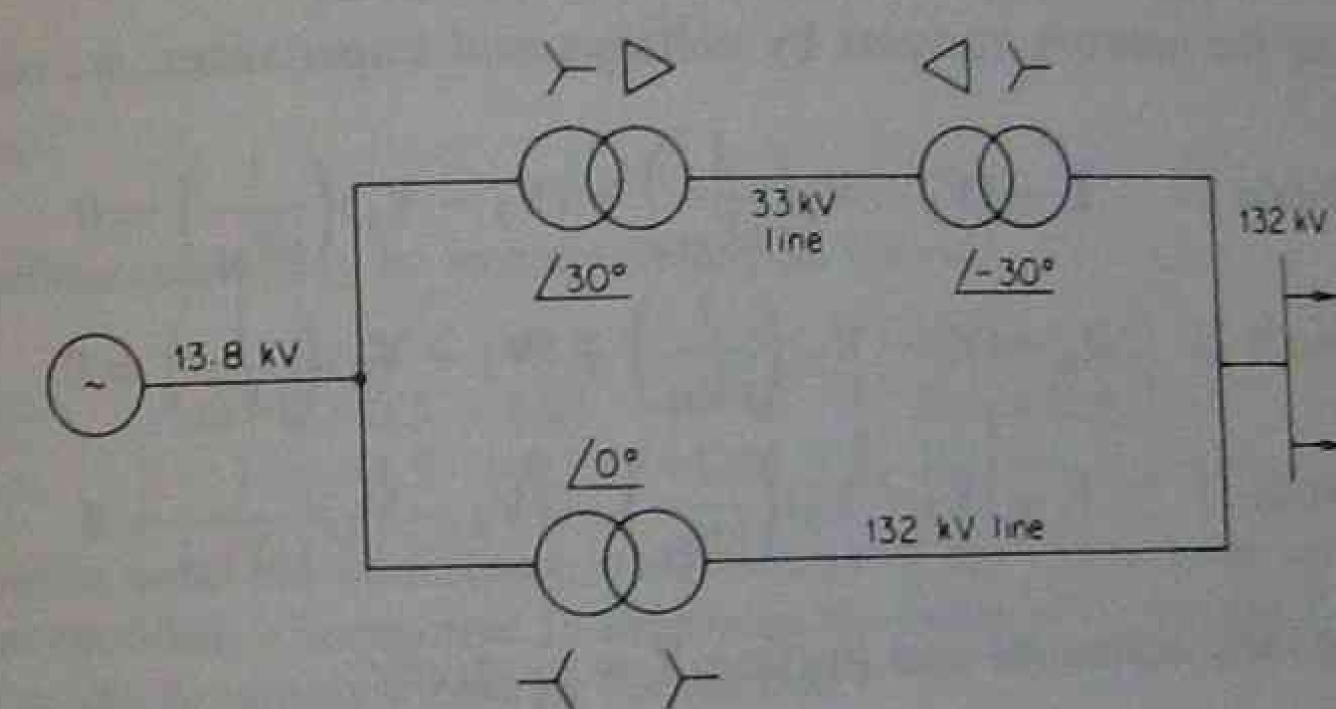

Figure 6.3 Loop with transformer phase shifts

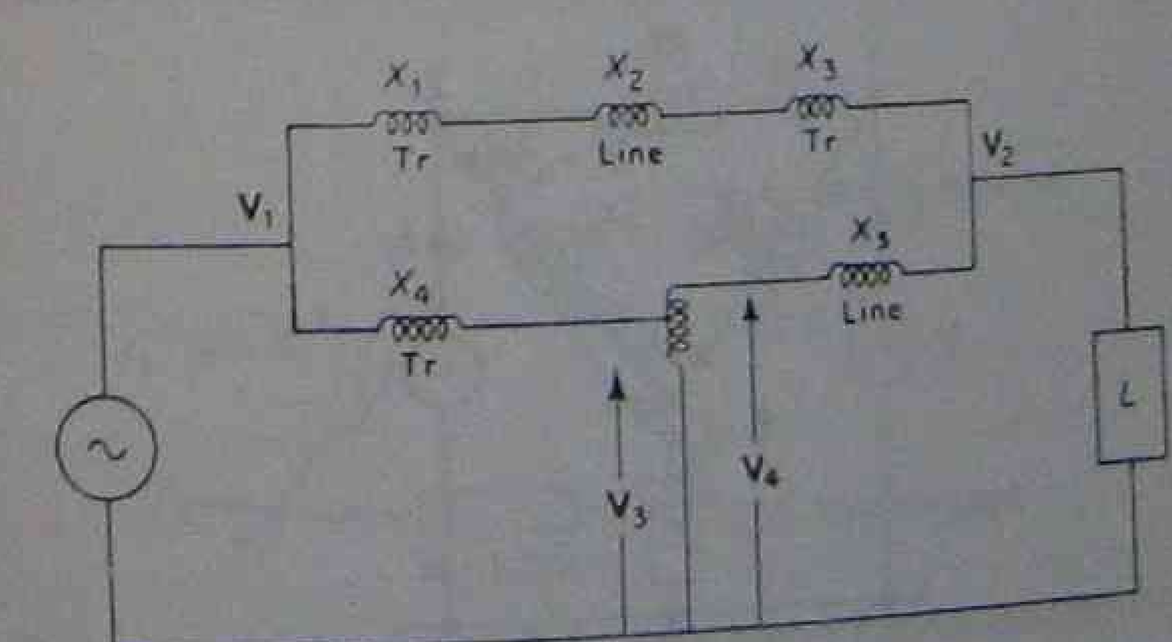

Figure 6.4 Equivalent circuit of network in Figure 6.3 showing autotransformer

Often, in lower voltage systems the out-of-balance voltage represented by the autotransformer can be neglected. If this is not the case, the best method of calculation is to determine the circulating current and consequent voltages due to the out-of-balance voltage acting alone, and then superpose these values on those obtained for operation with completely nominal voltage ratios.

6.3 Computation of Power Flows in a Network

Consider the three-node system of Figure 6.5.

With the voltages, currents, and line impedances as shown, we can formulate the following equations for each node using Kirchoff's laws:

$$\left.\begin{array}{ll}\text{Node 1:} & I_1 - I_{12} + I_{31} = 0 \\ \text{Node 2:} & I_2 - I_{23} + I_{12} = 0 \\ \text{Node 3:} & -I_3 + I_{23} - I_{31} = 0\end{array}\right\} \qquad (6.1)$$

(Note that currents flowing *into* each node are positive.)

Replacing the network currents by voltages and impedances, we obtain

$$\text{Node 1:} \quad \mathbf{I}_1 - (\mathbf{V}_1 - \mathbf{V}_2\left(\frac{1}{\mathrm{j}X_{12}}\right) + (\mathbf{V}_3 - \mathbf{V}_1)\left(\frac{1}{\mathrm{j}X_{31}}\right) = 0$$

$$\text{Node 2:} \quad \mathbf{I}_2 - (\mathbf{V}_2 - \mathbf{V}_3)\left(\frac{1}{\mathrm{j}X_{23}}\right) + (\mathbf{V}_1 - \mathbf{V}_2)\left(\frac{1}{\mathrm{j}X_{12}}\right) = 0$$

$$\text{Node 3:} \quad -\mathbf{I}_3 + (\mathbf{V}_2 - \mathbf{V}_3)\left(\frac{1}{\mathrm{j}X_{23}}\right) - (\mathbf{V}_3 - \mathbf{V}_1)\frac{1}{(\mathrm{j}X_{32})} = 0$$

Rearranging these equations and putting $\mathbf{Y} = \frac{1}{\mathrm{j}X}$ gives:

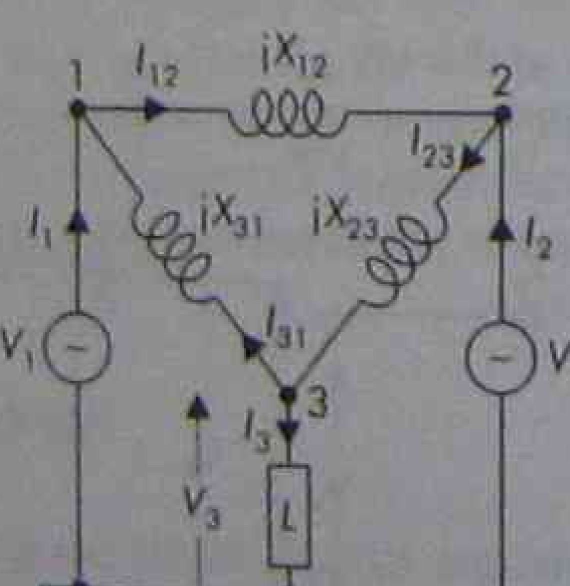

Figure 6.5 Three-node network for power-flow formulation

$$\mathbf{I}_1 = (\mathbf{Y}_{12} + \mathbf{Y}_{31})\mathbf{V}_1 + (-\mathbf{Y}_{12})\mathbf{V}_2 + (-\mathbf{Y}_{31})\mathbf{V}_3$$
$$\mathbf{I}_2 = (-\mathbf{Y}_{12})\mathbf{V}_1 + (\mathbf{Y}_{12} + \mathbf{Y}_{23})\mathbf{V}_2 + (-\mathbf{Y}_{23})\mathbf{V}_3$$
$$-\mathbf{I}_3 = (-\mathbf{Y}_{31})\mathbf{V}_1 + (-\mathbf{Y}_{23})\mathbf{V}_2 + (\mathbf{Y}_{31}\mathbf{Y}_{23})\mathbf{V}_3$$

These equations can be put into matrix form as:

$$\begin{bmatrix}\mathbf{I}_1 \\ \mathbf{I}_2 \\ -\mathbf{I}_3\end{bmatrix} = \begin{bmatrix}(\mathbf{Y}_{12} + \mathbf{Y}_{31}) & -\mathbf{Y}_{12} & -\mathbf{Y}_{31} \\ -\mathbf{Y}_{12} & (\mathbf{Y}_{12} + \mathbf{Y}_{23}) & -\mathbf{Y}_{23} \\ -\mathbf{Y}_{31} & -\mathbf{Y}_{23} & (\mathbf{Y}_{31} + \mathbf{Y}_{23})\end{bmatrix}\begin{bmatrix}\mathbf{V}_1 \\ \mathbf{V}_2 \\ \mathbf{V}_3\end{bmatrix} \qquad (6.2)$$

It will be seen that the diagonal elements are the *sum* of the admittances connected to the node (known as self-admittances) whilst the off-diagonal elements are the admittances connected between the relevant nodes, and are negative. Consequently, if the currents on the left-hand side are known, the voltages can be calculated by inverting the **Y** matrix.

Example 6.3

Perform a load flow on the interconnected busbars shown in Figure 6.6.

Solution

The nodal equations may be written directly as follows:

$$\begin{bmatrix}2 & -0.5 & -1.5 \\ -0.5 & 1.25 & -0.75 \\ -1.5 & -0.75 & +2.25\end{bmatrix}\begin{bmatrix}V_1 \\ V_2 \\ V_3\end{bmatrix} \begin{matrix}= \\ = \\ =\end{matrix} \begin{bmatrix}I_1 \\ I_2 \\ I_3\end{bmatrix}$$

It is usual to write the admittance matrix direct. If the values in the current column vector are specified, it is required to determine the values of V_1, V_2, and V_3. [Y] may be inverted or the equations solved by elimination.

$$\text{As} \quad [\mathbf{Y}][\mathbf{V}] = [\mathbf{I}] \quad \text{then} \quad [\mathbf{Y}]^{-1}[\mathbf{I}] = [\mathbf{V}]$$

Hence it is required to invert the admittance matrix. A simple method will be used in this case:

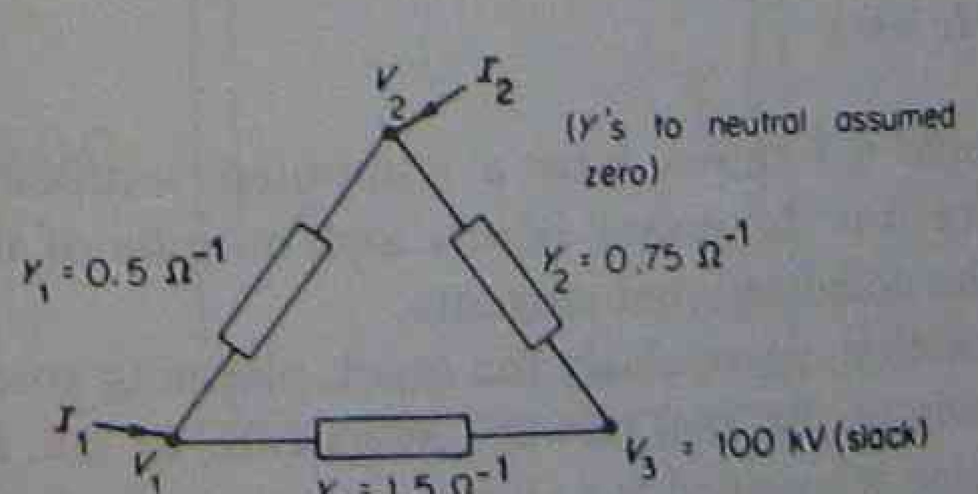

Figure 6.6 Three-busbar system–load flow using matrix inversion [note: Ω^{-1} is an alternative to S (siemen)]

$$V_1 = 0.5I_1 + 0.25V_2 + 0.75V_3$$

Substitute for V_1 in equations for I_2 and I_3:

$$-0.25I_1 + 1.125V_2 - 1.125V_3 = I_2$$
$$-0.75I_1 - 1.125V_2 + 1.125V_3 = I_3$$

Also, from the equation for I_2

$$V_2 = 0.222I_1 + 0.888I_2 + V_3$$

Substitute for V_2 in the other equations:

$$0.555I_1 + 0.222I_2 + V_3 = V_1$$
$$0.222I_1 + 0.888I_2 + V_3 = V_2$$
$$-I_1 - I_2 - 0V_3 = I_3$$

i.e.

$$\begin{bmatrix} 0.555 + 0.222 + 1.0 \\ 0.222 + 0.888 + 1.0 \\ -1 \quad -1 \quad 0.0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ I_3 \end{bmatrix}$$

The node at which the voltage is specified, the slack busbar, is node (3); hence the manipulation of the matrix into the form shown. Next, some loadings at the nodes will be specified:

$$I_1 = 1000 \text{ A (generator node)}$$
$$I_2 = 1500 \text{ A (load node)}$$
$$V_3 = 100\,\text{kV (phase value), slack busbar}$$

Dealing in kiloamperes and kilovolts,

$$\begin{aligned} V_1 &= 0.555(1) + 0.222(-1.5) + (+1)(100) \\ &= 100.222\,\text{kV} \\ V_2 &= 0.222(1) + 0.888(-1.5) + (1)(100) \\ &= 98.89\,\text{kV} \\ I_3 &= -(1)(1) + (1)(1.5) + 0(100) \\ &= +0.5\,\text{kA i.e. a generator node} \end{aligned}$$

(Note that $I_1 + I_2 + I_3 = 0$.)

Although Example 6.3 shows how a computed solution can be readily obtained, in practice it is the power inputs and outputs of the network that are specified, not the nodal injected currents.

A common calculation, particularly for quick planning studies and security calculations, is known as the 'real power', megawatt, or d.c. load flow. Here, the injected MW value is specified for all the nodes except the slack, where the voltage magnitude is given. It is required to find the MW flows in the network to satisfy the specified conditions.

Example 6.4

Perform a real-power load flow for the network shown in Figure 6.7.

Solution

An estimate may be obtained by assuming zero resistance in the network and using the fact that in a well-designed and well-operated system, all the voltages will be close to 1 p.u. such that P in p.u. is the same as I in p.u. Consequently, voltage drops given by PX around the network loops must sum to zero. Hence $\Sigma P_{ij}X_{ij} = 0$ where i and j refer to the branch between nodes i and j of the network. This follows from the fact that $\Delta V \propto \angle\delta \approx (XP/V)$ and the sum of the angular shifts round the loop is zero. Labelling the powers as in Figure 6.7, the following equations hold:

$$13.2P_1 + 2.06P_3 - 19.8P_4 - 6.6P_2 = 0$$
$$P_4 + P_3 = 100 \quad \text{or} \quad P_4 = 100 - P_3$$
$$P_2 - P_4 = 50 \quad P_1 = 60 + P_3$$
$$P_2 + P_1 = 210 \quad P_2 = 210 - 60 - P_3 = 150 - P_3$$
$$\therefore 13.2(60 + P_3) + 2.06(P_3) - 19.8(100 - P_3) - 6.6(150 - P_3) = 0$$

and

$$792 + 13.2P_3 + 2.06P_3 - 1980 + 19.8P_3 - 990 + 6.6P_3 = 0$$

From which $P_3 = 52.2\,\text{MW}$, which is a reasonably accurate estimate.

Once P in BC is known, the flows in the other branches follow readily, i.e.

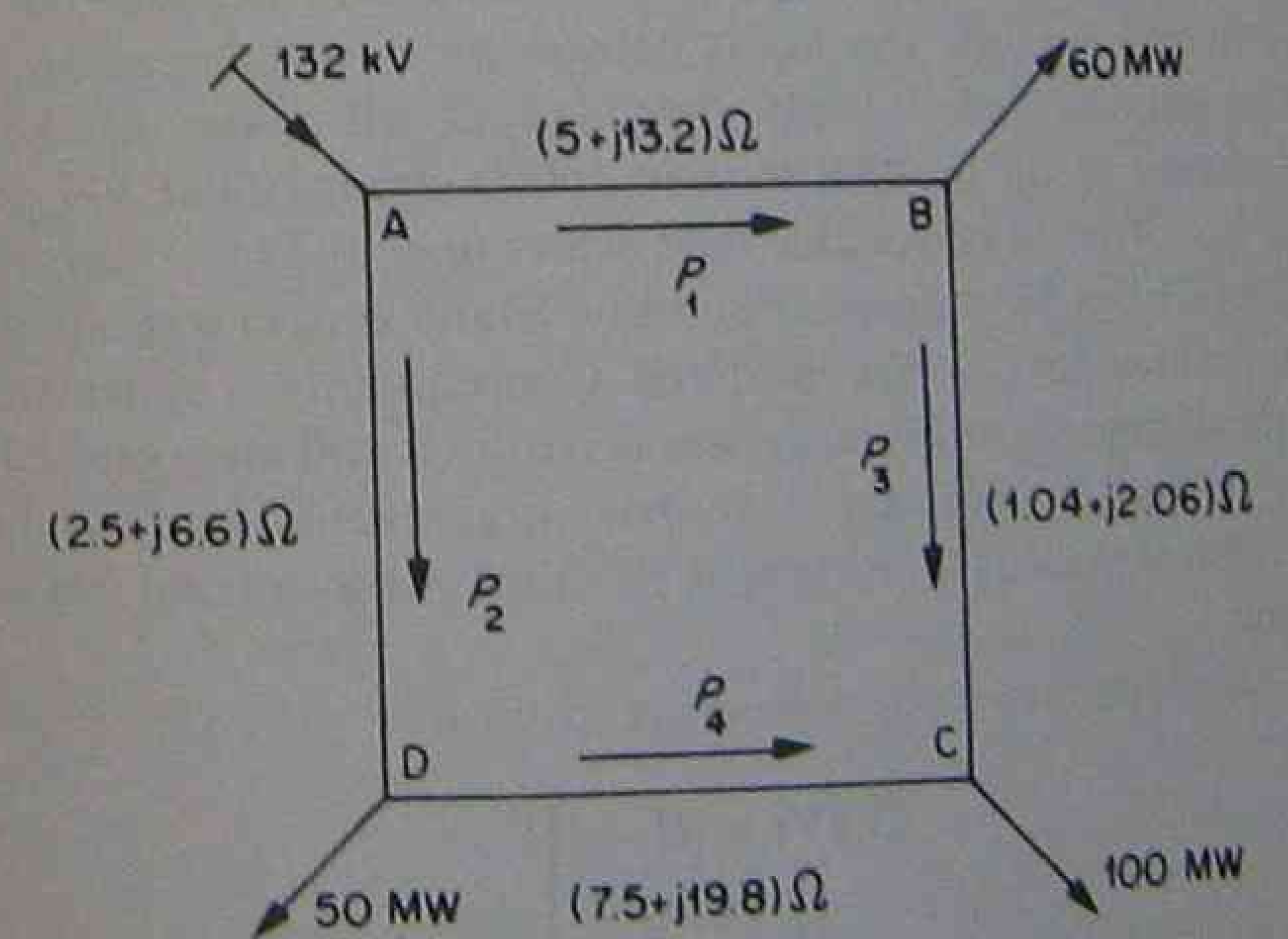

Figure 6.7 Line diagram of loop network for Example 6.4

$$P_4 = P_{DC} = 100 - 52.2 = 47.8\,\text{MW}$$
$$P_1 = P_{AB} = 60 + 52.2 = 112.2\,\text{MW}$$
$$P_2 = P_{AD} = 47.8 + 50 = 97.8\,\text{MW}$$

Quite often, for networks where $X/R > 10$, small changes to network flows, e.g. less than 10 per cent of existing flows, can be estimated by using the principle of superposition. For example, in the network of Figure 6.7, if node C supplied an extra 10 MW to the load, this would need to come from node A and flow over AD and DC in series, but in parallel with AB and BC in series. This 10 MW would divide in the ratio of (j13.2 + j2.06) to (j6.6 + j19.8) giving

$$10 \times \left(\frac{15.26}{15.26 + 26.4}\right)\text{MW} = 3.66\,\text{MW}$$

to be added to the path ADC. Hence P_2 would now become $97.8 + 3.66 = 101.46\,\text{MW}$ and $P_4 = 51.46\,\text{MW}$. The remaining flow of $10 - 3.66 = 6.33\text{MW}$ would be added to P_1 and P_3.

Using the same superposition principle it is possible to estimate the change in flows due to the removal of a circuit by injecting into the simplified network a flow equal and opposite to that previously in the removed circuit, and calculating the additional flows due to this negative injection. Such estimates are useful to assess the system security (see later in this chapter).

6.4 Complex Flows in Large Systems

In large practical power systems, for both planning and operational purposes it is necessary to carry out many load flows taking into account the complex impedances of the circuits, the limits caused by circuit capacities, and the voltages that can be satisfactorily provided at all nodes in the system. Systems consisting of up to 3000 busbars, 6000 circuits, and 500 generators may have to be solved in reasonable time scales (e.g. 1–2 min) with accuracies requiring 32 or 64 bits for numerical stability. Many system states possible in a day's operation may have to be considered. Consequently, a systematic way of computing the desired states of the system is required and an organized method for handling the data is necessary. Data base organization and rapid retrieval and modification are just as important as having an efficient and fast computation technique.

Basically, the equations to be solved are given by

$$\left.\begin{aligned} [\mathbf{Y}][\mathbf{V}] &= [\mathbf{I}] \\ [\mathbf{I}] &= \left[\frac{\mathbf{S}^*}{\mathbf{V}^*}\right] \end{aligned}\right\} \tag{6.3}$$

where * denotes conjugate. This solution can only be obtained by iteration as the problem is non-linear, but iterative matrix inversion using Gaussian elimination can be employed as indicated in the next section.

6.4.1 *Direct methods*

Triangulation and partitioning

Direct methods solve only linear systems, i.e. $[\mathbf{Y}][\mathbf{V}] = [\mathbf{I}]$ where $[\mathbf{I}]$ is specified. The fact that powers are specified in practice makes the problem non-linear. A new value for $\mathbf{I}$ must be obtained from $\mathbf{S} = \mathbf{VI}^*$ after each direct solution and this value used to obtain a new one.

Partitioning is useful for handling large problems on small computers, for network reduction, and the elimination of unwanted nodes. A matrix

$$[\mathbf{Y}] = \left[\begin{array}{cc|c} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ \hline Y_{31} & Y_{32} & Y_{33} \end{array}\right] \tag{6.4}$$

can be partitioned into four submatrices by the dividing lines shown in equation (6.4). Hence, in this typical admittance matrix for a network,

$$[\mathbf{Y}] = \begin{bmatrix} \mathbf{B} & \mathbf{C} \\ \mathbf{C}^t & \mathbf{D} \end{bmatrix}$$

where $\mathbf{C}^t$ is the transpose of $\mathbf{C}$,

$$\mathbf{B} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \quad \text{and so on}$$

A saving in computation time and storage is often obtained by manipulating the submatrices instead of the main one, and in the equation $[\mathbf{I}] = [\mathbf{Y}][\mathbf{V}]$ it is possible to eliminate nodes at which $\mathbf{I}$ is not injected. The matrix $[\mathbf{Y}]$ is partitioned so that the nodes to be eliminated are grouped together in a submatrix, e.g.

$$\begin{bmatrix} \mathbf{I}_w \\ \mathbf{I}_u \end{bmatrix} = \begin{bmatrix} \mathbf{B} & \mathbf{C} \\ \mathbf{C}^t & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{V}_w \\ \mathbf{V}_u \end{bmatrix} \tag{6.5}$$

where the subscript u indicates unwanted, and subscript w wanted, nodes. For example, if the currents at the unwanted nodes are zero,

$$\mathbf{I}_u = 0 \qquad -\mathbf{C}^t\mathbf{V}_w = \mathbf{D}\mathbf{V}_u \qquad \text{and} \qquad -\mathbf{D}^{-1}\mathbf{C}^t\mathbf{V}_w = \mathbf{V}_u$$

Substituting this value of $\mathbf{V}_u$ in the expression for $\mathbf{I}_w$ produces

$$\mathbf{I}_w = \mathbf{B}\mathbf{V}_w - \mathbf{C}\mathbf{D}^{-1}\mathbf{C}^t\mathbf{V}_w$$

which gives a new admittance matrix containing only wanted nodes, i.e.

$$[\mathbf{Y}] = \mathbf{B} - \mathbf{C}\mathbf{D}^{-1}\mathbf{C}^t \tag{6.6}$$

Methods are available for modifying the admittance matrix to allow for changes in the configuration of the network, say, due to line outages. These modifications can be performed such that a completely fresh inversion of the new admittance matrix is not necessary.

Most power-system matrices are symmetric, i.e. $\mathbf{Y}_{ij} = \mathbf{Y}_{ji}$, and it is only required to store elements on and above the diagonal. Frequently, most of the non-zero elements lie within a narrow band about the diagonal and, again, considerable savings in computer storage may be attained.

Accuracy When solving the set of equations $[\mathbf{A}][\mathbf{x}] = [\mathbf{b}]$, the residual vector $\mathbf{r} = [\mathbf{b}] - [\mathbf{A}][\mathbf{x}]$ is not zero because of rounding errors. Troubles arise when ill-conditioned equations are obtained in which, although the residual is small, the solution may be inaccurate. Such matrices are often large and sparse and for most rows the diagonal element is equal to the sum of the non-diagonal elements and of opposite sign.

6.4.2 Iterative methods

The two sets of equations of (6.3) are solved simultaneously as the iteration proceeds. A number of methods are available, and as three of these have gained wide adoption they will be described in detail. The first is the Gauss–Seidel method which has been widely used for many years and is simple in approach; the second is the Newton–Raphson method which, although more complex, has certain advantages. A third, which is almost an industry standard, is the fast decoupled load flow. The speed of convergence of these methods is of extreme importance as, apart from the cost of computer time, the use of these methods in schemes for the automatic control of power systems requires very fast load-flow solutions.

(i) The Gauss–Seidel method

In this method the unknown quantities are initially assumed and the value obtained from the first equation for, say V_1 is then used when obtaining V_2 from the second equation, and so on. Each equation is considered in turn and then the complete set is solved again until the values obtained for the unknowns converge to within required limits.

Application of the method to the simple network of Example 6.3 gives the nodal equations as

$$2V_1 - 0.5V_2 - 1.5V_3 = I_1 = 1$$
$$-0.5V_1 + 1.25V_2 - 0.75V_3 = I_2 = -1.5$$
$$-1.5V_1 - 0.75V_2 + 2.25V_3 = I_3$$

where the voltages are in kV and the currents in kA.

As V_3 is known, i.e. 100 kV (slack busbar voltage), it is necessary only to solve the first two equations. Initially, make $V_2 = 100\,\text{kV}$.

$$\therefore {}^1V_1 = 0.5(1 + 150 + 50) = 100.5000$$

Using this value to evaluate V_2,

$${}^1V_2 = \frac{1}{1.25}(-1.5 + 75 + 50.25) = 99.0000$$

Using this value of V_2 to evaluate V_1,

$${}^2V_1 = 0.5(1 + 150 + 49.5) = 100.2500$$

$${}^2V_2 = \frac{1}{1.25}(-1.5 + 75 + 50.125) = 98.9000$$

$${}^3V_1 = 100.2250 \qquad {}^2V_2 = 98.8900$$
$${}^4V_1 = 100.2225 \qquad {}^4V_2 = 98.8890$$
$${}^5V_1 = 100.22225 \qquad {}^5V_2 = 98.8888$$

Iterations are now producing changes only in the fourth place of decimals and the process may be stopped; hence

$$I_3 = -1.5 \times 100.22225 - 0.75 \times 98.8888 + 2.25 \times 100 = 0.5\,\text{kA}$$

The solution has been obtained with much less computation than the more direct method used previously.

In the three-node system of Example 6.3, the iterative form of the three nodal equations with the nodal constraints that $\mathbf{S} = \mathbf{VI}^*$ is obtained as follows (p indicates the iteration number and is *not* a power). For node 1:

$$\mathbf{I}_1 = \mathbf{V}_1\mathbf{Y}_{11} + \mathbf{V}_2\mathbf{Y}_{12} + \mathbf{V}_3\mathbf{Y}_{13}$$

$$\therefore \mathbf{V}_1 = -\mathbf{V}_2\frac{\mathbf{Y}_{12}}{\mathbf{Y}_{11}} - \mathbf{V}_3\frac{\mathbf{Y}_{13}}{\mathbf{Y}_{11}} + \frac{\mathbf{I}_1}{\mathbf{Y}_{11}}$$

$$\therefore \mathbf{V}_1^* = -\frac{\mathbf{Y}_{12}^*}{\mathbf{Y}_{11}^*}\mathbf{V}_2^* - \frac{\mathbf{Y}_{13}^*}{\mathbf{Y}_{11}^*}\mathbf{V}_3^* + \frac{\mathbf{I}_1^*}{\mathbf{Y}_{11}^*}$$

Substituting $\mathbf{I}_1^* = \mathbf{S}_1/\mathbf{V}_1$ and writing in the iterative form,

$$\mathbf{V}_1^{p+1} = -\frac{\mathbf{Y}_{12}}{\mathbf{Y}_{11}}\mathbf{V}_2^p - \frac{\mathbf{Y}_{13}}{\mathbf{Y}_{11}}\mathbf{V}_3^p + \frac{\mathbf{S}_1^*}{\mathbf{Y}_{11}}\frac{1}{\mathbf{V}_1^{*p}}$$

Similarly,

$$\mathbf{V}_2^{p+1} = -\frac{\mathbf{Y}_{21}}{\mathbf{Y}_{22}}\mathbf{V}_1^{p+1} - \frac{\mathbf{Y}_{23}}{\mathbf{Y}_{22}}\mathbf{V}_3^p + \frac{\mathbf{S}_2^*}{\mathbf{Y}_{22}}\frac{1}{\mathbf{V}_2^{*p}}$$

and

$$\mathbf{V}_3^{p+1} = -\frac{\mathbf{Y}_{31}}{\mathbf{Y}_{33}}\mathbf{V}_1^{p+1} - \frac{\mathbf{Y}_{32}}{\mathbf{Y}_{33}}\mathbf{V}_2^{p+1} + \frac{\mathbf{S}_3^*}{\mathbf{Y}_{33}}\frac{1}{\mathbf{V}_3^{*p}}$$

At any node i the already scanned nodes up to i will have new values appropriate to the $(p+1)$ iteration and the nodes yet to be scanned $(j > i)$ are appropriate to iteration p. Generally,

$$(\mathbf{V}_i^{p+1}) = +\frac{\mathbf{S}_i^*}{\mathbf{Y}_{ii}}\cdot\frac{1}{\mathbf{V}_i^{*p}} - \sum_{\substack{j>1\\ j\neq i}}\frac{\mathbf{Y}_{ij}}{\mathbf{Y}_{ii}}\mathbf{V}_j^p - \sum_{\substack{j>i\\ j\neq i}}\frac{\mathbf{Y}_{ij}}{\mathbf{Y}_{ii}}\mathbf{V}_i^{p+1} \tag{6.7}$$

In this particular case, node 3 is the slack-bus, and as $\mathbf{V}_i$ is known, the equation for it is not required. It is seen in the above equations that the new value of $\mathbf{V}_i$ in the preceding equation is immediately used in the next equation, i.e. $\mathbf{V}_1^{p+1}$ is used in the $\mathbf{V}_2$ equation. Each node is scanned, in turn, over a complete iteration.

Equation (6.7) refers to a busbar with P and Q specified. At a generator node (i), usually V_i and P_i are specified with perhaps upper and lower limits to Q_i.

The magnitude of $\mathbf{V}_i$ is fixed, but its phase depends on Q_i. The values of $\mathbf{V}_i$ and Q_i from the previous iteration are not related by equation (6.7) because $\mathbf{V}_i$ has been modified to give a constant magnitude. It is necessary at the next iteration to calculate the value of Q_i corresponding to $\mathbf{V}_i$ from equation (6.7), i.e. Q_i = imaginary part of $\mathbf{S}_i$, i.e. of

$$\mathbf{Y}_{ii}^*\mathbf{V}_i^{p-1}\left[\mathbf{V}_i^{*p} + \sum_{\substack{j\neq i\\ j>i}}\frac{\mathbf{Y}_{ij}^*}{\mathbf{Y}_{ii}^*}\mathbf{V}_j^{*p-1} + \sum_{\substack{j\neq i\\ j<i}}\frac{\mathbf{Y}_{ij}^*}{\mathbf{Y}_{ii}^*}\mathbf{V}_j^{*p}\right]$$

This value of Q_i holds for existing value of $\mathbf{V}_i$ and is then substituted into

$$-\sum_{\substack{j\neq i\\ j>i}}\frac{\mathbf{Y}_{ij}^*}{\mathbf{Y}_{ii}^*}\mathbf{V}_j^{*p} + \frac{P_i + \mathrm{j}Q_i}{\mathbf{Y}_{ii}^*\mathbf{V}_i^p}\sum_{\substack{j\neq i\\ j<i}}\frac{\mathbf{Y}_{ij}^*}{\mathbf{Y}_{ii}^*}\mathbf{V}_j^{*p+1}$$

to obtain $(\mathbf{V}_i^{p+1})^*$.

The real and imaginary components of $\mathbf{V}_i^{p+1}$ are then multiplied by the ratio V_i/V_i^{p+1}, thus complying with the constant V_i constraint. This final step is a slight approximation, but the error involved is small and the saving in computation is large. The phase of $\mathbf{V}_i$ is thus found and the iteration can proceed to the next node. The process continues until the value of V^{p+1} at any node differs from V^p by a specified amount, a common figure being 0.0001 p.u. The study is commenced by assuming 1 p.u. voltage at all nodes except one; this exception is necessary in order that current flows may be obtained in the first iteration.

Acceleration factors The number of iterations required to reach the specified convergence can be greatly reduced by the use of acceleration factors. The correction in voltage from V^p to V^{p+1} is multiplied by such a factor so that the new voltage is brought closer to its final value, i.e.

$$\begin{aligned} {}^1\mathbf{V}^{p+1} &= \mathbf{V}^p + \alpha(\mathbf{V}^{p+1} - \mathbf{V}^p) \\ &= \mathbf{V}^p + \alpha\Delta\mathbf{V}^p \end{aligned}$$

where ${}^1\mathbf{V}^{p+1}$ is the accelerated new voltage. It can be shown that a complex value of α reduces the number of iterations more than a real value. However, up to the present, only real values seem to be used; the actual value depends on the nature of the system under study but a value of 1.6 is widely used. The Gauss–Seidel method with the use of acceleration factors is known as the method of successive overrelaxation.

Transformer tap-changing

Further changes which must be accommodated in the admittance matrix are those due to transformer tap-changing. When the ratio is at the nominal value the transformer is represented by a single series impedance, but when off-nominal, adjustments have to be made as follows.

Consider a transformer of ratio $t:1$ connected between two nodes i and j; the series admittance of the transformer is $\mathbf{Y}_t$. Referring to Figure 6.8(a), the following nodal-voltage equation holds:

$$\mathbf{I}_j = \mathbf{V}_j(\mathbf{Y}_{jr} + \mathbf{Y}_{j0} + \mathbf{Y}_{js} + \mathbf{Y}_t) - (\mathbf{V}_r\mathbf{Y}_{jr} + \mathbf{V}_x\mathbf{Y}_t + \mathbf{V}_s\mathbf{Y}_{js} + \mathbf{V}_0\mathbf{Y}_{j0})$$

As $\mathbf{V}_0 = 0$ and $\mathbf{V}_x = \mathbf{V}_i/t$,

$$\mathbf{I}_j + \mathbf{V}_s\mathbf{Y}_{js} = \mathbf{V}_j(\mathbf{Y}_{jr} + \mathbf{Y}_{j0} + \mathbf{Y}_{js} + \mathbf{Y}_t) - \left(\mathbf{Y}_{jr}\mathbf{V}_r + \frac{\mathbf{Y}_t\mathbf{V}_i}{t}\right)$$

where x is an artificial node between the voltage transforming element and the transformer admittance. From this last equation it is seen that for the node on

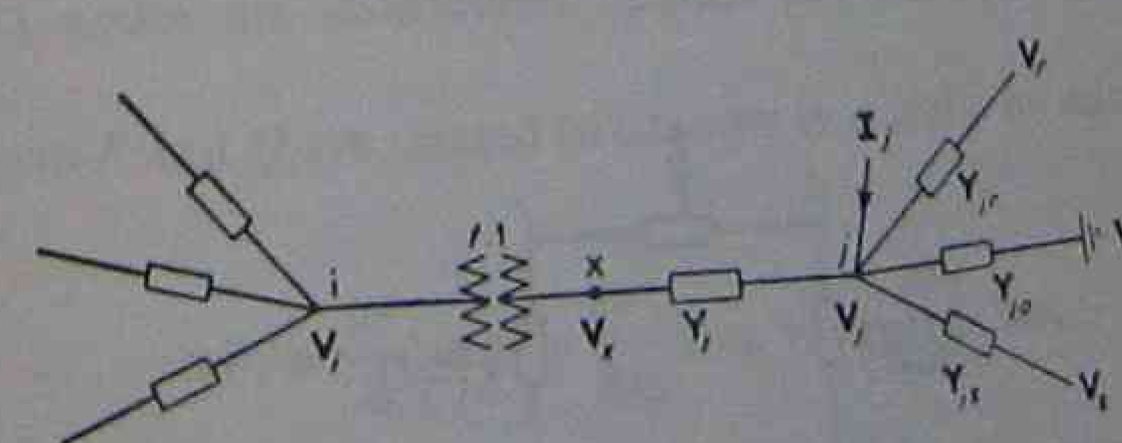

Figure 6.8(a) Equivalent circuit of transfofrmer with off-nominal tap ratio. Transformer series admittance on non-tap side

the off-tap side of the transformer (i.e. the more remote of the two nodes i and j), the following conditions apply:

when forming $\mathbf{Y}_{jj}$ use $\mathbf{Y}_t$ for the transformer, and when forming $\mathbf{Y}_{ij}$ use $\mathbf{Y}_t/t$ for the transformer.

It can similarly be shown for the tap-side node that the following conditions apply:

when forming $\mathbf{Y}_{ii}$ use $\mathbf{Y}_t/t^2$, and when forming $\mathbf{Y}_{ij}$ use $\mathbf{Y}_t/t$.

These conditions may be represented by the π section shown in Figure 6.8(b), although it is probably easier to modify the mutual and self-admittances directly.

(ii) The Newton–Raphson method

Although the Gauss–Seidel was the first popular method, the Newton–Raphson method was subsequently used increasingly. With some systems the latter gives a greater assurance of convergence and is at the same time economical in computer time.

The basic iterative procedure is as follows:

value at new iteration,

$$x^{p+1} = x^p - \frac{f(x^p)}{f'(x^p)}$$

extending this to a multi-equation system,

$$\mathbf{x}^{p+1} = \mathbf{x}^{(p)} - J^{-1}(\mathbf{x}^p)f(\mathbf{x}^p)$$

where the $\mathbf{x}$'s in f are column vectors and $J(\mathbf{x}^p)$ is a matrix known as the Jacobian matrix, of the form

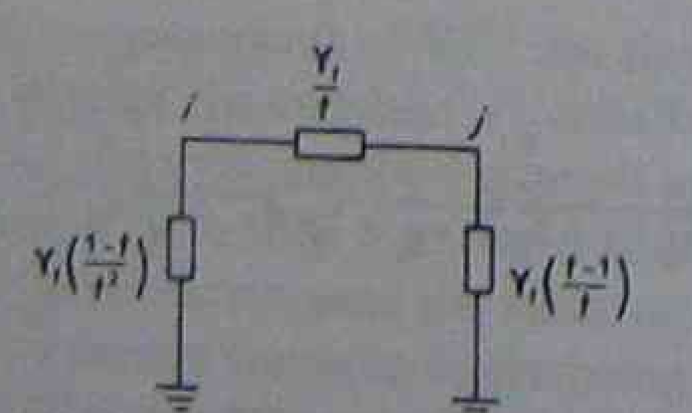

Figure 6.8(b) The π section to represent transformer with off-nominal tap ratio

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdot & \cdot & \cdot & \frac{\partial f_1}{\partial x_n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \frac{\partial f_k}{\partial x_k} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{\partial f_n}{\partial x_1} & \cdot & \cdot & \cdot & \frac{\partial f_n}{\partial x_n} \end{bmatrix}_{p\text{th iteration}}$$

Consider now the application to an n-node power system, for a link connecting nodes k and j of admittance $\mathbf{Y}_{kj}$,

$$P_k + jQ_k = \mathbf{V}_k\mathbf{I}_k^* = \mathbf{V}_k\sum_{j=1}^{n-1}(\mathbf{Y}_{kj}\mathbf{V}_j)^*$$

Let

$$\mathbf{V}_k = a_k + jb_k \quad \text{and} \quad \mathbf{Y}_{kj} = G_{kj} - jB_{kj}$$

Then,

$$P_k + jQ_k = (a_k + jb_k)\sum_{1}^{n-1}[(G_{kj} - jB_{kj})(a_j + jb_j)]^* \tag{6.8}$$

from which,

$$P_k = \sum_{j=1}^{n-1}[a_k(a_jG_{kj} + b_jB_{kj}) + b_k(b_jG_{kj} - a_jB_{kj})] \tag{6.9}$$

$$Q_k = \sum_{j=1}^{n-1}[b_k(a_jG_{kj} + b_jB_{kj}) - a_k(b_jG_{kj} - a_jB_{kj})] \tag{6.10}$$

Hence, there are two non-linear simultaneous equations for each node. Note that $(n-1)$ nodes are considered because the slack node n is completely specified.

Changes in P and Q are related to changes in a and b by equations (6.9) and (6.10), e.g.

$$\Delta P_1 = \frac{\partial P_1}{\partial a_1}\Delta a_1 + \frac{\partial P_1}{\partial a_2}\Delta a_2 + \cdots \frac{\partial P_1}{\partial a_{n-1}}\Delta a_{n-1}$$

Similar equations hold in terms of ΔP and Δb, and ΔQ in terms of Δa and Δb.

The equations may be expressed generally in the following manner:

$$
\begin{bmatrix} \Delta P_1 \\ \cdot \\ \cdot \\ \Delta P_{n-1} \\ \hline \Delta Q_1 \\ \cdot \\ \cdot \\ \Delta Q_{n-1} \end{bmatrix}
=
\left[\begin{array}{ccc|ccc}
\frac{\partial P_1}{\partial a_1} & \cdots & \frac{\partial P_1}{\partial a_{n-1}} & \frac{\partial P_1}{\partial b_1} & \cdots & \frac{\partial P_1}{\partial b_{n-1}} \\
 & \cdots & & & \cdots & \\
\frac{\partial P_{n-1}}{\partial a_1} & \cdots & \frac{\partial P_{n-1}}{\partial a_{n-1}} & \frac{\partial P_{n-1}}{\partial b_1} & \cdots & \frac{\partial P_{n-1}}{\partial b_{n-1}} \\
\hline
\frac{\partial Q_1}{\partial a_1} & \cdots & \frac{\partial Q_1}{\partial a_{n-1}} & \frac{\partial Q_1}{\partial b_1} & \cdots & \frac{\partial Q_1}{\partial b_{n-1}} \\
 & \cdots & & & \cdots & \\
\frac{\partial Q_{n-1}}{\partial a_1} & \cdots & \frac{\partial Q_{n-1}}{\partial a_{n-1}} & \frac{\partial Q_{n-1}}{\partial b_1} & \cdots & \frac{\partial Q_{n-1}}{\partial b_{n-1}}
\end{array}\right]
\begin{bmatrix} \Delta a_1 \\ \cdot \\ \cdot \\ \Delta a_{n-1} \\ \hline \Delta b_1 \\ \cdot \\ \cdot \\ \Delta b_{n-1} \end{bmatrix}
\tag{6.11}
$$

For convenience, denote the Jacobian matrix by

$$
\begin{array}{|c|c|} \hline J_A & J_B \\ \hline J_C & J_D \\ \hline \end{array}
$$

The elements of the matrix are evaluated for the values of P, Q, and V at each iteration as follows.

For the submatrix J_A and from equation (6.9),

$$\frac{\partial P_k}{\partial a_j} = a_k G_{kj} - b_k B_{kj} \tag{6.12}$$

where $k \neq j$, i.e. off-diagonal elements.

Diagonal elements,

$$\frac{\partial P_k}{\partial a_k} = 2a_k G_{kk} + b_k B_{kk} - b_k B_{kk} + \sum_{\substack{j=1 \\ j\neq k}}^{n-1} (a_j G_{kj} + b_j B_{kj}) \tag{6.13}$$

This element may be more readily obtained by expressing some of the quantities in terms of the current at node k, $\mathbf{I}_k$, which can be determined separately at each iteration

Let

$$\mathbf{I}_k = c_k + jd_k = (G_{kk} - jB_{kk})(a_k + jb_k) + \sum_{\substack{j=1 \\ j\neq k}}^{n-1} (G_{kj} - jB_{jk})(a_j + jb_j)$$

from which,

$$c_k = a_k G_{kk} + b_k B_{kk} + \sum_{\substack{j=1 \\ j\neq k}}^{n-1} (a_j G_{kj} + b_j B_{kj})$$

and

$$d_k = b_k G_{kk} - a_k B_{kk} + \sum_{\substack{j=1 \\ j\neq k}}^{n-1} (b_j G_{kj} - a_j B_{jk})$$

So that,

$$\frac{\partial P_k}{\partial a_k} = a_k G_{kk} - b_k B_{kk} + c_k$$

For J_B,

$$\frac{\partial P_k}{\partial b_k} = a_k B_{kk} + b_k G_{kk} + d_k$$

and

$$\frac{\partial P_k}{\partial b_j} = a_k B_{kj} + b_k G_{kj} \qquad (k \neq j)$$

For J_C,

$$\frac{\partial Q_k}{\partial a_k} = a_k B_{kk} + b_k G_{kk} - d_k$$

and

$$\frac{\partial Q_k}{\partial a_j} = a_k B_{kj} + b_k G_{kj} \qquad (k \neq j)$$

For J_D,

$$\frac{\partial Q_k}{\partial b_j} = -a_k G_{kj} + b_k B_{kj} \qquad (k \neq j)$$

and

$$\frac{\partial Q_k}{\partial b_k} = -a_k G_{kk} + b_k B_{kk} + c_k$$

The process commences with the iteration counter 'p' set to zero and all the nodes except the slack-bus being assigned voltages, usually 1 p.u.

From these voltages, P and Q are calculated from equations (6.9) and (6.10). The changes are then calculated:

$$\Delta P_k^p = P_k \text{ (specified)} - P_k^p \text{ and } \Delta Q_k^p = Q_k \text{ (specified)} - Q_k^p$$

where p is the iteration number.

Next the node currents are computed as

$$I_k^p = \left(\frac{P_k^p + jQ_k^p}{V^p}\right)^* = c_k^p + jd_k^p$$

The elements of the Jacobian matrix are than formed, and from equation (6.11),

$$\begin{bmatrix} \Delta a \\ \Delta b \end{bmatrix} = \begin{bmatrix} J_A & J_B \\ J_C & J_D \end{bmatrix}^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \tag{6.14}$$

Hence, a and b are determined and the new values, $a_k^{p+1}k = a_k^p + \Delta a_k^p$ and $b_k^{p+1} = b_k^p + \Delta b_k^p$, are obtained. The process is repeated ($p = p + 1$) until ΔP and ΔQ are less than a prescribed tolerance.

The Newton–Raphson method has better convergence characteristics and for many systems is faster than the Gauss–Seidel method; the former has a much larger time per iteration but requires very few iterations (four is general), whereas the Gauss–Siedel requires at least 30 iterations, the number increasing with the size of system.

Acceleration factors may be used for the Newton–Raphson method. The quantities are frequently expressed in polar form.

The polar form of the equations has advantages and the equations are:

$$\begin{aligned} P_k &= P(V, \theta) \\ Q_k &= Q(V, \theta) \end{aligned} \tag{6.15}$$

The power at a bus is

$$\begin{aligned} \mathbf{S}_k &= P_k + jQ_k = \mathbf{V}_k \mathbf{I}_k^* \\ &= V_k \sum_{m \neq k} Y_{km}^* V_m^* \end{aligned} \tag{6.16}$$

$$P_k = \sum_{m \neq k} V_k V_m (G_{km} \cos\theta_{km} + B_{km} \sin\theta_{km}) \tag{6.17}$$

$$Q_k = \sum_{m \neq k} V_k V_m (G_{km} \sin\theta_{km} - B_{km} \cos\theta_{km}) \tag{6.18}$$

where $\theta_{km} = \theta_k - \theta_m$.

For a load bus,

$$\begin{aligned} \Delta P_k &= \sum_{m \neq k} \frac{\partial P_k}{\partial \theta_m} \Delta\theta_m + \sum_{m \neq k} \frac{\partial P_k}{\partial V_m} \Delta V_m \\ \Delta Q_k &= \sum \frac{\partial Q_k}{\partial \theta_m} \Delta\theta_m + \sum \frac{\partial Q_k}{\partial V_m} \Delta V_m \end{aligned} \tag{6.19}$$

For a generator (P, V) busbar, only the ΔP_k equation is used as Q_k is not specified. The mismatch equation is

$$\begin{bmatrix} \mathbf{\Delta P}^{P-1} \\ \mathbf{\Delta Q}^{P-1} \end{bmatrix} = \begin{bmatrix} \mathbf{H}^{P-1} & \mathbf{N}^{P-1} \\ \mathbf{J}^{P-1} & \mathbf{L}^{P-1} \end{bmatrix} \begin{bmatrix} \mathbf{\Delta\theta}^P \\ \left(\dfrac{\mathbf{\Delta V}^P}{\mathbf{V}^{P-1}}\right) \end{bmatrix} \tag{6.20}$$

$\Delta\theta^P$ is the correction to P, Q, and PV buses and $\Delta V^P / V^{P-1}$ is the correction to P, Q buses.

For buses k and m,

$$\begin{aligned} H_{km} &= \frac{\partial P_k}{\partial \theta_m} = V_k V_m (G_{km} \sin\theta_{km} - B_{km} \cos\theta_{km}) \\ N_{km} &= V_m \frac{\partial P_k}{\partial V_m} = V_k V_m (G_{km} \cos\theta_{km} + B_{km} \sin\theta_{km}) \\ J_{km} &= \frac{\partial Q_K}{\partial \theta_m} = -V_k V_m (G_{km} \cos\theta_{km} + B_{km} \sin\theta_{km}) \\ L_{km} &= V_m \frac{\partial Q_m}{\partial V_m} = V_k V_m (G_{km} \sin\theta_{km} - B_{km} \cos\theta_{km}) \end{aligned}$$

Also,

$$\begin{aligned} H_{kk} &= -Q_k - B_{kk} V_k^2 \\ N_{kk} &= P_k + G_{kk} V_k^2 \\ J_{kk} &= P_k - G_{kk} V_k^2 \\ L_{kk} &= Q_k - B_{kk} V_k^2 \end{aligned}$$

In the above, admittance of the link km is $Y_{km} = G_{km} + jB_{km}$. The computational process can be enhanced by pre-ordering and dynamic ordering, defined as

- *Preordering*, in which nodes are numbered in sequence of increasing number of connections.
- *Dynamic ordering*, in which at each step in the elimination the next row to be operated on has the fewest non-zero terms.

(iii) Decoupled load flow

The coupling between P–θ an Q–V components is weak. Hence the equations can be reduced to

$$\begin{aligned} [\mathbf{P}] &= [\mathbf{T}][\theta] \\ [\mathbf{Q}] &= [\mathbf{U}][V - V_0] \end{aligned} \tag{6.21}$$

At reference node $\theta_0 = 0$ and $V_k = V_0$, so elements of $\mathbf{T}$ and $\mathbf{U}$ are given by

$$T_{km} = -\frac{V_k V_m}{Z_{km}^2 / X_{km}} \qquad U_{km} = -\frac{1}{Z_{km}^2 / X_{km}}$$

$$T_{kk} = -\sum_{m \neq k} T_{km} \qquad U_{kk} = -\sum_{m \neq k} U_{km}$$

where Z_{km} and X_{km} are the branch impedance and reactance and $[U]$ is constant valued. Then

$$[\Delta \mathbf{P}] = [\mathbf{T}][\Delta\theta]$$
$$[\Delta \mathbf{Q}/\mathbf{V}] = [\mathbf{U}][\Delta \mathbf{V}]$$

These are solved alternatively. Advantage is obtained by using the following:

$$[\Delta \mathbf{P}/\mathbf{V}] = [\mathbf{A}][\Delta\theta]$$
$$[\Delta \mathbf{Q}/\mathbf{V}] = [\mathbf{C}][\Delta \mathbf{V}] \qquad (6.22)$$

Fast decoupled load flow This makes the Jacobians of the decoupled method constant in value throughout the iteration. The following assumptions are made:

$$E_k = E_m = 1 \text{ p.u.}$$

$G_{km} \ll B_{km}$ and can be ignored; this is reasonable for lines and cables,

$$\cos(\theta_k - \theta_m) \approx 1$$
$$\sin(\theta_k - \theta_m) \approx 0$$

Hence,

$$[\Delta \mathbf{P}] = [\mathbf{B}][\Delta\theta]$$
$$[\Delta \mathbf{Q}] = [\mathbf{B}][\Delta \mathbf{V}]$$

where $B_{km} = -B_{km}$ for $m \neq k$, and

$$B_{kk} = \sum_{m \neq k} B_{km}$$

Further assumptions yield:

$$\left[\frac{\Delta P}{V}\right] = [B'][\Delta\theta]$$
$$\left[\frac{\Delta Q}{V}\right] = [B''][V] \qquad (6.23)$$

where

$$B'_{km} = -\frac{1}{X_{km}} (m \neq k)$$
$$B'_{kk} = \sum_{m \neq k} \frac{1}{X_{km}}$$
$$B''_{km} = -B_{km} (m \neq k)$$
$$B''_{kk} = \sum_{m \neq k} B_{km}$$

Matrices B' and B'' are real and constant in value and need to be triangulated only once.

Example 6.5

Using the fast decoupled method calculate the angles and voltages after the first iteration for the three-node network described by the following admittance matrix (from Figure 6.9). V_1 is 230 kV, initially $V_2 = 220$ kV, $\theta_2 = 0$, and V_3 is 228 kV, $\theta_3 = 0$. Node 2 is a load consuming 200 MW, 120 MVAr; node 3 is a generator node set at 70 MW and 228 kV. V_1 is an infinite busbar.

$$\mathbf{Y} = \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{bmatrix} 0.00819 - j0.049099 & -0.003196 + j0.019272 & -0.004994 + j0.030112 \\ -0.003196 + j0.019272 & 0.007191 - j0.043099 & -0.003995 + j0.02409 \\ -0.004994 + j0.030112 & -0.003995 + j0.02409 & 0.008989 - j0.053952 \end{bmatrix}$$

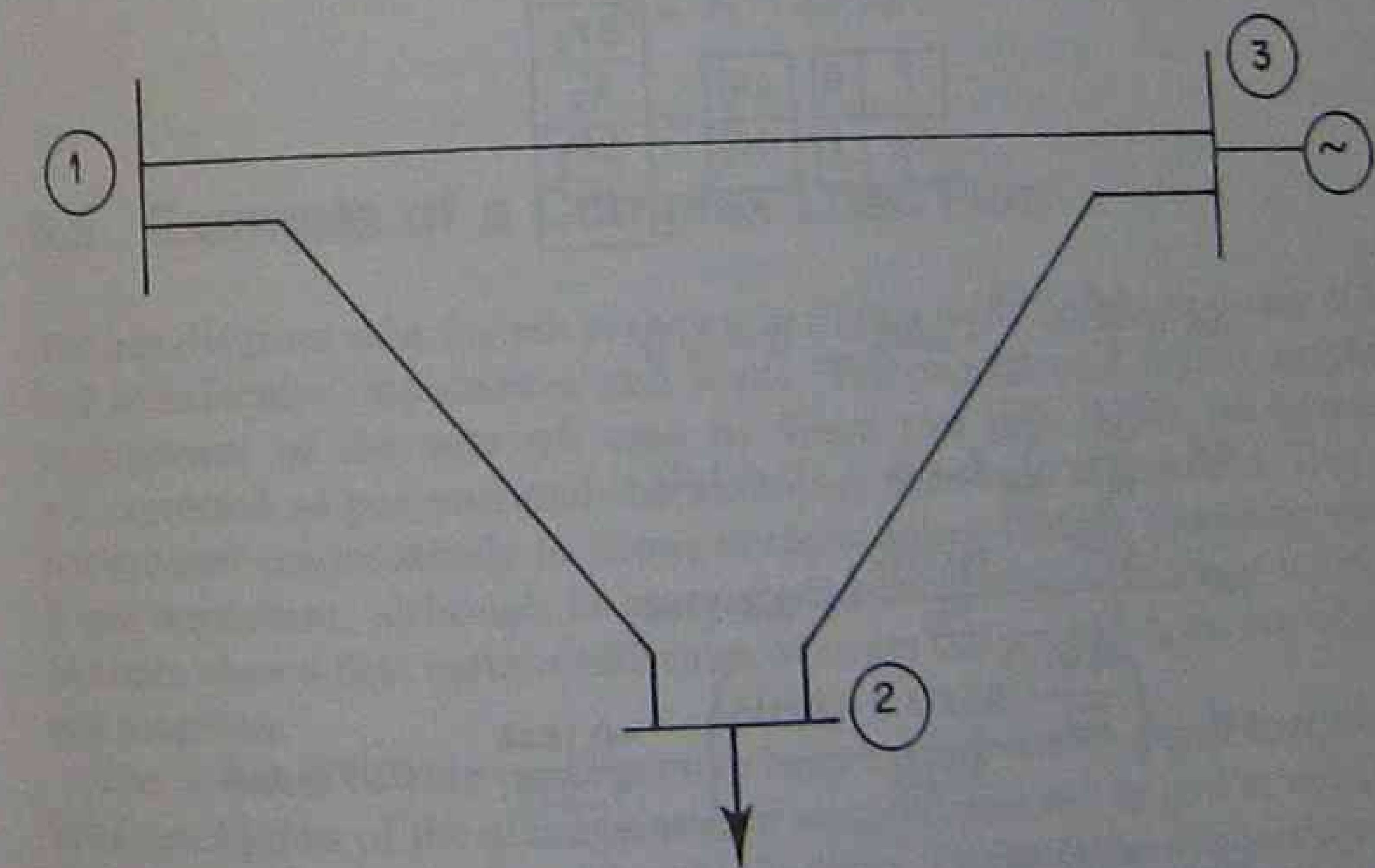

Figure 6.9 Three-node system for fast decoupled load flow

$$\begin{bmatrix} \dfrac{\Delta \mathbf{P}_2}{\mathbf{V}_2} \\ \dfrac{\Delta \mathbf{P}_3}{\mathbf{V}_3} \end{bmatrix} = \begin{bmatrix} \mathbf{B}'_{22} & \mathbf{B}'_{23} \\ \mathbf{B}'_{32} & \mathbf{B}'_{33} \end{bmatrix} \begin{bmatrix} \Delta\theta_2 \\ \Delta\theta_3 \end{bmatrix}$$

$$V_1 = 230\,\text{kV}, \qquad V_2 = 220\,\text{kV}, \theta_2 = 0, \qquad V_3 = 228\,\text{kV}, \theta_3 = 0$$

$$B'_{22} = \frac{V_1}{X_{12}} + \frac{V_3}{X_{23}} = \frac{220}{50.5} + \frac{220}{40.4} = 9.80198$$

$$B'_{23} = -\frac{V_3}{X_{23}} = -\frac{220}{40.4} = -5.4455$$

$$B'_{32} = -\frac{V_2}{X_{23}} = -\frac{220}{40.4} = -5.4455$$

$$B'_{33} = \frac{V_1}{X_{13}} + \frac{V_2}{X_{23}} = \frac{220}{32.32} + \frac{220}{40.4} = 12.25247$$

$$C_{32} = L_{32} = \frac{B'_{32}}{B'_{22}} = -\frac{5.4455}{9.80198} = -0.5555$$

$$D_{32} = -B'_{22} = 9.80198$$

$$D_{33} = B'_{33} - \frac{B'_{32}B'_{23}}{B'_{22}} = 12.25247 - \frac{5.4455^2}{9.80198} = 9.2272$$

$$\Delta P_2^0 = -200 - 220(-0.003196 \times 230 + 0.00719 \times 220 - 0.003995 \times 228)$$
$$= -185.937\,\text{MW}$$

$$\Delta P_3^0 = 70 - 228(-0.004994 \times 230 - 0.003995 \times 220 + 0.008989 \times 228)$$
$$= 64.99\,\text{MW}$$

$$\begin{bmatrix} D_{22} & \\ & D_{33} \end{bmatrix} \begin{bmatrix} 1 & L_{23} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta\theta_2 \\ \Delta\theta_3 \end{bmatrix} = \begin{bmatrix} \Delta\theta'_2 \\ \Delta\theta'_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ C_{32} & 1 \end{bmatrix} \begin{bmatrix} \Delta\theta'_2 \\ \Delta\theta'_3 \end{bmatrix} = \begin{bmatrix} \dfrac{\Delta P_2}{V_2} \\ \dfrac{\Delta P_3}{V_3} \end{bmatrix}$$

$$\Delta\theta'_i = \frac{\Delta P_i}{V_i} - \sum_{k=2}^{i-1} C_{ik}\Delta\theta'_k \qquad (i = 2, 3)$$

$$\Delta\theta_i = \frac{\Delta\theta'_i}{D_{ii}} - \sum_{k=3}^{i+1} L_{ik}\Delta\theta_k \qquad (i = 3, 2)$$

$$\Delta\theta'_2 = \frac{\Delta P_2^{(0)}}{V_2} = \frac{-185.937}{220} = -0.845168$$

$$\Delta'\theta_3 = \left(\frac{\frac{64.99}{228} - 0.5555 \times 0.84516}{9.2272}\right) = \frac{-0.1844}{9.2272} = -0.1998\text{ rad}$$

$$\Delta\theta'_2 = \frac{-0.845168}{9.80198} - 0.5555 \times 0.01988 = -0.09732\text{ rad}$$

Hence,

$$\theta'_2 = \theta_2^{(0)} + \Delta\theta_2^{(0)} = -0.09732\text{ rad} = -5.576^\circ$$
$$\theta'_3 = \theta_3^0 + \Delta\theta'_3 = -0.012\text{ rad} = -1.14477^\circ$$

Considering reactive power at node 2,

$$\Delta Q_2^0 = Q_2 - V_2^0\{V_1[G_{21}\sin(\theta'_2 - \theta_1) - B_{21}\cos(\theta'_2 - \theta_1)] - V_2^0 B_{22}$$
$$+ V_3^0[G_{23}\sin(\theta'_2 - \theta'_3) - B_{23}\cos(\theta'_2 - \theta'_3)]\}$$
$$= \quad -120 - 220\{230[-0.003196\sin(-5.576 - 0)$$
$$- 0.019272\cos(-5.576 - 0)] - 220 \times 0.043099$$
$$+ 228[-0.003995\sin(-5.576 + 1.14477)$$
$$- 0.02409\cos(-5.576 + 1.14477)]\} = -59.476\,\text{MVAr}$$

Hence,

$$\Delta V'_2 = \frac{\Delta Q_2^0}{B''_{22}V_2^0} = \frac{-59.476}{0.043049 \times 220} = -6.28\,\text{kV}$$

and

$$V_1 = 230\,\text{kV}$$
$$V'_2 = V_2^0 + \Delta V'_2 = 220 - 6.28 = 213.72\,\text{kV}$$
$$V'_3 = V_3^0 = 228\,\text{kV}$$

The process is repeated for the next iteration, and so on, until convergence is reached, when,

$$P_1 = 134.389\,\text{MW}$$
$$Q_1 = 56.77\,\text{MVAr}$$

6.5 Example of a Complex Load Flow

The line diagram of a system is shown in Figure 6.10, along with details of line and transformer impedances and loads. This represents a slightly simplified arrangement of the network used by Ward and Hale (1956). All quantities are expressed as per unit and the nodes are numbered as indicated. They are not ordered consecutively to show, in the solution, that the numbering system is not important, although in more sophisticated studies on larger systems it has been shown that certain orderings of nodes can produce faster convergence and solutions.

The solution of this problem has been carried out by digital computer; a brief description of the arrangement of essential data will be givn as well as the first iteration performed on a hand calculator. This program was developed for instructional purposes and is not as refined or sophisticated as commercial

Table 6.1

+5 ----------------------- n			
+8 ----------------------- l			
+1			Off-diagonal elements per row
+2			
+2			
+1			
+2			
+2			Column numbers
+3	+1		
+5	+2		
+5			
+4	+3		
+0.708238	−0.033932	Y_{12}/Y_{11}	Off-diagonal elements $Y_{ij}/\mathbf{Y}_{ii}$
+0.081853	+0.050025	Y_{23}/Y_{22}	
+1.010063	−0/055033	Y_{21}/Y_{22}	
+0.646931	−0.043097	Y_{35}/Y_{33}	
+0.353069	+0.043097	Y_{32}/Y_{33}	
+0.640699	−0.052397	Y_{45}/Y_{44}	
+0.726081	−0.090184	Y_{54}/Y_{55}	
+0.292797	+0.087839	Y_{53}/Y_{55}	
Generation Table 1			
+1.000000	+0.000000		Voltage estimates, v_i
+1.000000	+0.000000		
+1.000000	+0.000000		
+1.000000	+0.000000		
+1.000000	+0.000000		
−0.000369	+0.007706		$P+jQ/Y_{ii}^*$ values
−0.019534	+0.066298		
+0.179357	−0.162088		
−0.014702	+0.076068		
−0.046094	+0.058905		
+0.234539	+0.039069		$\mathbf{V}_S\mathbf{Y}_{is}/\mathbf{Y}_{ii}$ values slack-bus connections
+0	+0		
+0	+0		
+0.366206	+0.055921		
+0	+0		
+0.558269	−11.652234		$\mathbf{Y}_{ii}$ values
+0.444860	−8.164860		
+1.021401	−1.954524		
+0.433934	−5.306045		
+0.576541	−4.641795		
+0.223371	+0.037209	+101.000000	$\mathbf{Y}_{is}/Y_{ii}$ values + slacknumbers
+0.000000	+0.000000	+0.000000	
+0.000000	+0.000000	+0.000000	
+0.348767	+0.053259	+101.000000	
+0.000000	+0.000000	+0.000000	

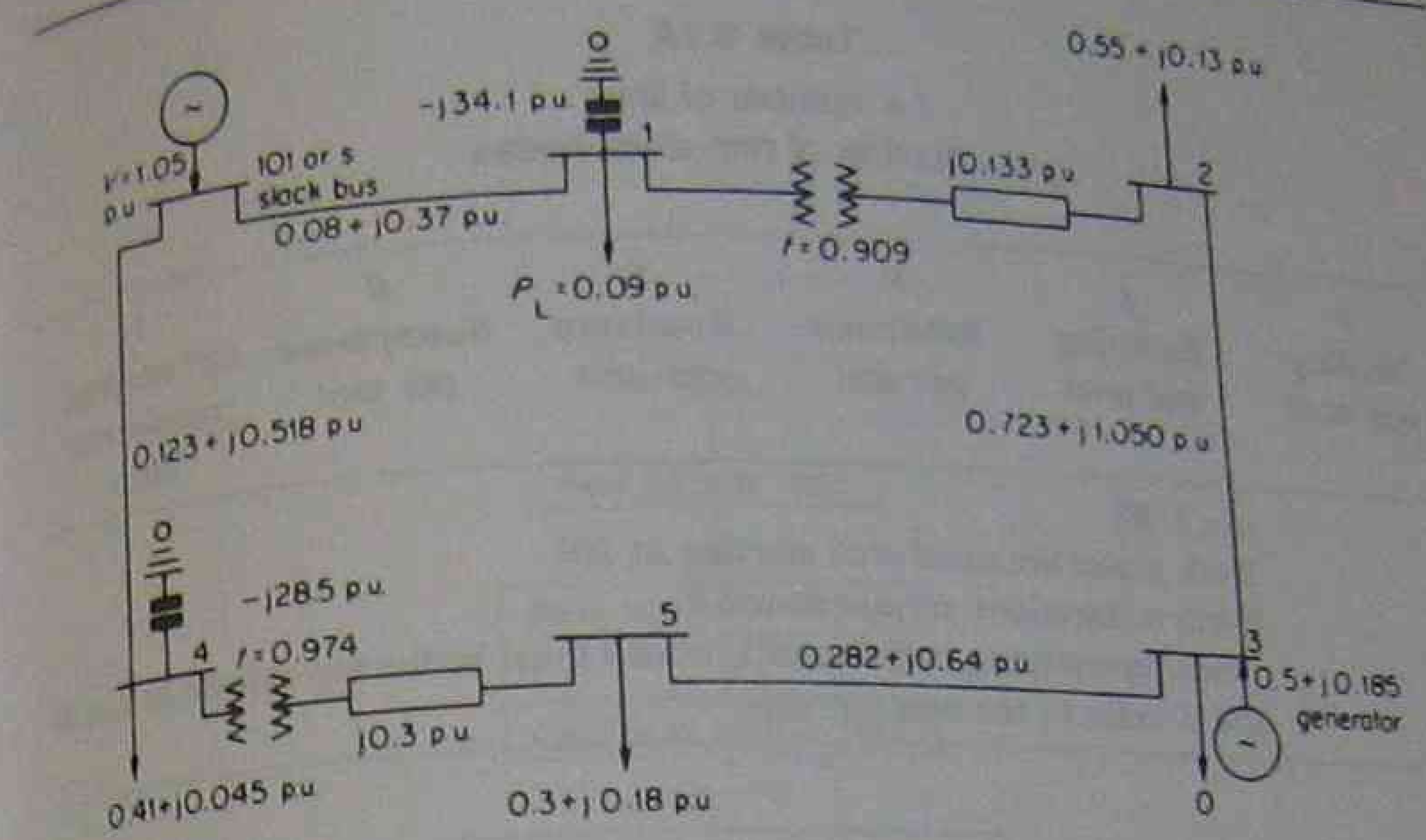

Figure 6.10 System for complex load flow using iterative method

programs; it is hoped, however, that the beginner will find it easier to understand than the latter. In view of the nature of equation (6.7), the basic system data will be modified to allow less computation during the actual solution of the equations; this will be obvious as the method is described. The system admittance-matrix [Y] is stored by specifying the following: order of matrix (n), number of off-diagonal elements, number of off-diagonal elements per row, column numbers of the off-diagonal elements in each row, a list of off-diagonal elements (Y_{ij}/Y_{ii}). Tables 6.1A and 6.1B show the tabular arrangement for input data and Table 6.1 shows the actual data.

In the tables, P and Q are considered positive for watts and lagging vars generated or supplied, and negative when received. It should be noted that the $\mathbf{Y}_{ij}$ values are, in fact, negative although they are shown as positive in the tables; this is useful as the $\mathbf{Y}_{ij}$ terms appear as negative in the nodal equations. The connections to the slack-busbar (V_s) are shown separately, no equation is necessary for the slack-nodes as the voltages are fully specified. The input data are shown in Table 6.1 and identification of the various quantities will be made easier by reference to the following system matrix:

$$\begin{bmatrix} \mathbf{Y}_{11} & \mathbf{Y}_{12} & \mathbf{Y}_{13} & \mathbf{Y}_{14} & \mathbf{Y}_{15} & \mathbf{Y}_{1s} \\ \mathbf{Y}_{21} & \mathbf{Y}_{22} & \mathbf{Y}_{23} & \mathbf{Y}_{24} & \mathbf{Y}_{25} & \mathbf{Y}_{2s} \\ \mathbf{Y}_{31} & \mathbf{Y}_{32} & \mathbf{Y}_{33} & \mathbf{Y}_{34} & \mathbf{Y}_{35} & \mathbf{Y}_{3s} \\ \mathbf{Y}_{41} & \mathbf{Y}_{42} & \mathbf{Y}_{43} & \mathbf{Y}_{44} & \mathbf{Y}_{45} & \mathbf{Y}_{4s} \\ \mathbf{Y}_{51} & \mathbf{Y}_{52} & \mathbf{Y}_{53} & \mathbf{Y}_{54} & \mathbf{Y}_{55} & \mathbf{Y}_{5s} \end{bmatrix}$$

Table 6.1A

l = number of links
n = number of non-slack nodes

i Sending end note	j Receiving end note	R Resistance per unit	X Reactance per unit	B Susceptance per unit	t Off-nominal transformer ratio

n = 1–99
Slack nodes are numbered starting at 101
Earth connections are numbered 0
When representing transformers, node i must be the tap-side node and R and X refer to the non-tap side

Table 6.1B

l = number of links
n = number of non-slack nodes
s = number of slack nodes
(subscripts G = generation, subscript L = loads)

Node number	V_{real} per unit	V_{imag} per unit	P_G per unit	Q_G per unit	P_L per unit	Q_L per unit

The initial part of the computer flow-diagram is shown in Figure 6.11.

Before proceeding with the first iteration, the calculation of the admittances associated with one of the transformers will be shown. Consider the transformer 1–2 of impedance j0.133 per unit and having $t = 0.909$. For the off-tap-side node 2, $\mathbf{Y}_{22}$ is formed using $\mathbf{Y}_t$ only, i.e.

$$\mathbf{Y}_{22} = \frac{1}{0.723 + \mathrm{j}1.05} + \frac{1}{\mathrm{j}0.133}$$
$$= 0.444860 - \mathrm{j}8.164860$$
$$\mathbf{Y}_{21} = \frac{1}{\mathrm{j}0.133} \cdot \frac{1}{0.909} = -\mathrm{j}8.278000$$

The formation of the admittance matrix is necessary for both the Gauss–Seidel and Newton–Raphson methods. At this point, the methods differ, and the application of the Gauss–Seidel method will be given first with a complete computer solution, and then an outline will be given of the Newton–Raphson method applied to the problem. For node 1:

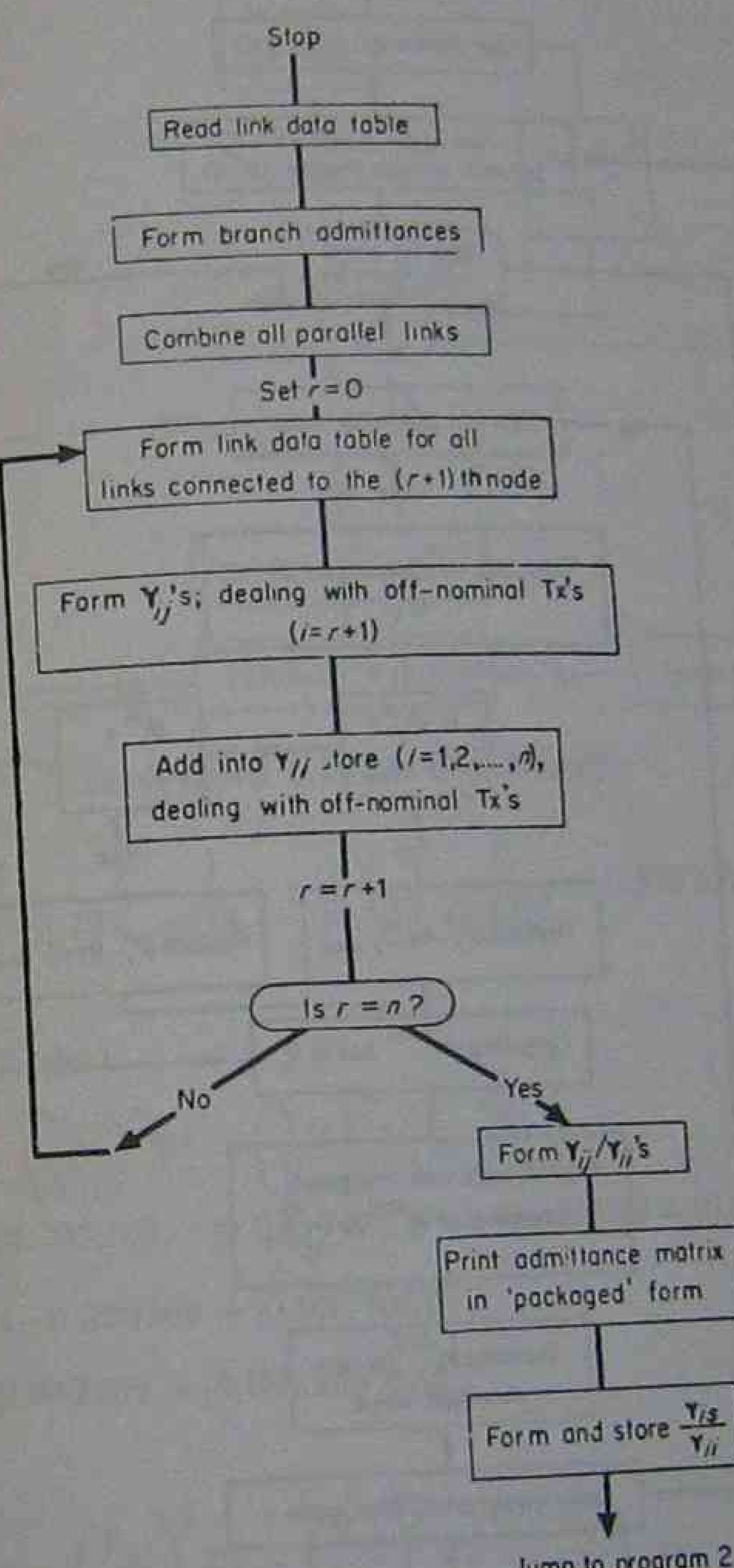

Figure 6.11(a) Initial part of the flow diagram for data processing of the load-flow program

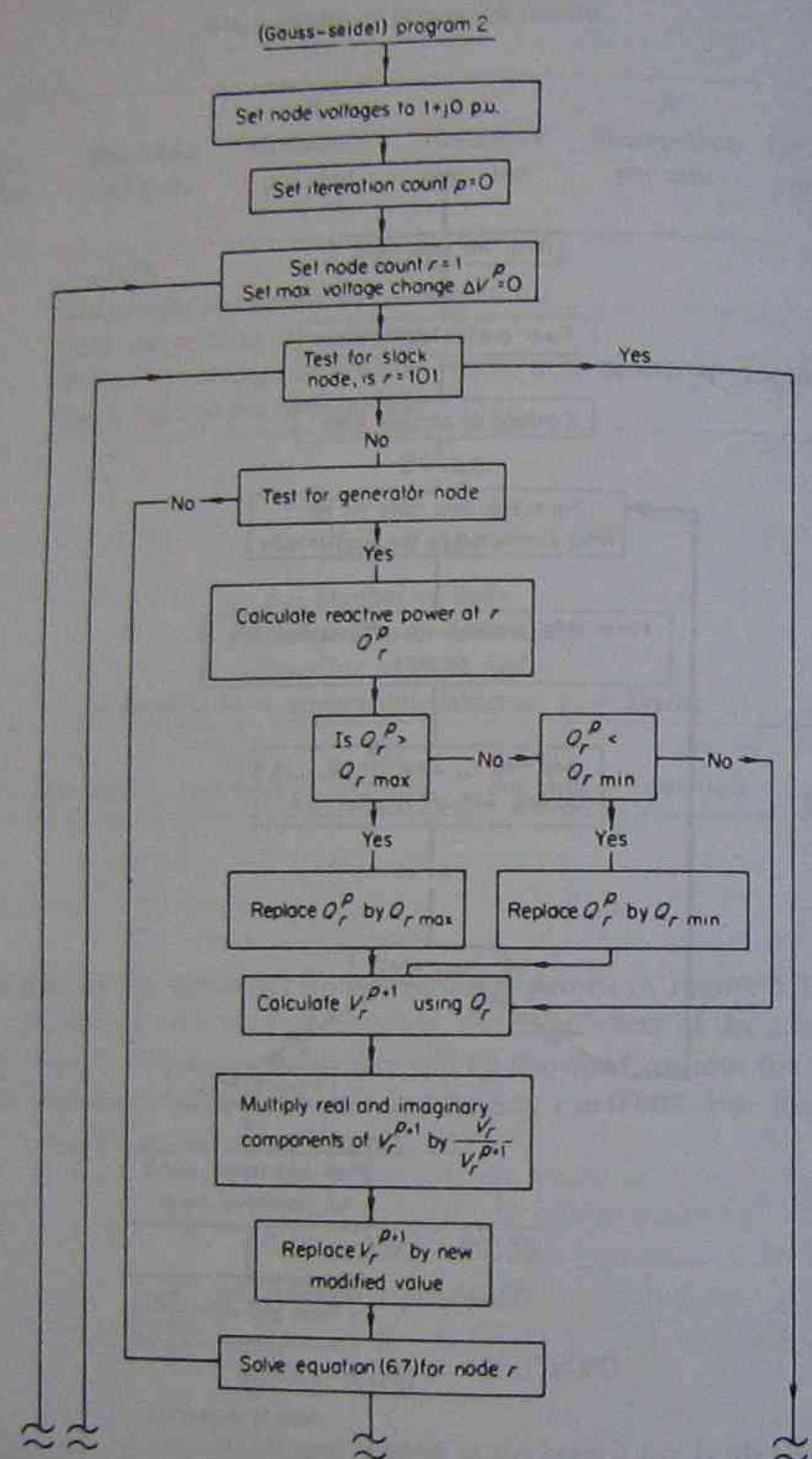

Figure 6.11(b) Flow diagram for Gauss–Seidel method *(continued opposite)*

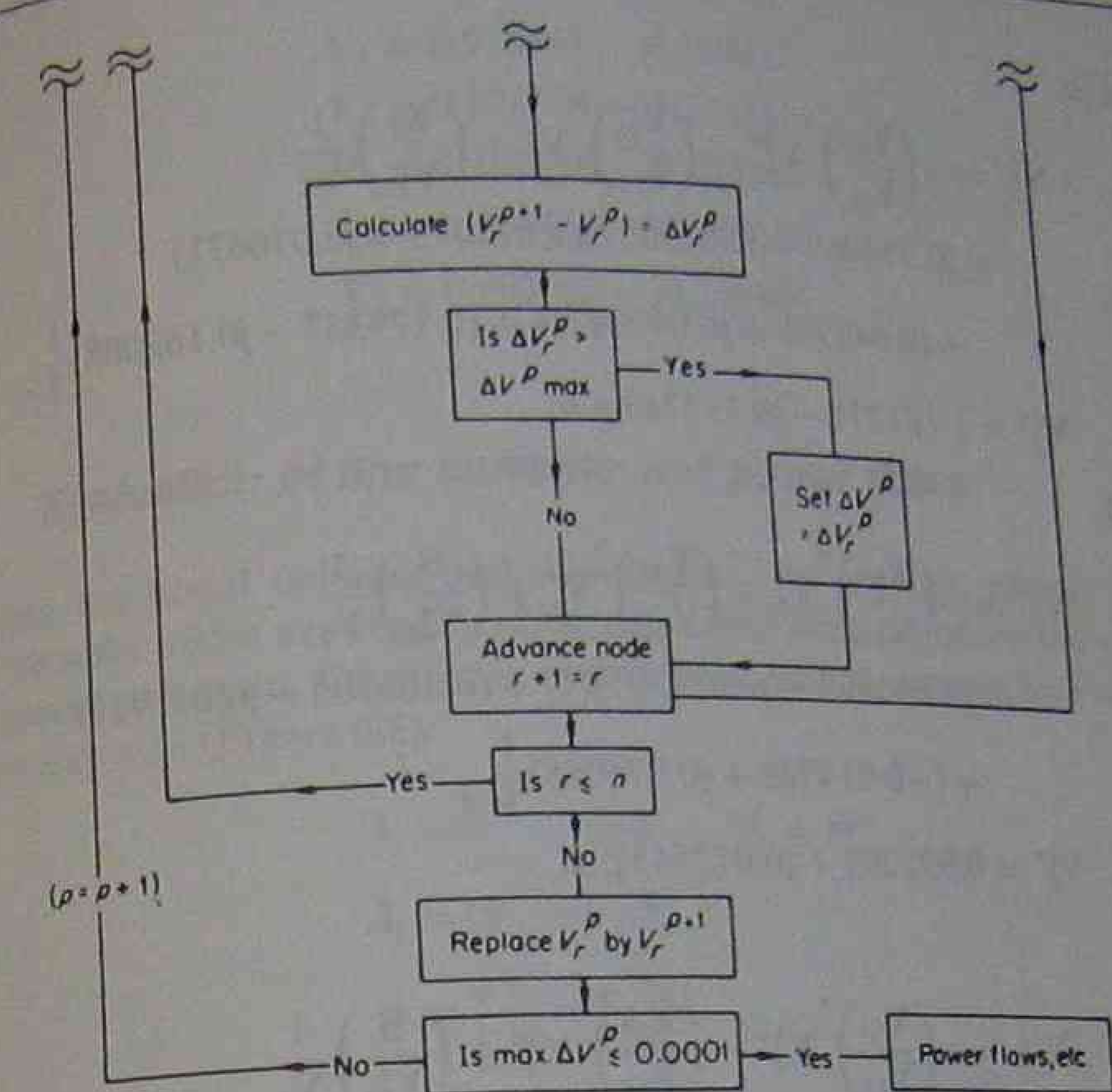

Figure 6.11(b) *(continued)*

$$\mathbf{Y}_{11} = \frac{1}{0.08 + j0.37} + \frac{1}{-j34.1} + \frac{1}{j0.133} \cdot \frac{1}{0.909^2} = 0.558269 - j11.652234$$

$$\mathbf{Y}_{12} = \mathbf{Y}_{21}$$

First iteration. Node 1:

$$\begin{aligned}
\mathbf{V}_1^{1*} &= -\left(\frac{\mathbf{Y}_{12}}{\mathbf{Y}_{11}}\right)^* \mathbf{V}_2^* - \left(\frac{\mathbf{Y}_{1s}}{\mathbf{Y}_{11}}\right)^* \mathbf{V}_s^* + \frac{\mathbf{S}_1}{\mathbf{Y}_{11}^*}\frac{1}{\mathbf{V}_1} \\
&= (0.708238 - j0.033932)^* 1^* + (0.234539 + j0.039069)^* 1.05^* \\
&\quad + (-0.000369 + j0.007706)\frac{1}{1} \\
\therefore \mathbf{V}_1^{1*} &= 0.942409 + j0.002569 \text{ p.u.}
\end{aligned}$$

Node 2:

$$\begin{aligned}
\mathbf{V}_2^{1*} &= -\left(\frac{\mathbf{Y}_{21}}{\mathbf{Y}_{22}}\right)^* \mathbf{V}_1^1 - \left(\frac{\mathbf{Y}_{23}}{\mathbf{Y}_{22}}\right)^* \mathbf{V}_3^* + \left(\frac{\mathbf{S}_2}{\mathbf{Y}_{22}^*}\right)\frac{1}{\mathbf{V}_2} \\
&= (1.010063 + j0.055033)(0.942409 + j0.002569) \\
&\quad + (0.081853 - j0.050025)1 + (-0.019534 + j0.066298)\frac{1}{1} \\
\therefore \mathbf{V}_2^{1*} &= 1.014073 + j0.070671 \text{ p.u.}
\end{aligned}$$

Node 3:

$$\mathbf{V}_3^{1*} = -\left(\frac{\mathbf{Y}_{32}}{\mathbf{Y}_{33}}\right)^* \mathbf{V}_2^{1*} - \left(\frac{\mathbf{Y}_{35}}{\mathbf{Y}_{33}}\right)^* \mathbf{V}_5^* + \left(\frac{\mathbf{S}_3}{\mathbf{Y}_{33}^*}\right)\frac{1}{\mathbf{V}_3}$$

$$= (0.353069 - \mathrm{j}0.043097)(1.014073 + \mathrm{j}0.070671)$$

$$+ (0.646931 + \mathrm{j}0.043097)1 + (0.179357 - \mathrm{j}0.162088)\frac{1}{1}$$

$$\mathbf{V}_3^{1*} = 1.187371 - \mathrm{j}0.137743 \text{ p.u.}$$

Node 4:

$$\mathbf{V}_4^{1*} = -\left(\frac{\mathbf{Y}_{45}}{\mathbf{Y}_{44}}\right)^* \mathbf{V}_5^* - \left(\frac{\mathbf{Y}_{4s}}{\mathbf{Y}_{44}}\right)^* \mathbf{V}_s^* + \left(\frac{\mathbf{S}}{\mathbf{Y}_{44}^*}\right)\frac{1}{\mathbf{V}_4}$$

$$= (0.640699 - \mathrm{j}0.052397)(1) + (0.366206 - \mathrm{j}0.055921)$$

$$+ (-0.014702 + \mathrm{j}0.076068)\frac{1}{1}$$

$$\mathbf{V}_4^{1*} = 0.992202 + \mathrm{j}0.072543 \text{ p.u.}$$

Node 5:

$$\mathbf{V}_5^{1*} = -\left(\frac{\mathbf{Y}_{54}}{\mathbf{Y}_{55}}\right)^* \mathbf{V}_4^{1*} - \left(\frac{\mathbf{Y}_{53}}{\mathbf{Y}_{55}}\right)^* \mathbf{V}_3^{1*} + \left(\frac{\mathbf{S}}{\mathbf{Y}_{55}^*}\right)\frac{1}{\mathbf{V}_5}$$

$$= (0.726081 + \mathrm{j}0.090184)(1.010513 + \mathrm{j}0.069748)$$

$$+ (0.292797 - \mathrm{j}0.087839)(1.187371 - \mathrm{j}0.137743)$$

$$+ (-0.046094 + \mathrm{j}0.058905)\frac{1}{1}$$

$$\therefore \mathbf{V}_5^{1*} = 1.003342 - \mathrm{j}0.056430 \text{ p.u.}$$

$$\mathbf{V}_s \text{ or } \mathbf{V}_{101} = 1.05000 + \mathrm{j}0.000000 \text{ p.u.}$$

It will be noticed that the latest value of each nodal voltage is used. In this iteration no acceleration factor has been used (i.e. the factor = 1); if a factor of 1.6 is used, $\mathbf{V}_1^1$ becomes

$$\mathbf{V}_1 + 1.6(\mathbf{V}_1^1 - \mathbf{V}_1)$$

i.e.

$$1 + 1.6(-0.057591 - \mathrm{j}0.002569)$$

or

$$0.907854 - \mathrm{j}0.004110 \text{ p.u.}$$

This modified value of $\mathbf{V}_1^1$ should then be used to evaluate $\mathbf{V}_2^1$ when $\mathbf{V}_2^1$ is modified to $\mathbf{V}_2 + 1.6(\mathbf{V}_2^1 - \mathbf{V}_2)$, and so on. The busbar voltages after 30 iterations are

$$\mathbf{V}_1 = 0.918345 - \mathrm{j}0.159312$$
$$\mathbf{V}_2 = 0.978674 - \mathrm{j}0.221811$$
$$\mathbf{V}_3 = 1.101718 - \mathrm{j}0.065242$$
$$\mathbf{V}_4 = 0.901468 - \mathrm{j}0.194617$$
$$\mathbf{V}_5 = 0.903003 - \mathrm{j}0.196604$$

6.5.1 Evaluation of line currents and power flows

Knowing the nodal voltages and admittances, the current, power, and var flows between nodes are readily obtained. Links with transformers, however, need special attention. Consider a transformer with its impedance referred to the non-tap side (Figure 6.8):

$$\mathbf{I}_i = \frac{\mathbf{I}_j}{t} \quad \text{and} \quad \mathbf{V}_i = t\mathbf{V}_x$$

$$\therefore \mathbf{I}_j = (\mathbf{V}_x - \mathbf{V}_j)\mathbf{Y}_t$$

$$= \left(\frac{\mathbf{V}_i}{t} - \mathbf{V}_j\right)\mathbf{Y}_t$$

The power at j is

$$P_j = \mathbf{V}_j\left[\left(\frac{\mathbf{V}_i}{t} - \mathbf{V}_j\right)\mathbf{Y}_t\right]^*$$

Also,

$$\mathbf{I}_i = \left(\frac{\mathbf{V}_i}{t^2} - \frac{\mathbf{V}_j}{t}\right)\mathbf{Y}_t$$

$$\therefore P_i = \mathbf{V}_i\left[\left(\frac{\mathbf{V}_i}{t^2} - \frac{\mathbf{V}_j}{t}\right)\mathbf{Y}_t\right]^*$$

Therefore power transferred i to j

$$= P_i - P_j$$

6.5.2 Application of the Newton–Raphson method to the system in Figure 6.10

The elements of the admittance matrix are determined as in the previous method, and it should be noted that for off-diagonal elements both G and B will be (-1) times the values derived from the network.

Let all the busbar voltages be assigned a voltage of $(1 + \mathrm{j}0)$ p.u.

At node 3:

$$P_3 = 0.5 \qquad Q_3 = 0.185\text{(generated)}$$

$$\mathbf{Y}_{32} = 0.444860 - j0.646063 \text{ p.u.}$$
$$\mathbf{Y}_{35} = 0.576541 - j1.308461 \text{ p.u.}$$
$$\mathbf{Y}_{33} = 1.021401 - j1.954524 \text{ p.u.}$$

$$a_3 = 1, b_3 = 0 \qquad a_2 = 1, b_2 = 0 \qquad a_5 = 1, b_5 = 0$$

First iteration ($p = 0$):

$$\begin{aligned} P_3^0 &= 1 \times 1 \times (-0.444860) + 1 \times 0 \times (-0.646063) \\ &\quad + 0 \times 0 \times (-0.444860) - 0 \times 1 \times (-0.646063) \\ &\quad + 1 \times 1 \times (+1.021401) + 1 \times 0 \times (1.954524) \\ &\quad + 0 \times 0 \times (1.021401) - 0 \times 1 \times (1.954524) \\ &\quad + 1 \times 1 \times (-0.576541) + 1 \times 0 \times (-1.308461) \\ &\quad + 0 \times 0 \times (-0.576541) - 0 \times 1 \times (-1.308461) \\ &= 0.0 \text{ p.u.} \end{aligned}$$

Note that $\mathbf{Y}_{31}$, $\mathbf{Y}_{34}$ do not exist. (The above result would be expected in the initial case as all the involved voltages are equal to 1 p.u.) Similarly,

$$Q_3^0 = 0.0 \text{ p.u.}$$

Therefore,

$$\Delta P_3^0 = 0.5 - 0 = 0.5 \text{ p.u.}$$

and

$$\Delta Q_3^0 = 0.185 - 0 = 0.185 \text{ p.u.}$$

ΔP and ΔQ for the remaining non-slack nodes are similarly obtained.

$$\mathbf{I}_k^p = \frac{P_k^p - jQ_k^p}{(V_k^p)^*} \qquad \text{and} \qquad \mathbf{I}_3^0 = \frac{0.5 - j0.185}{1 - j0}$$

hence

$$c_3^0 = 0.5 \qquad \text{and} \qquad d_3^0 = -0.185$$

The elements of the Jacobian are determined next:

$$\begin{aligned} \frac{\partial P_3}{\partial a_3} &= a_3^0 G_{33} - b_3^0 B_{33} + C_3^0 \\ &= 1(1.021401) - 0.0 + 0.5 = 1.521401 \\ \frac{\partial P_3}{\partial a_2} &= a_3^0 G_{32} - b_3^0 B_{32} \\ &= 1(-0.444860) + 0.0 \\ \frac{\partial P_3}{\partial a_5} &= -0.576541 \\ \frac{\partial P_3}{\partial b_3} &= a_3^0 B_{33} + b_3^0 G_{33} + d_3^0 \\ &= 1(1.954520) + 0.0 + (-0.185) = 1.76952 \\ \frac{\partial P_3}{\partial b_2} &= a_3^0 B_{32} + b_3^0 G_{32} = 1(-0.646063) + 0.0 \\ \frac{\partial P_3}{\partial b_5} &= 1(-1.308461) + 0.0 \end{aligned}$$

Similarly, we obtain

$$\frac{\partial Q_3}{\partial a_3}, \quad \frac{\partial Q_3}{\partial a_2}, \quad \frac{\partial Q_3}{\partial a_5}, \quad \frac{\partial Q_3}{\partial b_3}, \quad \frac{\partial Q_3}{\partial b_2}, \quad \frac{\partial Q_3}{\partial b_5}$$

The Jacobian matrix for the first iteration is thus formed and inverted, and then Δa_k^0 and Δb_k^0 ($k = 1 \ldots 5$) are evaluated. hence, $V_k^{0+1} = V_k^0 + \Delta a^0 + j\Delta b^0$. The process is repeated until changes in real and reactive power at each bus are less than a prescribed amount, say 0.01 p.u.

6.5.3 Summary

The direct method involving matrix inverstion and a final iterative procedure to deal with the restraints at the nodes has advantages for smaller networks. For large networks the fast decoupled load flow (FDLF) method is preferable. System information (such as specified generation and loads, line and transformer series, and shunt admittances) is fed into the computer. The data are expressed in per unit on arbitrary MVA and voltage bases. The computer formulates the self- and mutual-admittances for each node.

If the system is very sensitive to reactive power flows, i.e. the voltages change considerably with change in load and network configuration, the computer program may diverge. It is preferable to allow the reactive-power outputs of generators to be initially without limits to ensure an initial convergence. Convergence having been attained, the computer evaluates the real and reactive power flows in each branch of the system, along with losses, absorption of vars, and any other information that may be required. Programs are available which automatically adjust the tap settings of transformers to optimum values. Also, facilities exist for outputting only information regarding overloaded and underloaded lines; this is very useful when carrying out a series of load flows investigating the outages of plant and lines for security assessment purposes.

6.6 Optimal Power Flows

In Chapter 4, economic loading of generating plant, using 'B' coefficients to represent losses, was detailed using a differentiation and minimization method. In practice, non-linear minimization techniques with constraints are used to achieve a minimum of some function associated with the most economic operation of the system. Typical functions to be minimized are:

- total cost of system operation;
- total system losses;
- reactive requirements.

Nowadays, mathematical minimization techniques which guarantee a solution for many thousands of equations are available commercially. The emphasis is on speed, minimum data storage, accuracy, and easy data handling and interfacing. Normally, it is necessary to set up a scalar function, $f(x)$, to be minimized subject to constraints expressed as:

$$\min f(x)$$

subject to

$$g_i(x) \geqslant 0 \qquad \text{where } i = 1, 2 \ldots m$$

and

$$h_j(x) = 0 \qquad \text{where } j = 1, 2 \ldots p$$

In the case of a power system, $f(x)$ is often expressed as a cost function comprising the sum of generator costs plus the system losses; thus.

$$f(x) = \sum_{i=1}^{G} C_i P_i + \sum_{b=1}^{B} C_L P_L$$

where C_i is cost per unit of output of generation P_i and $C_L P_L$ is cost of losses in each circuit of B branches, subject to

$$\left.\begin{aligned} &\sum P_K = 0 \\ \text{and} \quad &\sum Q_K = 0 \end{aligned}\right\} \text{at each node } k \text{ (Kirchoff's laws)}$$

and the following limits or constraints apply:

$$\left.\begin{aligned} {}_{\min}P_i < P_i < P_{i\max} \\ {}_{\min}Q_i < Q_i < Q_{\max} \end{aligned}\right\} \quad \text{each generator } i$$

$${}_{\min}V_k < V_k < V_{k\max} \qquad \text{each node } k$$

$$S_{mn} < S_{mn\max} \qquad \text{for each circuit } m \text{ to } n$$

Additional constraints can be as follows:

$${}_{min}T_t < T_t < T_{t\max} \qquad \text{for each tap-change transformer t}$$

$${}_{\min}Q_c < Q_c < Q_{c\max} \qquad \text{for each compensator c}$$

Limits could also be added to ensure transient stability in the form:

$${}_{\min}\theta_{mn} < \theta < \theta_{mn\max} \qquad \text{where } \theta \text{ is an angle between node } m \text{ and node } n$$

More generally, it may be necessary to ensure that power P and var Q flows across defined boundaries in a network do not exceed pre-determined limits for security reasons. For example:

$$\sum_{mn\,\mathrm{SetS}} P^s_{mn} \leqslant P_{\mathrm{SpeC}} \qquad \text{where } P^S_{mn} \text{ is a set of flows whose sum must not exceed } P_{\mathrm{SpeC}}.$$

It can readily be seen that obtaining an optimum (minimum) solution is a complicated task, even with modern computers. The mathematics appropriate to this form of optimization is continually evolving and improving, including the use of novel techniques such as fuzzy logic, artificial neural networks, and genetic algorithms. Consequently, the employment of a computer library or commercial packages best suited to individual requirements is recommended.

Problems

6.1. A single-phase distributor has the following loads at the stated distances from the supply end: 10kW at 10m, 10kW at 0.9 p.f. lagging at 16m, 5kW at 0.8 p.f. lagging at 91.5m, and 20kW at 0.95 p.f. lagging at 137m. The loads may be assumed to be constant current at their nominal voltage values (240V). If the supply voltage is 250V and the maximum voltage drop is 5 per cent of the nominal value, determine the nearest commercially available conductor size.
(Answer: $r + 0.376x = 0.7 \times 10^{-3}$)

6.2 In the d.c. network shown in Figure 6.12, calculate the voltage at node B by inverting the admittance matrix. Check the answer by Thevenin's theorem.
(Answer: 247.7 V)

6.3 In the interconnected network shown in Figure 6.13, calculate the current in feeder BC.
(Answer: 26.75 A)

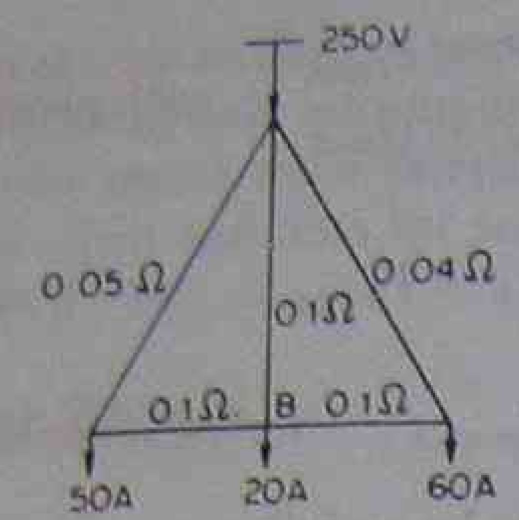

Figure 6.12 Network for Problem 6.2

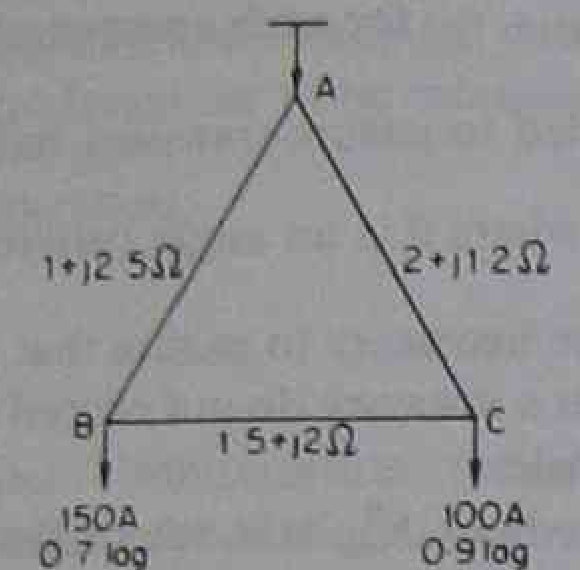

Figure 6.13 Network for Problem 6.3

6.4 In the network shown in Figure 6.14, the loads are represented by constant impedances 1+j1 p.u. Determine the current distributions in the network (a) when the transformer has its nominal ratio; (b) when the transformer is tapped up to 10 per cent.
(Note: determine the distribution with the off-nominal voltage alone and use superposition.)
(Answer: Transformer branch (a) 0; (b) 0.0735 – j0.075 (p.u.))

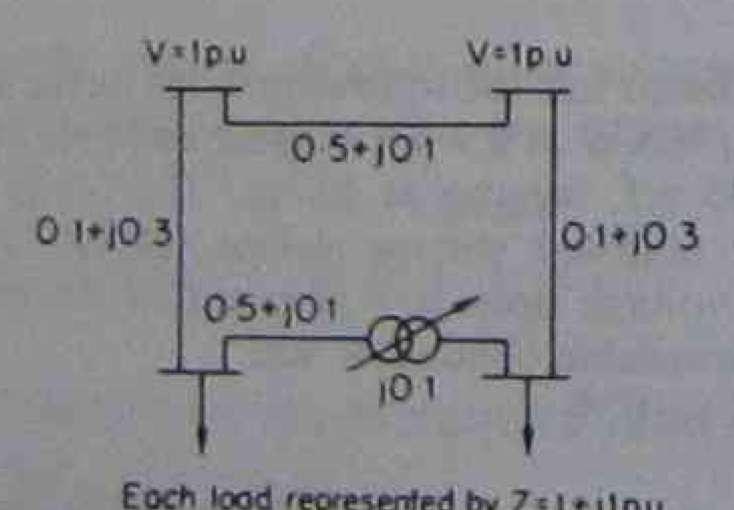

Figure 6.14 Network for Problem 6.4

6.5 Enumerate the information which may be obtained from a load-flow study. Part of a power system is shown in Figure 6.15. The line-to-neutral reactances and values of real

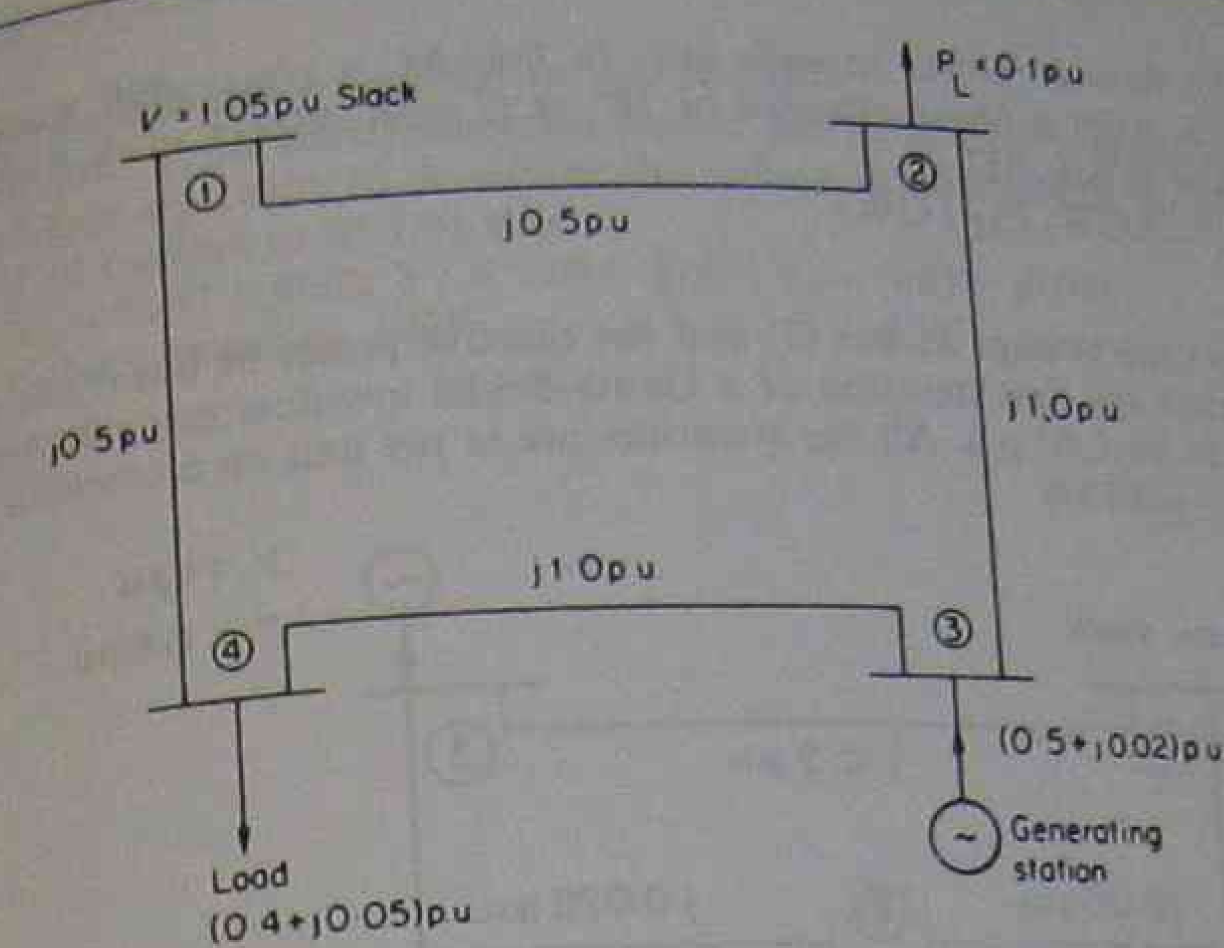

Figure 6.15 System for Problem 6.5

and reactive power (in the form $P \pm jQ$) at the various stations are expressed as per unit values on a common MVA base. Resistance may be neglected. By the use of an iterative method suitable for a digital computer, calculate the voltages at the stations after the first iteration without the use of an accelerating factor.
(Answer: $V_2$1.03333 – j0.03333 p.u.; $V_3$1.11666 + j0.23333 p.u.; $V_4$1.05556 + j0.00277 p.u.)

6.6 A 400 kV interconnected system is supplied from bus A, which may be considered to be an infinite busbar. The loads and line reactances are as indicated in Figure 6.16.

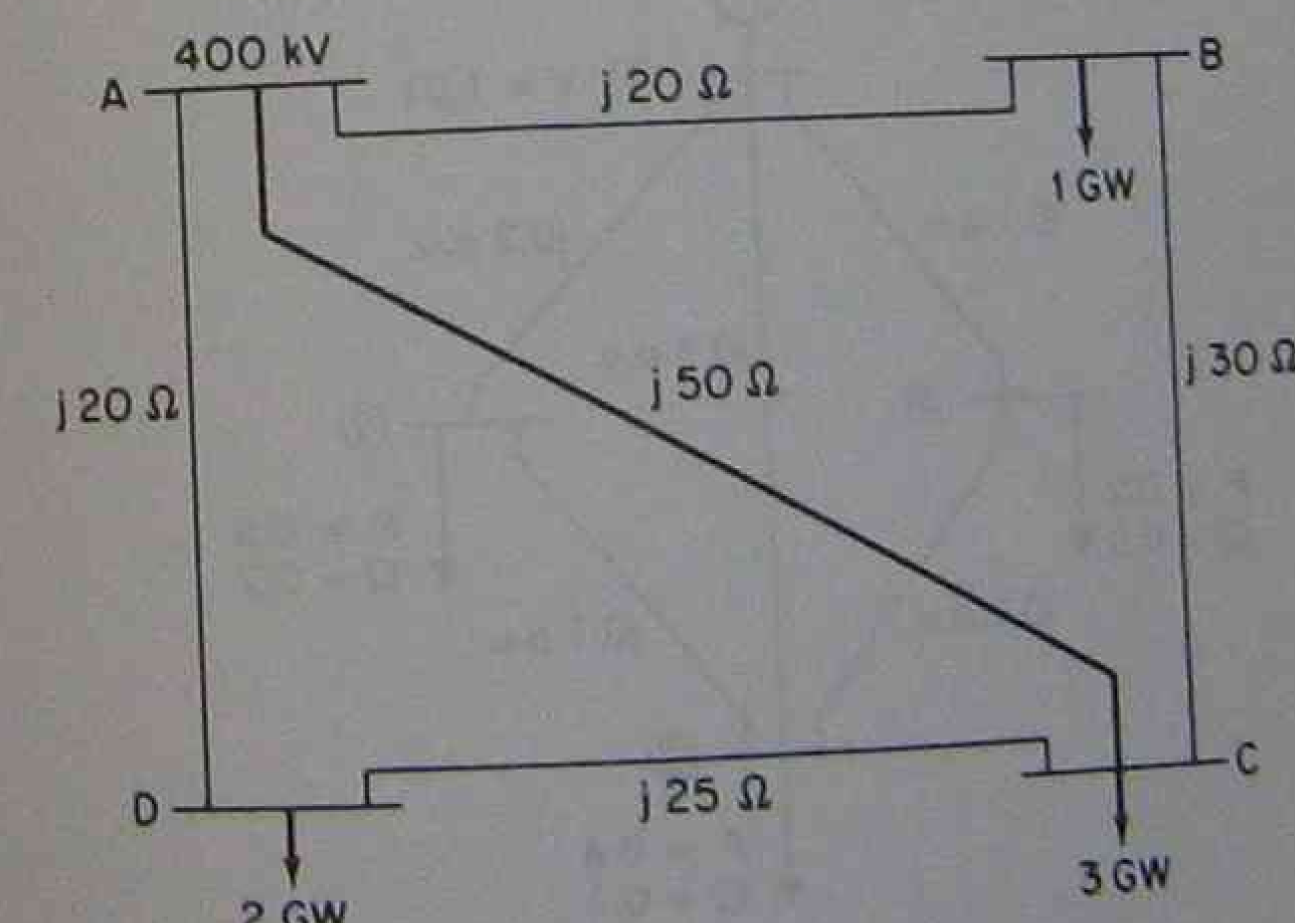

Figure 6.16 System for Problem 6.6